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SHAPE STABILIZATION OF STEEL SUSPENSION BRIDGE

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Abstract. Stabilization of primary shape of suspension bridge is one of the governing design problems of the structure. The form, i.e. shape changes of suspension steel bridge, is prescribed more by kinematic vs elastic displacements. Kinematic displacements developed due to asymmetric loadings. Total displacements can be relatively split into kinematic and elastic components. Firstly, kinematic displacements are determined, secondly elastic displacements are evaluated taking into account the shape of cable to be changed. Such a relative splitting of displacements allow an evaluating the contribution of both total displacement components and an employing of the efficient engineering stabilization tools. The developed analytical expressions for determining of total and kinematic displacements for cable any cross are presented. The suspension bridge primary shape stabilization methods and their efficiency are analyzed. It is proposed to introduce the engineering tools for constraining horizontal displacements of suspension cable, resulting not only in reduction of vertical cable displacements, but also in stabilizing of the primary shape of the bridge.

Keywords: suspension steel bridge, cable, symmetric and asymmetric loadings, kinematic displacements, elastic displacements, total displacements, bridge, shape stabilization tools.

1. Introduction

Suspension bridges correspond to the type of the most efficient bridge structures (Gimsing 1997; Kulbach 2007; Lewis 2003; Михайлов 2002; Ryall *et al.* 2000; Troyano 2003; Walter *et al.* 1988). However, one deals with large shape changes, the main disadvantage of such structures. This peculiarity is conditioned mainly not by deforming the structural members, but by the kinematic displacements of cable (main load-carrying member of structure), being developed during adaptation process of cable to asymmetric loadings to resist loadings in a specific way (Bangash 1999; Gimsing 1997; Krishna 2001; Palkowski 2006; Ryall *et al.* 2000).

The cable adaptation to loadings is conditioned by the feature that the actual cable flexural stiffness is rather small compared to its axial stiffness. Thus, the cable takes the form to carry main portion of loading via tension. The pure displacements, compatible with the cable shape changes to adapt the loading via tension, are denoted as kinematic ones. Generally, these displacements are the governing ones when considering total displacements consisting of kinematic and elastic (caused by deforming structure under loadings) components (Gimsing 1997; Jennings 1987; Kulbach 1999; Ryall *et al.* 2000).

Analysis and design of suspension bridges, the most efficient load carrying structures, naturally was considered in many investigations (Gimsing 1997; Качурин *et al.* 1971; Ryall *et al.* 2000). However, the most investigations considered the total displacements ignoring their nature and influence of adapted cable shape for actual distributions of stresses and strains (Москалев, Попова 2003; Palkowski 2006). We remind the reader that distribution of stresses and strains (or inner forces and displacements) depends on the actual cable form.

Probabilistic approach is an important up-to-date tool for actual analysis of suspension steel bridge behaviour and the subsequent design of structural elements (Kala 2007, 2008).

Conventional design procedures introduces two main requirements for suspension bridge structures; namely, the strength and the stiffness. They state that suspension bridge structure cannot violate them under all considered loading cases (load combinations). Stiffness conditions actually are dominating in design of a suspension bridge. Usually they are expressed via constraints for the max magnitudes of

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cable total displacements $\omega_{max} \le \omega_{lim}$ in respect of all load combinations.

Thus an identification of governing factors, conditioning development of max displacements is of actual necessity in design of rational suspension bridge structure. The second step is employing certain engineering tools to reduce them in the most efficient way. One can refer to many investigations (e.g. Kulbach 1999; Михайлов 2002; Москалев, Попова 2003; Petersen 1993), recommending to restrict displacement analysis only in respect of kinematic displacements (as governing ones), when describing the actual displacements of asymmetrically loaded cable. Most of these investigations in design practice are treated as the engineering ones. Such investigations are based on applying the superposition principle. This principle leads to splitting of actual loading to symmetric and asymmetric components for total displacement analysis of asymmetrically loaded cable (Беленя *et al.* 1991; Москалев, Попова 2003). But such a too simplified approach leads to inadmissible errors under certain loading (when identifying actual displacements of cable) as strong geometrical nonlinearity of special adaptation nature is recognized. The investigation (Качурин *et al.* 1971) presents an analysis of errors obtained when replacing the asymmetric loading by the sum of symmetric and asymmetric items and employs the superposition principle for cable displacement analysis. The correcting equivalent load aiming to reduce this systemic error was proposed (Москалев, Попова 2003) to improve engineering methods. Despite the introduced efforts, the general and clear algorithm how this equivalent load is created and how the kinematic displacements are identified was not presented.

One must note, that one can list only some investigations (not comprehensive) assigned to analysis of kinematic displacements. Therefore, the extended analysis of kinematic nature displacements, that of the development general techniques for evaluating the cable shape are still an actual necessity. Obviously, a proper evaluation of the kinematic displacements (proper evaluation of the cable shape changes) results in a proper distribution of the total displacements and the inner forces. Having identified the nature of the governing factors, contributing to the development of max displacements, one can create and introduce certain engineering tools vs each factor, that subsequently reduce the total displacements of a suspension bridge. A designer can dispose a certain package of "engineering tools" ready for employing under certain circumstances. One can also note, that the analysis of kinematic displacements clears out the application limits of engineering methods aiming to avoid the admissible error of a calculation method.

2. Total displacements of suspension steel bridge

The large displacements of suspension steel cable under asymmetric loadings are explained by the kinematic nature of cable displacements, as it was mentioned above (Gimsing 1997; Москалев, Попова 2003; Palkowski 2006). The suspension members of the bridge are also sensitive to horizontal displacements of supports. These displacements increase the cable sag, resulting in the increment of total vertical displacements.

In design calculations of suspension bridges one must identify the total displacements in all stages of loading. One proposes to split the total displacements to kinematic and elastic ones, aiming to obtain clear and proper expressions for evaluating cable behaviour in all stages of deformation. The following analysis algorithm is proposed: first kinematic displacements are determined; second, the elastic displacements are determined following the actually changed shape of the cable. Such a relative splitting the displacements allows to identify an individual contribution of both displacement components to the changed shape of the bridge and the subsequent usage of proper suspension bridge stabilization tools.

When calculating the suspension bridge displacements, it is supposed that the cable is equally loaded via axial forces of hangers. It is assumed that the lengths of hangers prior and after loading remain the same. Thus, vertical displacements of cable and beam coincide.

When calculating the total cable displacements, it is important to identify a ratio of permanent symmetric and temporary asymmetric loads γ. When employing the conventional mounting methods, it is assumed that all permanent load is transmitted to cable (Gimsing 1997; Качурин *et al.* 1971). Then the thrusting (horizontal) force of the cable is:

$$H_0 = \frac{ql^2}{8f_0}, \tag{1}$$

where f_0 – primary sag of cable; l – span of cable.

The temporary load p of the bridge is transmitted both to beam of the bridge and to the cable. The part of asymmetric load p_c, corresponding to the cable, depends not only on cable geometric parameters, but also on flexural stiffness of the bridge beam EJ_b. By a simplified solution in case of asymmetric loading (Качурин *et al.* 1971) one can identify a part of temporary load, corresponding to the cable:

$$p_c = \frac{\frac{\Delta f_b}{16} - \frac{(q + p_{c,sim})l^2}{32H_1} + 0.5f_1}{\frac{\Delta f_b}{16} + \frac{l^2}{32H_1}}, \tag{2}$$

where $\Delta f_b = \frac{5pl^4}{384EJ_b}$ – the deflection of analogical beam resisting the whole temporary load; $p_{c,sim}$ – equivalent symmetric temporary load; H_1 – thrusting force of cable in case of symmetric load; f_1 – sag of cable under an asymmetric load.

Any point total cable displacement in case of permanent symmetric q and temporary asymmetric p loads are obtained by expressions:

$$\omega_l(x) = \omega_{l,k}(x) - \omega_{l,el}(x), \tag{3}$$

$$\omega_r(x) = \omega_{r,k}(x) - \omega_{r,el}(x), \tag{4}$$

where $\omega_{l,k}(x)$, $\omega_{r,k}(x)$ – the cable left and right parts vertical kinematic displacements, respectively (Fig. 1). Note, that only the most dangerous asymmetric loading case, when a half span is subjected by temporary load, is investigated. It is found that max total displacements develop in length quarters of span ($x \approx l/4$ and $x \approx 3l/4$) (Juozapaitis, Norkus 2004).

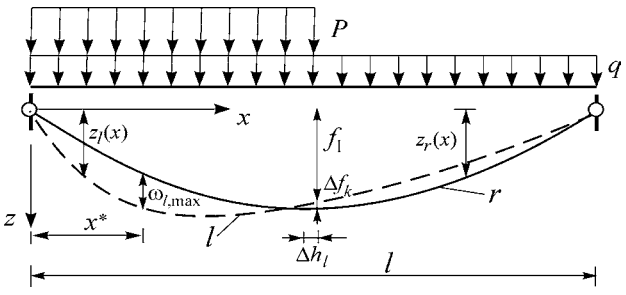

Fig. 1. Deformed scheme of asymmetrically loaded cable

The total vertical cable displacement in case of asymmetric load is calculated by:

$$\Delta f = \Delta f_k + \Delta f_{el}, \tag{5}$$

where Δf_k – cable kinematic displacement; Δf_{el} – cable elastic displacement.

Let us consider kinematic and elastic displacements separately.

3. Kinematic displacements of asymmetrically loaded cable

3.1. Kinematic vertical displacements

An equilibrium form of symmetrically loaded cable fits a quadratic parabola. This case of loading causes in cable only elastic displacements, with the max one at the middle span (Москалев, Попова 2003; Palkowski 2006). The supplement asymmetric load, applied to the half span of cable, forces the cable to change the primary form according to a new bending moment diagram. This change results in large displacements (Fig. 1). Assuming that cable axial stiffness $EA \to \infty$, one can eliminate cable elastic displacements from the total ones. Now the primary form change of the cable is prescribed only by kinematic displacements. The max displacements develop in both left and right parts of the cable.

Due to the above-mentioned engineering (simplified) calculation method (Беленя *et al.* 1991; Москалев, Попова 2003), the max kinematic displacement are equal in absolute magnitudes in both cable parts, the displacement in the middle span being equal to zero. Such an approach allows to obtain simpler analytic solutions, but leads to a systemic error when determining kinematic displacements.

Consider the form changes (displacements) (Fig. 1) of the cable, loaded by distributed loads: the symmetric load q, subjected per total span and the asymmetric load p, subjected at left middle span, i.e. at $l/2$.

The primary cable form fits parabola:

$$z(x) = f_0\left[\left(\frac{4x}{l}\right) - \left(\frac{4x^2}{l^2}\right)\right]. \tag{6}$$

The displaced shape of cable can be expressed by (Juozapaitis *et al.* 2004):

$$z_{lk}(x) = \frac{f_{k1}}{\left(1+\frac{\gamma}{2}\right)} \times \left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{3x}{l} - \frac{4x^2}{l^2}\right)\right], \tag{7}$$

$$z_{rk}(x) = \frac{f_{k1}}{\left(1+\frac{\gamma}{2}\right)} \times \left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(\frac{x}{l} - 1\right)\right], \tag{8}$$

for cable parts $x \le l/2$ and $l/2 \le x \le l$, respectively, where f_{k1} – the cable sag (deflection or vertical displacement) of cable middle span ($x \le l/2$) due to asymmetric load; $\gamma = p_c/q$ – ratio of intensities of asymmetric and symmetric loads. Subscripts l and r denote left and right middle spans of the cable, respectively.

The kinematic sag of the cable is expressed by a sum of the primary sag f_0 and the kinematic displacement Δf_k:

$$f_{k1} = f_0 + f_k. \tag{9}$$

A displacement of the left cable part (7) consists of a sum of two items fitting parabola, the displacement of right cable part (8) consists of two items fitting parabola and the line.

The position of the max cable displacement can be identified by equaling the cable function $z'_{lk}(x) = 0$ and subsequently having sold it in respect of coordinate x. The variation of γ, e.g. within the interval [1, 10], results in the variation of max displacement within the bounds [0.437l, 0.386l].

A method of identifying the kinematic displacement value via (7) and (8) expressions is a simple procedure in case for known f_{k1}. The latter can be obtained by employing the static and geometric Eqs of loaded cable (Juozapaitis, Norkus 2004; Москалев, Попова 2003; Palkowski 2006).

When the cable shape is prescribed only by kinematic displacements, one can employ the total lengths of the cable, compatible with the primary and secondary cable shapes $s_0 = s_{1k}$. So after certain transformations, one can obtain:

$$f_{k1} = f_0 \frac{\left(1+\frac{\gamma}{2}\right)}{\sqrt{1+\gamma+\frac{5\gamma^2}{16}}}. \tag{10}$$

One can find from (10), that the kinematic sag f_{k1} under $\gamma \ne 0$ is less of the primary sag f_0. It is obvious that the kinematic displacement is negative, i.e. directed up vs the primary cable shape. Then Δf_k, taking into account (9), can be identified by:

$$\Delta f_k = -f_0\left(1-\frac{1+\frac{\gamma}{2}}{\xi}\right), \quad (11)$$

where $\xi = \sqrt{1+\gamma+\frac{5\gamma^2}{16}}$.

The numerical simulations proved that for any γ magnitude the displacement Δf_k always is directed up. Note that relation of Δf_k vs γ is non-linear. From static Eqs one can obtain that thrusting force depends on γ, i.e.:

$$H_{k1} = \frac{ql^2\left(1+\frac{\gamma}{2}\right)}{8\left(f_0+\Delta f_k\right)}. \quad (12)$$

The analysis of formulae (11) and (12) yields that engineering methods (i.e. stating that $\Delta f_k = 0$) for evaluating cable kinematic displacements always results in an error when determining displacements of asymmetrically loaded cable (Juozapaitis, Norkus 2004).

3.2. Kinematic horizontal displacements

The vertical kinematic displacements of asymmetrically loaded cable are always accompanied by horizontal ones (Качурин *et al.* 1971; Москалев, Попова 2003; Palkowski 2006). These are directed to the cable part, loaded by asymmetric load *p*. Analyzing the new shape of cable in terms of lengths contributing both parts, one can state that the left cable part relatively increases and the right cable part relatively decreases. Taking this into account and the cable left part and right parts lengths, the horizontal displacement at cable middle span are expressed by:

$$\Delta h_{lk} = (s_{lk} - s_{l0}) \times \cos\phi_x, \quad (13)$$

$$\Delta h_{rk} = (s_{rk} - s_{r0}) \times \cos\phi_x, \quad (14)$$

where ϕ_x – cable slope angle with horizontal.

Taking that $\phi_x \approx 0$ at middle span and having employed the geometrical Eqs, one obtains the expressions for determining the horizontal kinematic displacements (Juozapaitis, Norkus 2004):

$$\Delta h_{lk} = \frac{4}{3l}\left[\left(f_0+\Delta f_k\right)^2\frac{\left(1+\frac{5\gamma}{4}+\frac{7\gamma^2}{16}\right)}{\left(1+\frac{\gamma}{2}\right)^2} - f_0^2\right], \quad (15)$$

$$\Delta h_{rk} = \frac{4}{3l}\left[\left(f_0+\Delta f_k\right)^2\frac{\left(1+\frac{3\gamma}{4}+\frac{3\gamma^2}{16}\right)}{\left(1+\frac{\gamma}{2}\right)^2} - f_0^2\right], \quad (16)$$

corresponding to the left and right cable parts, respectively.

An analysis of the formulae (15) and (16) shows that both vertical and horizontal kinematic displacements directly depend on the cable primary sag f_0 and the loads ratio γ. From the above formulae one can find that horizontal kinematic displacement at cable middle span is related to the vertical kinematic one. The performed numerical simulations proved that: Δh_{lk} and Δh_{rk} are of the same order magnitudes as Δf_k; the horizontal displacement of both parts are equal in absolute values, i.e. $|\Delta h_l| = |\Delta h_r|$.

One can note that kinematic vertical and horizontal displacements at cable middle span prescribe the stress and strain state of the whole cable. It is obvious, that aiming to reduce the max vertical displacements, one must constrain the horizontal ones by employing certain engineering tools.

4. Max vertical kinematic displacements of asymmetrically loaded cable

4.1. Left part displacements

The vertical kinematic displacements of the left part (subjected by complementary load *p*) can be treated as the difference of primary and final cable shapes (Juozapaitis, Norkus 2004; Москалев, Попова 2003):

$$\omega_{lk}(x) = z_{lk}(x) - z_{l0}(x). \quad (17)$$

By combining the expressions (6), (7) and (17) one can obtain an expression for identifying the vertical kinematic displacements of the left ($x \le l/2$) cable part (Juozapaitis, Norkus 2004):

$$\omega_{lk}(x) = \frac{f_0+\Delta f_k}{\left(1+\frac{\gamma}{2}\right)}\left[\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right)+\gamma\left(\frac{3x}{l}-\frac{4x^2}{l^2}\right)\right] - f_0\left(\frac{4x}{l}-\frac{4x^2}{l^2}\right). \quad (18)$$

Assuming the max cable displacement to be a quarter of the cable ($x \approx l/2$), one can obtain an approx formula for its magnitude:

$$\omega_{lk,\max}(x) = 0.75 f_0\left[\frac{(1+\frac{2\gamma}{3})}{\xi}-1\right]. \quad (19)$$

This formula is a relative compact one and does not require many computational efforts vs the exact solution. An analysis of results, obtained by an application of the formula (19), proved that the max error when determining the max displacements vs the results obtained by formula (18), does not exceed 1.6% in case of γ =10 and 0.14% in case of γ = 1.

4.2. Right part displacements

The kinematic displacements of the right, free from asymmetric load, part ω_{rk} are always (for γ > 0) negative, i.e. directed up. They can be determined in analogous way as for the cable left part:

$$\omega_{rk}(x) = z_{rk}(x) - z_{r0}(x). \quad (20)$$

By combining the expressions (6), (7) and (20), one can obtain a relation for determining the cable right part vertical kinematic displacements (Juozapaitis, Norkus 2004):

$$\omega_{rk}(x) = \frac{f_0 + \Delta f_k}{\left(1+\frac{\gamma}{2}\right)}\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \gamma\left(1 - \frac{x}{l}\right)\right] - f_0\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right). \quad (21)$$

Formula (21) shows that the right part kinematic displacements (analogously to these of the left part) depend on the primary sag f_0, the vertical kinematic displacement at half span of cable Δf_k and the loads ratio γ.

The right, free from temporary loading, cable part kinematic displacements can be determined in an analogous way, as for the left part, taking that $\omega_{rk,\max} \cong \omega_{rk}(3l/4)$ (Juozapaitis, Norkus 2007):

$$\omega_{rk,\max}(x) = 0.75 f_0\left[\left(\frac{1}{\xi} - 1\right) + \frac{\gamma}{3\xi}\right]. \quad (22)$$

An analysis of the formulae (21) and (22) shows that the right part displacements are greater (in absolute magnitudes) comparing to the ones of the left unloaded part ($\omega_{rk,\max} > \omega_{lk,\max}$). This, from the first view unexpected result, can be explained by the negative displacement at the middle span Δf_k. Such a distribution of kinematic displacements was described in (Качурин *et al.* 1971).

We remind the reader that an employment of the engineering methods results kinematic displacements of both parts to be identical.

Aiming to identify the difference of max displacements $\omega_{lk,\max}$ and $\omega_{rk,\max}$, an analysis of relative error vs loads ratio γ was performed. Results of the analysis are presented in Fig. 2. In case of $\gamma = 1$, the right displacement $\omega_{rk,\max}$ is by 28% greater than the left one $\omega_{lk,\max}$. This difference in error gradually increases, i.e. for $\gamma = 5$ it is 70%, and for $\gamma = 10$ it reaches even 86%.

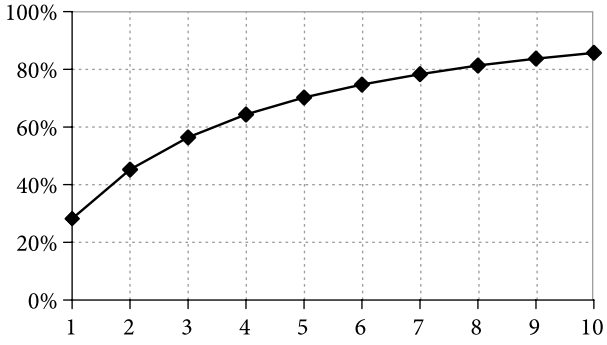

Fig. 2. Relative difference in % of cable right and left parts max displacements vs loads ratio γ

An analogous verification of the difference between cable right and left parts max displacements and the displacement values was also performed by authors applying the FEM package COSMOS/M. The simulated results were adequate to these, obtained via techniques proposed by authors (Juozapaitis *et al.* 2005).

5. Elastic cable displacements

When the actual adapted to loading cable shape is identified (via kinematic displacements), one can determine the elastic displacements (the second component of the total displacements). The cable total sag at the middle span can be treated as the sum of a kinematic sag and the elastic displacement, namely:

$$f_1 = f_{k1} + \Delta f_{el}. \quad (23)$$

The kinematic sag is obtained by the formula (10). By employing the known compatibility Eq for displacements and strains (Juozapaitis, Norkus 2004), one can obtain the formula for determining the elastic displacement at cable middle span:

$$\Delta f_{el}^2 + 2 f_k \Delta f_{el} - \frac{H s_{k0}}{EA}\frac{3l\psi}{8} = 0, \quad (24)$$

where

$$H = \frac{ql^2\left(1+\frac{\gamma}{2}\right)}{8\left(f_k + \Delta f_{el}\right)}. \quad (25)$$

The formula (25) is identical to the formula (12), but the first one evaluates additionally the influence of the elastic deformations on thrusting force magnitude.

Solution of the expressions (24) and (25) in respect of Δf_{el} results the known (Juozapaitis, Norkus 2004; Москалев, Попова 2003) and complicated 3rd order (cubic) Eq. Let us present (omitting the derivation) an approx formula for determining the elastic cable displacement:

$$\Delta f_{el} \approx \frac{3}{128}\frac{ql^4\left(1+\frac{\gamma}{2}\right)\psi}{EA f_k^2\left[1+\frac{3}{128}\frac{ql^4\left(1+\frac{\gamma}{2}\right)\psi}{EA f_k^3}\right]}. \quad (26)$$

This formula is analogous to the known simplified expression for determining cable middle span elastic displacement Δf_{el} in case of symmetric distributed loading (Беленя *et al.* 1991; Москалев, Попова 2003).

6. Stabilization of kinematic displacements

The largest contribution to the total displacement magnitude is caused by the kinematic displacement. Thus, aiming to satisfy the stiffness conditions $\omega_{\max} \le \omega_{\lim}$, one must introduce certain means reducing the magnitudes of kinematic displacements. The developed elastic displacements can be reduced by increasing cable cross-sectional area and the

primary sag (Eq (26)). An analysis of the nature of kinematic displacements (described above) enables to apply various engineering for this purpose.

An increment of the beam flexural stiffness EJ_b one can list amongst the simplest and the mostly employed in engineering practice methods for reducing displacements under symmetric loadings. But one must keep in mind that the increment of cross-sectional area results in the increment of the bridge mass.

An employment of the relatively rigid cable members of finite magnitude flexural stiffness EJ_c one can mention as the new method for stabilizing primary shape of the bridge (Grigorjeva *et al.* 2006; Juozapaitis, Norkus 2007). Such suspension cable bridge combines the cable and flexural beam properties, i.e. resists the change of primary form via tension and bending. Such cable structural units are produced from hot rolled or welded profiles. It is obvious that aiming to stabilize e.g. primary form of bridge one must choose a carrying member of required flexural stiffness. An efficiency of so "modified" cables, when comparing with absolutely flexible cables, is directly dependent on flexural stiffness EJ_c and the primary sag f_0 magnitudes (Grigorjeva *et al.* 2006; Juozapaitis, Norkus 2007; Москалев, Попова 2003). This method is not analyzed in a more detail in this investigation.

Amongst the engineering tools, which could be eventually employed in engineering practice of suspension steel cables, subjected by asymmetric load p, one can list: an increment of the symmetric load q magnitude; a reduction of the primary cable sag f_0. Both methods are not efficient technical/economical tools as they cause the significant side effect: an enlargement of the horizontal (thrusting) force $H(H_{k1})$. This case subsequently requires increasing the cable cross-sectional area and the mass of anchors.

Introduce the parameter $m = q/(q + p)$ $1/(1 + \gamma)$, denoting the ratio of the symmetric load and the total load. Then one can investigate a variation of kinematic displacement magnitude vs the increment of asymmetric load part. Fig. 3 illustrates the relative reduction of the max kinematic displacement vs parameter m. Kinematic displacement at $\gamma = 10$ (then $m \approx 0.091$) was taken as a starting point. By increasing the symmetric load part, i.e. by reducing the parameter γ from 10 till 1, m changes from 0.091 till 0.50. Aiming to reduce γ from 8 till 10, one must increase the symmetric load by 1.25 times. Thus, m will increase from 0.091 till 0.111 magnitudes. Fig. 3 shows that for $m = 0.111$ the max displacement of cable left part reduces by 2% and that of right part reduces by 4%. Having increased the q magnitude twice, the max displacements reduces by: for left part by 7.5% and for right part by 15%. A significant reduction of kinematic displacements are obtained only by increasing the symmetric load by 5 times ($m = 0.333$). Then the max kinematic displacements are reduce to 26.5% and 42.5% for left and right cable parts, respectively. If the symmetric load is increased by 10 times, an analogous reduction by 46% and 63% is obtained.

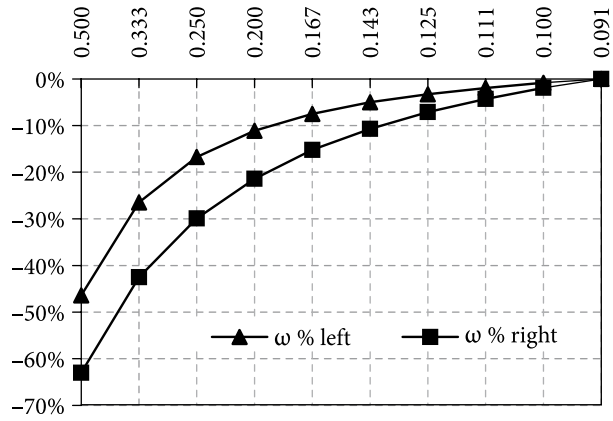

Fig. 3. Relation of $\omega_{l,\max}$and $\omega_{r,\max}$ relative increments vs variation of m

Fig. 4 illustrates a relative changes (in %) of horizontal force H_{k1}vs the increment of symmetric load q. Having increased the load q by 1.25 times ($m = 0.111$), the horizontal force increases by 3.6%. Having increased the load q twice ($m = 0.333$), the horizontal force increment is 14%. An increment of the symmetric load from 5 till 10 times, causes an increment of H_{k1} by 59% and 134%, respectively.

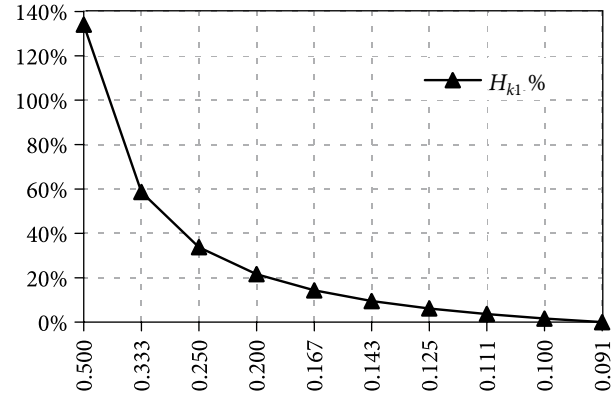

Fig. 4. Relation of H_{k1} relative increments vs m

When the primary sag f_0 is reduced, stabilization effect is analogous to an increment of symmetric load magnitude as the reduction of f_0 results in an increment of thrusting force H_{k1}. One must note that an increment of load q (for $m \geq 0.2$) induces the relatively larger horizontal force magnitude vs the relative reduction of max kinematic displacements.

One must note that the increased thrusting force results in the increased elastic deformations/displacements. Therefore such a method of the stabilization of kinematic displacements requires to design relatively larger cross-sectional areas for reducing the Δf_{el}.

We remind the reader that the vertical kinematic and horizontal displacements are in direct relationship (Juozapaitis, Norkus 2004; Москалев, Попова 2003). The max horizontal displacements develop at a half span of cable.

An analysis of the formulae (14) and (15) yields that horizontal displacements as well as vertical ones depend on the cable primary sag f_0 and on the loads ratio γ.

Having constrained (via certain constructional tools) the horizontal displacement Δh_k at the cable middle span, one can obtain the necessary reduction of cable max vertical displacements (aiming to satisfy stiffness conditions $\omega_{max} \le \omega_{lim}$). Such engineering approach is employed in practice for bridges (Gimsing 1997). Let us consider the case when a cable at middle span is connected with a horizontal bar-tie, aiming to reduce the vertical displacements. This case is realized via a connection of the main cable and girder at the middle span. Investigate the cable left and right parts separately. They are considered as suspension cables with a flexible lower support. The primary sag is $\Delta f_{0l} = \Delta f_{0k} = f_0/4$. Horizontal forces in these parts are not equal in case of asymmetric loading. The difference of their magnitudes depends on deformation of bar-tie, i.e. it depends on the actual horizontal displacement magnitude. In separate cases it makes sense to connect the cable with the beam via elastically flexible ties. Such connection can be constructed from bars of continual flexibilities. An increment of the γ increases the difference of the horizontal forces. Kinematic displacements of the left cable part are calculated by:

$$\omega_{l,\max} = \Delta f_{1l} - \Delta f_{0l}, \tag{27}$$

where Δf_{1l} – deflection of suspension of the left cable after deformation.

The relationship of horizontal displacement vs sags of left and right cable parts is described by:

$$\Delta h_k \cos\varphi_0 \cong \frac{4}{3l} \times \left(\frac{\Delta f_{1l}^2}{0.5l - \Delta h_k} \cos^3 \varphi_l - \frac{\Delta f_{1r}^2}{0.5l - \Delta h_k} \cos^3 \varphi_r \right). \tag{28}$$

Fig. 5 illustrates a relationship of the cable left part relative displacements vs the horizontal displacement Δh_k. The case when a cable is not constrained by tie (not connected with the beam), as starting the point is taken. The horizontal displacement was gradually reduced from 1 till 10 times vs the starting point displacement. Fig. 5 shows the strong non-linear reduction of the max vertical displacements vs the reduction of the horizontal displacement (obtained by increasing the axial stiffness of tie E_tA_t). One can find that such constraint is very efficient in early stages. When the horizontal displacement is reduced twice, the vertical horizontal displacement reduced by 50%, comparing with the starting one.

The gradual reduction of horizontal displacement from 2 till 10 times results in the reduction of vertical displacement by 90%.

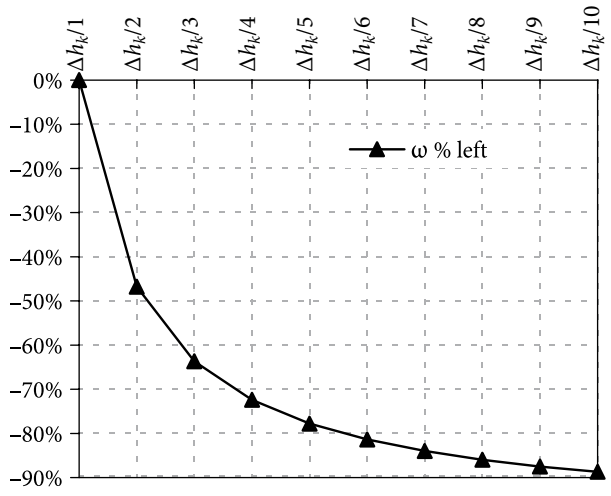

Fig. 5. The cable left part kinematic displacement $\omega_{l,max}$ vs horizontal displacement Δh_k

This stabilization tool is very efficient too, as it causes a small increment of horizontal force H_{k1}. Fig. 6 shows a relationship of the horizontal force relative values, when γ = 1. When horizontal displacement is reduced by 10 times, one obtains the 26% increment of the horizontal force. One must note, that an increment of the loads ratio causes an increase of the horizontal force.

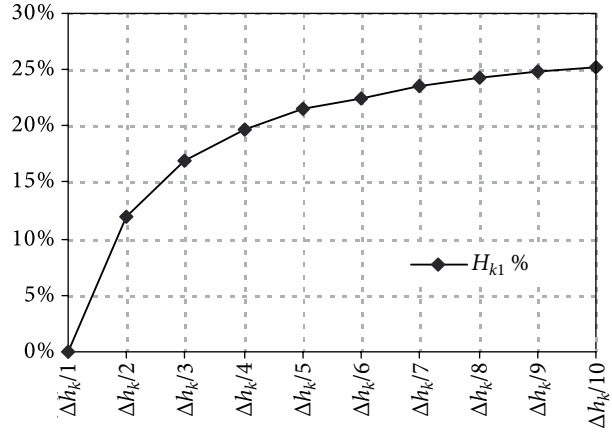

Fig. 6. Relation of horizontal force H_{k1} relative magnitude vs horizontal displacement Δh_k

One must note, that by connecting the cable with beam via a rigid connection, one relatively divides the cable into two independent parts. If the axial stiffness of beam EA_b is sufficiently large, then one can assume that $\Delta h_k \to 0$. In this case the vertical kinematic displacements are almost zero magnitudes and the thrusting forces of left and right cable parts will differ. The difference of these forces depends directly on loads ratio γ.

7. Conclusions

An analysis of kinematic, elastic and total displacements of asymmetrically loaded suspension steel bridge was performed. The improved kinematic displacement calculation method is presented.

It was proved that max kinematic displacement appears in free from temporary loaded cable part of the cable and that the cable middle span is always lifted up. The direct relationship of the vertical and the horizontal displacements has been shown.

The simplified expression for determining the elastic displacement at the middle span of bridge is presented.

The main techniques and their efficiency for stabilizing the kinematic displacements of cable have been analyzed. It was proved that the cable stabilization via increasing the symmetric load magnitude and/or via reducing the primary sag causes a significant increment of the thrusting force, resulting the increment of the elastic displacements. The method to stabilize the suspension cable bridge displacements via constraining its horizontal ones was proposed. The connection of the suspension cable with the beam of the bridge is proposed, as the rational engineering tool is a relatively small increment of thrusting force in various parts of the cable is induced.

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