A MODEL FOR GENERATING MULTI-LAYER ANTI-CORROSION PROTECTION FOR ROAD INFRASTRUCTURES

Zenonas Kamaitis[1], Stasys Čirba[2]

Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
[1]Dept of Bridges and Special Structures, e-mail: zenonas.kamaitis@ts.vgtu.lt
[2]Dept of Mathematical Modeling, e-mail: jmmat@fm.vgtu.lt

Abstract. Various protective barriers are used in practice to protect reinforced concrete and steel structures in severely aggressive environments. In this paper, we consider a multi-layer corrosion protection system (CPS) which is modeled as a three-unit of non-identical components cold standby system. The system can be maintained or non-maintained. In this system it is assumed that degradation and renewal (if necessary) rates of all components are both exponential and different. After repair the components are "as good as new". Under these assumptions, using the Markovian degradation/renewal process some important reliability indices such as the system reliability, the point and steady-state or limiting availability, mean time to failure are defined. The performance of corrosion protection system is illustrated by numerical example.

Keywords: civil infrastructures, anti-corrosion protection, multi-layered systems, standby system, Markovian process, reliability, availability, time to failure.

1. Introduction

Methods for improving the performance of reinforced concrete and steel structures by surface corrosive resistant barriers have been used for many years. The type of protective barrier depends on the resistance of the barrier materials to the aggressive environment involved. Where reliable protection is required combined barriers composed from 2–3 layers of different materials (coatings, mastics, mortars, ceramics, sheets), having different protection ability were used in reinforced concrete [1–7] and steel [8–15] structures. Multi-layer systems are a powerful concept to fulfil requirements on their protection properties. Combined barriers are frequently used in chemical process plants, chemical storage tanks, silos, pipes, industrial chimneys, sewers. In special circumstances protection might be necessary in such ordinary locations as foundations, bridge structures or dams.

A popular method used to protect the steel bridges is a three-coat system, namely the metallised primer (zinc, aluminium), epoxy or urethane intermediates and aliphatic methane or other polymer composition topcoats [13–15]. Anti-corrosion polymer painting systems comprising epoxy and polyurethane resins were applied in some steel bridges in Lithuania. In reinforced concrete bridge structures the protection ability can be achieved by taking into account the resistance of protective surface treatment or coating, including primer and main coat as well as concrete cover [1]. It is obvious that each layer of protection system of steel or reinforced structures has different properties and is applied to perform specific functions. Together with corrosive resistance of individual layers the primer is to assure good adhesion, the main coat flexibility, and the topcoat hardness, erosion resistance or appropriate colour.

Much work has been carried out on the mechanical properties and durability of barrier materials, surface preparation for coatings and different coating systems evaluation in laboratory or "in situ" conditions. Despite continued improvements in reliability at the component level, multi-component corrosion protection system's level strategies retain an essential role for providing higher system reliability. However, there are only limited attempts to provide satisfactory analytical methods to assess the durability of protective measures as a whole.

Protective barriers in aggressive environments in gen-

eral have limited service lives. Three-layer coat systems in steel bridges need the rehabilitation every 15 years [16]. The protection systems, particularly organic coatings, are continuously deteriorating by corrosion and ageing. The condition of the protection system has a great effect on the condition and safety of the structural component. On the other hand, the protection cost, for example, in steel bridges takes a high portion of the maintenance budget [16]. Therefore it is important to search the methods for reliable assessment of protection system performance as a whole with the aim for economic and efficient protection design and appropriate maintenance.

Degradation of multi-layer protection system begins from the top layer. The other layers are not stressed, until the top layer has failed. After degradation of topcoat, the next layer continues to accomplish its protection function until all layers of the system are deteriorated and the limit states of degrading structure are reached. When all components fail, does the protection system fail. From a probabilistic point of view, multi-layer protection system can be generated with the cold-standby models. These models involve the use of redundant components that are in intact (not loaded) reserve and are activated when operating unit fails. Standby systems are widely used in electronic devices, energy and textile plants, alarm and satellite systems. Reliability and availability of cold standby systems have been extensively studied for many different system structures, objective functions, and distribution assumptions [eg 17, 18–21]. As far as the authors know, the modelling of combined protection barriers for civil engineering structures as standby redundancy until now was not carried out.

In this paper, an approach is developed and used to exploit the capabilities of components to realise a complex model of corrosion protection system (CPS). Another approach used is a cold standby model and Markovian degradation and renewal theory to represent the system deterioration as transition process from state to state leading to a gradual decrease of system performance. The probability-based Markovian models are often considered to represent

the bridge component deterioration process from the condition ratings data collected during the bridge inspections [22–24]. The deterioration and renewal (if required) of components are described by the exponential distribution assuming that the system after repair is "as good as new". Exponential distribution was found to be well fitted to the polymer protective coatings deterioration [25]. For illustrative purposes, a numerical example of reliability-based assessment of corrosion protection system is presented.

2. System description and assumptions

Consider a three-unit standby redundant parallel system with intact (not loaded) reserve, which comprises three independent parallel-connected elements (Fig 1a). Let S_i for $i = 0, 1, 2, 3$ be the state of the system where $\lambda_i > 0$ and $\mu_i \geq 0$ represent the rate of deterioration and renewal, respectively. If the sequence of component 1 failure then component 2 then component 3 is considered than the system will successively reach the damaged states S_1, S_2, and S_3 (Fig 1b). The reserve component is brought in operation when the previous unit fails with final state S_3 corresponding to system failure. Only when all elements fail, the protection system fails. The failure rates λ_i and repair rates μ_i for $i = 1, 2, 3$ indicate the rates at which the transitions are made from one state to another.

The basic assumptions made to model the durability of system are:

a – the system consists of three non-identical components in cold standby configuration; all components are activated sequentially in order upon failure of an operative component;

b – all components of system are repairable; the repaired components are restored immediately to an as-good-as-new condition (to initial performance level) each time repair is applied;

c – if the system is not easily accessible for repair, repair is costly or the time to failure is longer than required by design lifetime, the system is considered as non-repairable (without repair);

d – component failure and repair (if required) rates λ_i and μ_i are constants and time independent but different for components 1, 2 and 3; repair rates are large compared to failure rates, ie $\mu_i \gg \lambda_i$;

e – component failures and repairs are mutually independent;

f – system fails when all the components are in a failed state.

3. State transition probabilities

Let's first consider continuously operating cold standby system which components after failure are immediately restored. All possible states of the system can be modelled as the Markov process. Using assumptions *a*, *b*, *d* and *e*
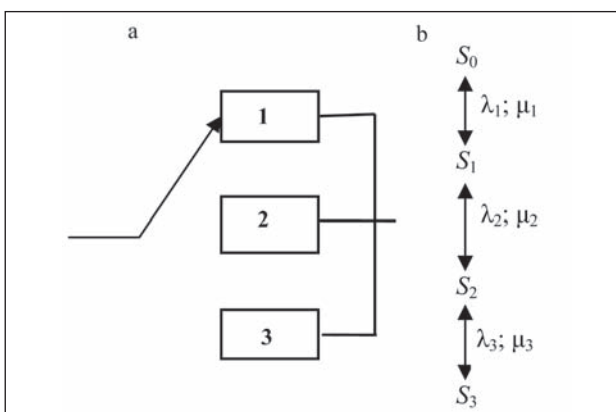

Fig 1. Standby parallel system: a – structural system; b – state transition diagram

from the state transition diagram (Fig 1) we may construct the Markov equations for 4 states:

$$\frac{d}{dt} p_0(t) = -\lambda_1 p_0(t) + \mu_1 p_1(t), \qquad (1)$$

$$\frac{d}{dt} p_1(t) = \lambda_1 p_0(t) - (\lambda_2 + \mu_1) p_1(t) + \mu_2 p_2(t), \qquad (2)$$

$$\frac{d}{dt} p_2(t) = \lambda_2 p_1(t) - (\lambda_3 + \mu_2) p_2(t) + \mu_3 p_3(t), \qquad (3)$$

$$\frac{d}{dt} p_3(t) = \lambda_3 p_2(t) - \mu_3 p_3(t), \qquad (4)$$

where $p_i(t)$ is probability that the system is in state i at time t, for $i = 0, 1, 2, 3$; $\lambda_i > 0$ and $\mu_i > 0$, for $i = 1, 2, 3$ are the constant failure and repair rates of components, respectively.

Thus for system consisting of 3 components there are 4 possible states where the state S_0 is the state at $t = 0$ for which all the components are safe and state 3 corresponds to system failure (S_3).

The state transition differential equations can be written in the matrix form

$$\frac{d}{dt} P(t) = MP(t), \qquad (5)$$

where $P(t)$ is a column vector with components $p_0(t)$, $p_1(t)$, $p_2(t)$, and $p_3(t)$; M is the Markov transition matrix:

$$M = \begin{bmatrix} -\lambda_1 & \mu_1 & 0 & 0 \\ \lambda_1 & -\lambda_2 - \mu_1 & \mu_2 & 0 \\ 0 & \lambda_2 & -\lambda_3 - \mu_2 & \mu_3 \\ 0 & 0 & \lambda_3 & -\mu_3 \end{bmatrix}. \qquad (6)$$

For passive parallel system the initial conditions are:

$$\begin{aligned} p_0(0) &= 1, \\ p_1(0) &= p_2(0) = p_3(0) = 0. \end{aligned} \qquad (7)$$

Since at any time the system can only be in one state, we have $\sum_{i=0}^{3} p_i(t) = 1$.

The objective is to calculate the probability $p_i(t)$ that the system is in state i at time t. Once this is known, the system reliability or availability can be calculated as the sum of state probabilities taken over by all the operating states.

By solving differential equations (1)–(4) associated with Fig 1, we obtain the following state probabilities:

$$p_0(t) = C_1 B_1 + C_2 B_2 e^{k_2 t} + C_3 B_3 e^{k_3 t} + C_4 B_4 e^{k_4 t}, \quad (8)$$

$$p_1(t) = C_1 D_1 + C_2 D_2 e^{k_2 t} + C_3 D_3 e^{k_3 t} + C_4 D_4 e^{k_4 t}, \quad (9)$$

$$p_2(t) = C_1 F_1 + C_2 F_2 e^{k_2 t} + C_3 F_3 e^{k_3 t} + C_4 F_4 e^{k_4 t}, \quad (10)$$

$$p_3(t) = C_1 + C_2 e^{k_2 t} + C_3 e^{k_3 t} + C_4 e^{k_4 t}, \qquad (11)$$

where

$$B_1 = \frac{\mu_1 \mu_2 \mu_3}{\lambda_1 \lambda_2 \lambda_3},$$

$$B_2 = \frac{\lambda_3 k_2 \mu_1 + \mu_1 (\mu_2 + k_2)(\mu_3 + k_2)}{\lambda_2 \lambda_3 (\lambda_1 + k_2)},$$

$$B_3 = \frac{\lambda_3 \mu_1 k_3 + \mu_1 (\mu_2 + k_3)(\mu_3 + k_3)}{\lambda_2 \lambda_3 (\lambda_1 + k_3)},$$

$$B_4 = \frac{1}{\lambda_1 \lambda_2 \lambda_3} \{ \lambda_2 \lambda_3 k_4 + (\mu_2 + k_4)[\lambda_3 k_4 + (\mu_2 + k_4)(\mu_3 + k_4)] + \lambda_2 k_4 (\mu_3 + k_4) \},$$

$$D_1 = \frac{\mu_2 \mu_3}{\lambda_2 \lambda_3},$$

$$D_2 = \frac{\lambda_3 k_2 + (\mu_2 + k_2)(\mu_3 + k_2)}{\lambda_2 \lambda_3},$$

$$D_3 = \frac{\lambda_3 k_3 + (\mu_2 + k_3)(\mu_3 + k_3)}{\lambda_2 \lambda_3},$$

$$D_4 = \frac{\lambda_3 k_4 + (\mu_2 + k_4)(\mu_3 + k_4)}{\lambda_2 \lambda_3},$$

$$F_1 = \frac{\mu_3}{\lambda_3}, \quad F_2 = \frac{\mu_3 + k_2}{\lambda_3}, \quad F_3 = \frac{\mu_3 + k_3}{\lambda_3}, \quad F_4 = \frac{\mu_3 + k_4}{\lambda_3}.$$

Using initial conditions [Eq (7)], we obtain the system of equations to determine the constants C_i

$$\begin{cases} C_1 + C_2 + C_3 + C_4 = 0, \\ C_2 k_2 + C_3 k_3 + C_4 k_4 = 0, \\ C_2 k_2^2 + C_3 k_3^2 + C_4 k_4^2 = 0, \\ C_1 B_1 + C_2 B_2 + C_3 B_3 + C_4 B_4 = 1. \end{cases}$$

The roots k_i are found from the characteristic equation

$$k^4 + bk^3 + ck^2 + dk = 0,$$

where

$$b = \sum_{i=1}^{3} \lambda_i + \sum_{i=1}^{3} \mu_i,$$

$$c = \lambda_1 (\lambda_2 + \mu_2 + \mu_3) + \lambda_3 (\lambda_1 + \lambda_2 + \mu_1) + \mu_3 (\lambda_2 + \mu_1 + \mu_2) + \mu_1 \mu_2,$$

$$d = \lambda_1 (\mu_2 \mu_3 + \lambda_2 \lambda_3 + \lambda_2 \mu_3) + \mu_1 \mu_2 \mu_3.$$

The 4 roots of the resolving equation are given by

$$k_1 = 0,$$

$$k_2 = G + H + \frac{b}{3},$$

$$k_3 = -\frac{G+H}{2} + i\frac{G-H}{2}\sqrt{3} - \frac{b}{3},$$

$$k_4 = -\frac{G+H}{2} - i\frac{G-H}{2}\sqrt{3} - \frac{b}{3},$$

where $i = \sqrt{-1}$ is the imaginary unit;

$$G = \sqrt[3]{\frac{bc}{6} - \frac{b^3}{27} - \frac{d}{2} + \sqrt{q}},$$

$$H = \sqrt[3]{\frac{bc}{6} - \frac{b^3}{27} - \frac{d}{2} - \sqrt{q}},$$

$$q = (\frac{c}{3} - \frac{b^2}{9})^3 + (\frac{bc}{6} - \frac{b^3}{27} - \frac{d}{2})^2.$$

Solving Eq (5) under condition that $\mu_1 = \mu_2 = \mu_3 = 0$, we can obtain the state transition probabilities $p_i(t)$ for standby system without repair:

$$p_0(t) = e^{-\lambda_1 t}, \tag{12}$$

$$p_1(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t}), \tag{13}$$

$$p_2(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \times$$
$$\left[\frac{e^{-\lambda_1 t}}{\lambda_3 - \lambda_1} - \frac{e^{-\lambda_2 t}}{\lambda_3 - \lambda_2} + \frac{(\lambda_2 - \lambda_1)e^{-\lambda_3 t}}{(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)} \right], \tag{14}$$

$$p_3(t) = \frac{\lambda_1 \lambda_2 \lambda_3}{\lambda_2 - \lambda_1} \times$$
$$\left[-\frac{1 - e^{-\lambda_2 t}}{(\lambda_3 - \lambda_2)\lambda_2} + \frac{1 - e^{-\lambda_1 t}}{(\lambda_3 - \lambda_1)\lambda_1} + \frac{(\lambda_2 - \lambda_1)(1 - e^{-\lambda_3 t})}{(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)\lambda_3} \right]. \tag{15}$$

4. Reliability indices

From Eqs (8), (9) and (10) the availability of the system with repair is expressed as:

$$A(t) = p_0(t) + p_1(t) + p_2(t), \tag{16}$$

and unavailability as:

$$UA(t) = 1 - A(t) = p_3(t). \tag{17}$$

The steady-state availability of the system is defined at $t = \infty$. Hence from Eqs (8)–(10) follows that:

$$A = \lim_{t \to \infty} A(t) = \sum_{i=0}^{2} p_i(t) = C_1(B_1 + D_1 + F_1), \tag{18}$$

where $p_i(t)$ – under the steady-state condition.

Now the steady state unavailability of the system is given by:

$$UA = 1 - A = 1 - C_1(B_1 + D_1 + F_1). \tag{19}$$

From Eqs (12), (13), and (14) the reliability for non-reparable system using assumptions a, c, d, e and f and solving Eq (5) under condition that $\mu_1 = \mu_2 = \mu_3 = 0$, we can obtain the state transition probabilities $p_i(t)$ and system performance indices:

$$\text{reliability} - R(t) = \sum_{i=0}^{2} p_i(t) \tag{20}$$

and from Eq (15) the unreliability

$$UR(t) = p_3(t) = 1 - \sum_{i=0}^{2} p_i(t). \tag{21}$$

Mean time to failure:

$$MTTF = \int_0^\infty R(t)dt = \int_0^\infty [p_0(t) + p_1(t) + p_2(t)]dt. \tag{22}$$

Using Eqs (12)–(14) and (22), the following expression is obtained:

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}. \tag{23}$$

5. Illustrative example

Let $\lambda_1 = 0,1$/year; $\lambda_2 = 0,15$/year; $\lambda_3 = 0,2$/year; two levels of $\mu_1 = \mu_2 = \mu_3 = 0,5$/year and 5/year. According to the differential Eqs (8)–(11) and (12)–(15) and the initial conditions (7), the state transition diagrams of the system with repair and without repair for specified values of member's failure and repair rates are shown in Fig 2. As expected, as the time increases the probability of both systems being in state p_0 decreases but increases the probability of being in states p_1, p_2 and p_3.

The plots of Eqs (16), (17), (20) and (21) are shown in Fig 3. The results clearly show that as the value of time becomes large, the system availability and unavailability approach a steady state while the reliability is decreasing very fast with time. Obviously, if components are repaired as they fail, we have continuously operating system, which availability will appear constant.

The mean time to failure (MTTF) of the system if any component is no failure is infinitive. The MTTF for not repairable system is shown in Fig 4. In this figure we can
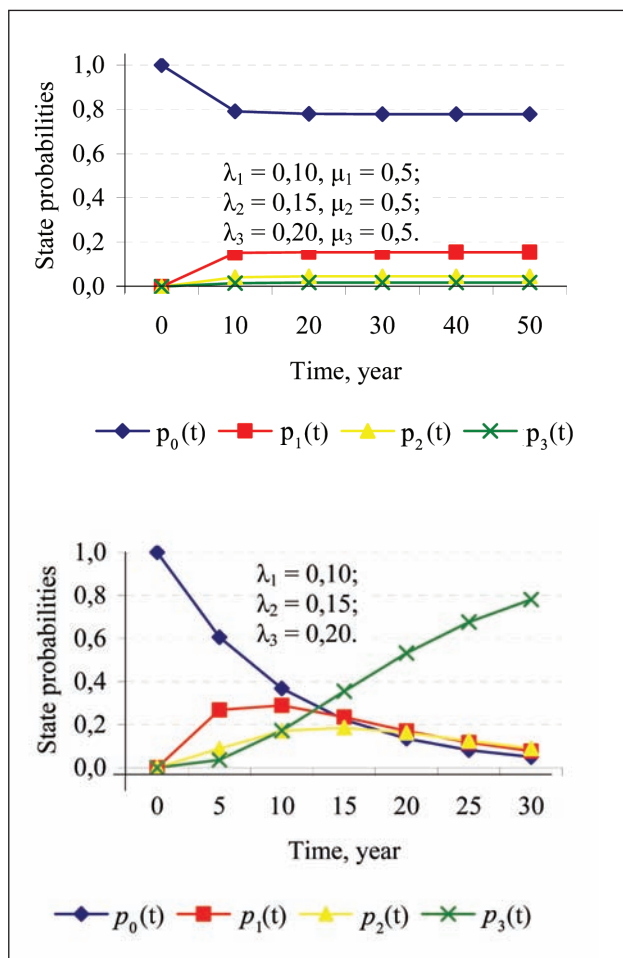
Fig 2. Time dependent state probabilities for the system with repair (above) and the system without repair (below)

Fig 3. Availability of the system with repair and reliability of the system without repair versus time
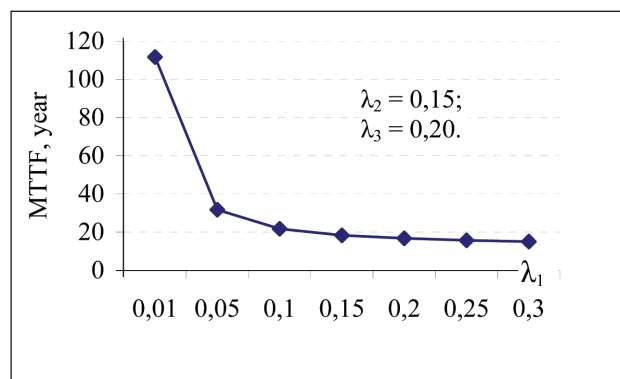
Fig 4. MTTF of the system without repair for the varying λ_1

see that the system MTTF is monotonically decreasing function of λ_1, which indicates that higher value of topcoat corrosion resistance will result in longer system MTTF.

The analysis of the influence of component repair rate values shows that when $\mu_1 = \mu_2 = \mu_3 = 0,5$ and $\lambda_1 = \lambda_2 = \lambda_3 = 5$ the results of steady state availability are $A = 0,981308$ and $A = 0,999976$, respectively. We can see that the value of system availability is increasing when rate of repair increases. In practice, the ratio of μ/λ for polymer coatings can be as high as 10–50. On the other hand, the reliability of 0,95 governed by SLS is a good choice for corrosive protection barriers.

6. Conclusions

1. A model of multi-layer corrosion protection system (CPS) for concrete or steel structures in aggressive environments is developed and combines the non-identical with different properties of individual layer material and thickness components. The performance of multi-layer corrosion protection system is proposed to generate with cold standby models.
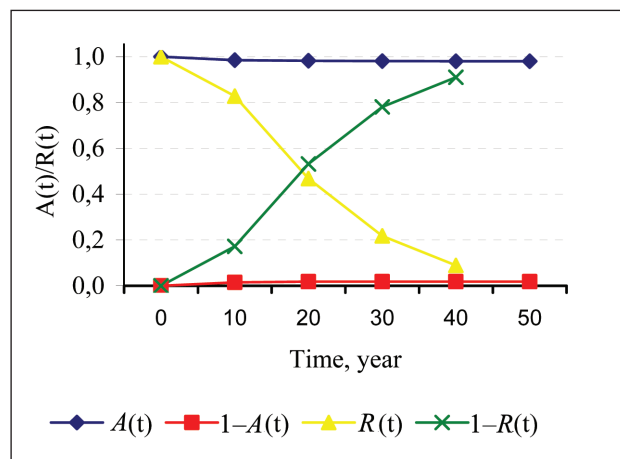
2. The system of differential equations for three-component system with one active unit and two spares in cold standby is set up to describe the transition states of protection system. The standby system can be repaired or not repaired. Components of the system are repairable (if any) to "as good as new" condition with different failure and repair rates exponentially distributed.

3. The reliability indices such as reliability, point wise and steady state availability, mean time to failure of multi-layer corrosion protection system with repair or without repair are analysed and defined by using Markovian deterioration/renewal process. The performance of corrosion protection system is illustrated by a numerical example.

4. Application of cold standby systems and Markov modelling is a suitable tool to assess the overall reliability of protection barriers. Results of investigation presented in this paper are the first attempt to model the performance of multi-layer corrosion protection of infrastructures as redundant standby system. Further work is needed on the probabilistic behaviour, maintenance scenarios and cost benefit analysis of various protective systems for particular applications.

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