



## ANALYTICAL AND EXPERIMENTAL APPROACH FOR ASSESSING VIBRATION SERVICEABILITY OF HIGHWAY BRIDGES DUE TO HEAVY VEHICLE TRAFFIC

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**Abstract.** The major goals of this paper are to evaluate the serviceability of bridges with vibration problems among these structures in actual use, and to establish the evaluation criteria based on the evaluation results. To accomplish these goals, the vehicle's weight, moving velocity, displacement from the measurement position, and acceleration were measured through the field test to evaluate the bridge's serviceability with respect to its vibration characteristics. Furthermore, the vibration characteristics of the bridge were analyzed with respect to various factors that influence its vibration. For this purpose, the three-axle truck used in the field test was modelled as the moving vehicle with 8 degrees of freedom, and the bridge considered was modelled using the frame element. According to the comparison of the analytical and experimental results, the analytical method that was used to evaluate the bridge's vibration serviceability as suggested in this paper based on the results of the analysis and field test is feasible.

**Keywords:** dynamic response, vibration serviceability, heavy vehicle, evaluation procedure, moving vehicle model.

### 1. Introduction

With rapid economic development, the number of heavy vehicles running on roads is rapidly increasing. The excessive vibration of bridges in actual use due to the scaling-up of vehicles is leading to the bridges' lower serviceability and threatened safety. In addition, with the rapid advances in the field of high performance, light materials and construction techniques, these bridges have a trend towards light and flexible. This means a considerable increase of vibration serviceability problems in bridge system. There are many cases that users feel discomfort or displeasure due to vibration, especially in bridges with heavy traffic congestion such as those in urban areas. Therefore, the vibration serviceability of bridges should be considered during their design or maintenance as necessary, even though there is no special structural problem with a bridge. Among the many factors that bring about bridges' vibration and thus affect their serviceability, vibration due to the effect of running vehicles causes poor riding comfort. Such vibrations have various frequencies. These frequencies affect vehicles or people crossing the bridge in various types depending on the value of the frequency. They also increase the dynamic wheel force of the running vehicle, resulting in damage to the pavement surface and amplification of its dynamic response. Reis *et*

*al.* (2008) investigates dynamic analysis of a bridge supported with many vertical supports under a moving load. In addition, pedestrians and vehicle passengers crossing the bridge feel displeasure and insecurity. Pimentel *et al.* (2001) evaluates the performance of currently used codes of practice regarding vibration serviceability of footbridges under human-induced loads due to walking. The evaluation is supported by experimental evidence from tests carried out by the authors on potentially lively footbridges. McCrea *et al.* (2002) investigated periodical inspection for possible deterioration of bridges and a program of maintenance and restoration tasks in order to prolong their life and vibration serviceability. da Silva *et al.* (2007), Figueiredo *et al.* (2008) developed four different loading models conducted parametric study to incorporate the dynamic effects, induced by pedestrian walking, in footbridge system. The results to reach high vibration levels in this footbridge could compromise the user's comfort limit state.

Currently, however, there is no consistent method of and criteria for evaluating the vibration levels of highway bridges due to the moving vehicles. The current Highway Bridge Design Standards of Korea and USA includes a requirement for deflection against live loads and impact loads among the vibration criteria for highway bridges. Moreover, such standard focuses on the static serviceabil-

ity and structural safety of bridges rather than on their vibration serviceability. In foreign countries, such standards include vibration requirements for the human body and structures to solve vibration problems that affect serviceability. Moreover, more studies on this are being conducted. For example, in Canada *Canadian Highway Bridge Design Code* (CSA), the effect of bridge deflections on the human body is included in the specifications for improving the serviceability of bridges. In this study, the method of evaluating bridges' vibration serviceability and the proper criteria for bridges' vibration serviceability will be suggested by measuring the vibration characteristics of an actual bridge using public traffic and a test vehicle, and by analyzing parameters such as velocity, surface roughness, vehicle weight, and measurement position using 3-dimensional vehicle-bridge interaction program developed in this study to determine the effect of the bridge's vibration characteristics due to vehicles on vibration perception.

## 2. Characteristics of vibration perception and criteria

A vibration is a strong swing of the ground or a structure due to the use of a machine or a structure. Criteria for assessing a vibration depending on its magnitude are necessary to evaluate a structure's serviceability in the face of this vibration. Several types of evaluation criteria have been conducted in order to investigate structural serviceability so far, although they are being applied in other fields. This study, thus, applies the most apt evaluation criteria for the evaluation of a bridge's vibration serviceability among existing criteria. For this purpose, the existing evaluation criteria were analyzed, and the applicability of the most apt criteria was reviewed among these existing criteria.

### 2.1. Reiher-Meister curve

The human body has a natural frequency, since it is also a vibration object, and is resonated by vibrations from outside. Therefore, many studies have long been conducted on vibration perception by considering the human body as the mechanical transport system (whole-body vibration). A representative study among the studies conducted so far is Reiher-Meister's (1931) curve as shown in Figs 1 and 2. This criterion was prepared by measuring and classifying the vibration perception level for people. The objectivity of this criterion was acknowledged in attributions to it in various publications. To apply the Reiher-Meister curve to the analysis of a bridge's vibration serviceability in this study, ISO criteria and various Eqs and specifications were compared and analyzed to determine whether or not it is appropriate.

#### 2.1.1. Proposed Eq for serviceability

In order to calculate the limit-frequency for unpleasant perception, Postlethwaite (1944) suggested the following equation with the function of frequency:

$$x_0 = 0.076 \times \left( 1 + \frac{197}{f^2} \right), \text{ mm} \quad (1)$$

The Eq for max allowable frequency is also provided by Oehler (1957) to avoid the complaints of passengers who cross the bridge. The Eqs are as below:

$$x_0 = \frac{50.8}{f^3}, \text{ mm } f = 1-6 \text{ Hz} \quad (2a)$$

$$x_0 = \frac{25.4}{f^2}, \text{ mm } f = 6-20 \text{ Hz} \quad (2b)$$

For another expression, Dieckmann (1958) investigated the perception of the whole-body vibration to evaluate the vibration. As the structure becomes larger and heavier, he proposed the extent of unpleasant perception to evaluate the serviceability of vibration more effectively. The corresponding relationship is given by Eq (3):

$$x_0 = 0.076 \times \left( 1 + \frac{125}{f^2} \right), \text{ mm} \quad (3)$$

where  $f$  = frequency, Hz.

Fig. 1 shows that the Eqs mentioned above are compared with the criterion of Reiher-Meister (1931). All of the Eqs above lie in the level of B (unpleasant) to A (annoying). In general, the criterion proposed by Postlethwaite, Oehler and Dieckmann, as well as other vibration criterion stated in ISO, show that vibration level of Reiher-Meister is reasonable to be applied to the evaluation of vibration serviceability in bridge. In this study, the evaluation of vibration serviceability in bridge will be studied with the respect of the vibration level of Reiher-Meister.

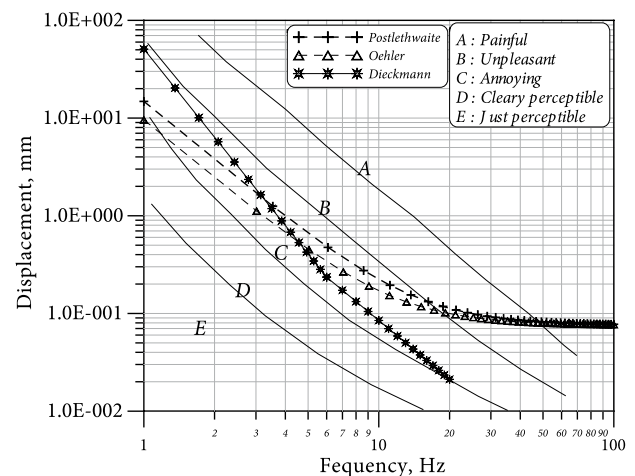


Fig. 1. Comparison with other vibration criteria

#### 2.1.2. ISO 2631:1997 "International Organization for Standardization"

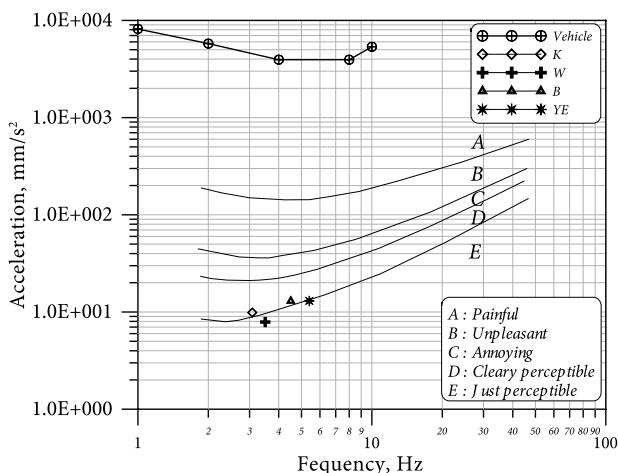
ISO 2631:1997 for vibration evaluation include criteria for random and transient vibrations during travel on bridge and for buildings. To compare with Reiher-Meister curve,

acceleration for 60s, which is the exposure time of vehicle passengers, while the vehicle is crossing the bridge under a transient vibration, was used. And with respect to the criteria for the building, the allowable vibration for the factory office, which is the highest and is considered closest to the expected level of the bridge's vibration serviceability, was applied among the allowable criteria for vibrations of 1–80 Hz, as shown in Table 1.

**Table 1.** Max RMS frequency-weighted acceleration (ISO 2631:1997)

Place	Time, s	Continuous/ intermittent vibration, RMS
Critical working areas (e.g. hospital operating room)	Day	0.0036
	Night	0.0036
Residences	Day	0.072/time
	Night	0.005
Offices	Any	0.14/time
Workshops	Any	0.28/time

The exposure time, which is the time spent crossing a bridge, can be obtained by assuming that a pedestrian walks at 1 m/s velocity. The value that applied this exposure time in the Eq for a factory office, as shown in Table 1, and the Reiher-Meister criteria were compared. With respect to the criteria for vehicle passengers and buildings, the root-mean-square (RMS) acceleration amplitude was used. The RMS value was converted, however, to a normal acceleration amplitude, so that it could be applied to the Reiher-Meister criteria, which use normal acceleration amplitude, as shown in Fig. 2. The ISO criteria for buildings is *D*, a very good level in the Reiher-Meister criteria, and the ISO criteria for vehicle passengers is much higher than *A*, a very poor vibration serviceability level. Thus, the Reiher-Meister classification criterion falls between the



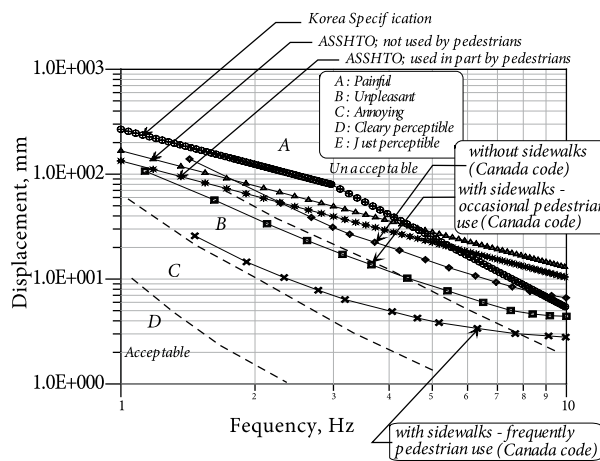
**Fig. 2.** Comparison between Reiher-Meister and ISO 2631

two ISO criteria. The Reiher-Meister curve in this section, therefore, seems to be considered vibration serviceability criteria that can be expected in normal highways.

**2.1.3. Various specifications**

The current Highway Design Standards for Steel Bridges include only criteria that restrict the deflection of bridges to attain at least a certain level of stiffness while maintaining the stress of each part of the bridge below the allowable stress of the material used, considering structural safety with respect to the moving vehicles, the effect of secondary stress due to deformation, and users' displeasure. The CSA, on the other hand, specifies the max deflection at the centre of the walkway, if there is a walkway, and the max deflection at the end, if there is no walkway, under the CSA truck load depending on the frequency. The AASHTO LFRD Bridge Design Specifications criteria are suggested to restrict the deflection of steel, aluminium and concrete bridges, in which the design vehicle load is loaded to produce max deflection.

The standards of various countries usually include restriction criteria for frequency-deflection and for span length-deflection. The span length and frequency have a relatively high degree of relationship. The span length-deflection was converted to frequency-deflection using this relationship to compare it with the Reiher-Meister curve, as shown in Fig. 3. Based on this comparison, the figure shows a significant difference to be attributed to displacement. The Reiher-Meister curve specifies dynamic displacement, whereas other standards specify max static deflection. Accordingly, it is considered inappropriate to apply the existing specifications, as they are to evaluate the vibration serviceability of bridges. The criteria for restricting deflection in the existing specifications other than CSA fell in area *A* in the Reiher-Meister curve, which was still excessive, whereas CSA fell in area *A* if there is no pedestrian, between areas *A* and *B* if there are few pedestrians, and between areas *B* and *C* if there are many pedestrians. Therefore, it would be reasonable to use CSA, which sug-



**Fig. 3.** Comparison between Reiher-Meister and various specification

gests relatively proper criteria and considering displacement and frequency, to evaluate the vibration serviceability of bridges among existing specifications.

Based on the comparison of the aforementioned standards, the Reiher-Meister curve will be used as criteria for bridges' vibration serviceability. Area *B* of the Reiher-Meister curve will be applied to bridges with relatively many pedestrians such as urban bridges and pedestrian bridges, and the area between areas *A* and *B* of the curve will be applied to bridges with no pedestrians and with maintenance staff and vehicle passengers, such as highway bridges.

### 3. Evaluation of vibration serviceability of bridges from the field test

In this study, 8 bridges with severe vibrations were selected for the evaluation of a highway's vibration serviceability. The bridges displacements and accelerations were measured under public traffic as shown in Table 2. The measurements were made at the walkway at the center of each span, and in the absence of a walkway, at the end, near the curb (Fig. 4).

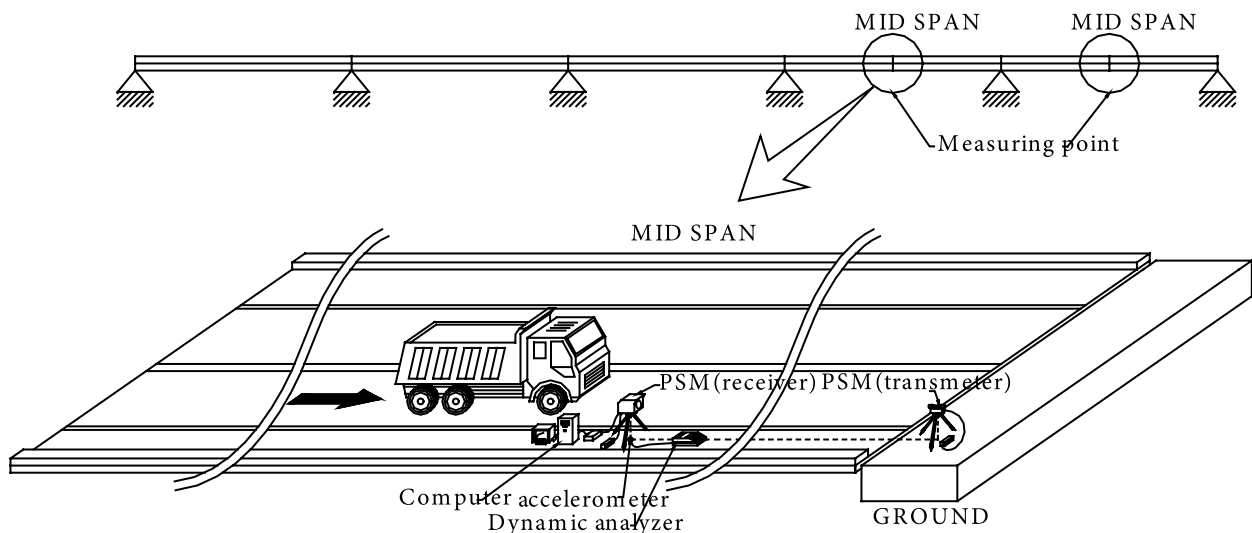
### 3.1. Data processing and analysis

The dominant frequency and amplitude of the vibration data of the bridge were used to evaluate the vibration serviceability of bridges under public traffic loads. The data obtained from the field test on the bridge considered, which was conducted to determine the dominant frequency, was converted into an ASCII file, and an FFT analysis was conducted (Bendat 1986).

The high-frequency range of more than 20 Hz was filtered using the Bessel function because the dominant mode of bridge considered is created at the low-frequency range. The results of the analysis of the filtered measurement data can be shown as various frequencies depending on the measurement responses, as shown in Fig. 5. The highest frequency was selected as the dominant frequency from the FFT analysis results because the highest frequency will be the major frequency in the vibration serviceability evaluation with respect to the vibration perception of the bridge users. The method used to obtain the amplitude of the bridge's vibration data that corresponds to the major frequency of the bridge is shown in Fig. 6.

**Table 2.** Bridges considered for measuring vibrations

Bridge	Structural type	Span length, m	Bridge	Structural type	Span length, m
J-bridge	continuous steel plate girder	50+50+50	YE-bridge	prestressed concrete girder	30+30
I-viaduct	continuous steel box girder	37+50+50+50+10	S-bridge	prestressed concrete girder	30+30
Y-bridge	continuous steel plate girder	50+50+50	K-bridge	Preflex girder	40+40
B-bridge	continuous steel box girder	30+30+30	W-bridge	continuous steel plate girder	40+50+50+50+40



**Fig. 4.** Examples of measuring point (W-bridge)

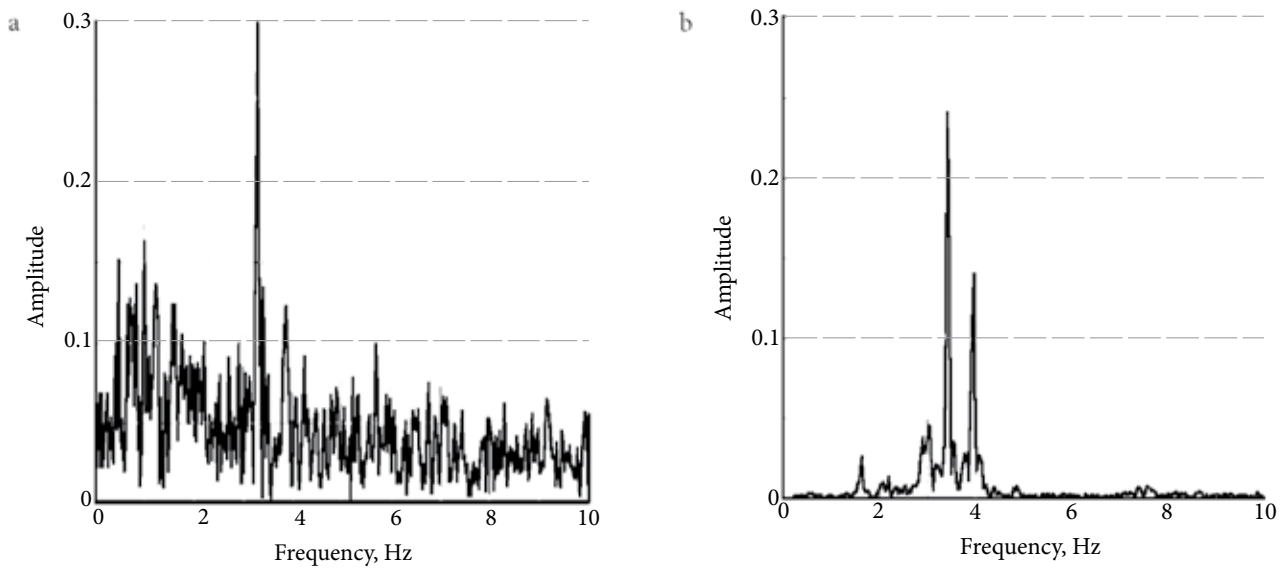


Fig. 5. FFT analysis: a – displacement; b – acceleration

The dominant frequency of the original data measured was obtained, then the period was obtained by taking the reciprocal of this frequency, and the max and min of amplitude values were obtained at the time interval of a half cycle. The amplitude was obtained from the data gathered by removing the noise from the micro-vibration using such method.  $L_{10}$  (90% upper limit) is used for the vehicle with many vibration sources and with an irregular size and vibration

time interval. This value was also determined and selected as the amplitude in the vertical direction in this study.

### 3.2. Characteristics of vibration perception in each bridge

Table 3 summarizes the results of the vibration measurements by level using the Reiher-Meister curve to evaluate the vibration serviceability of the bridge considered using

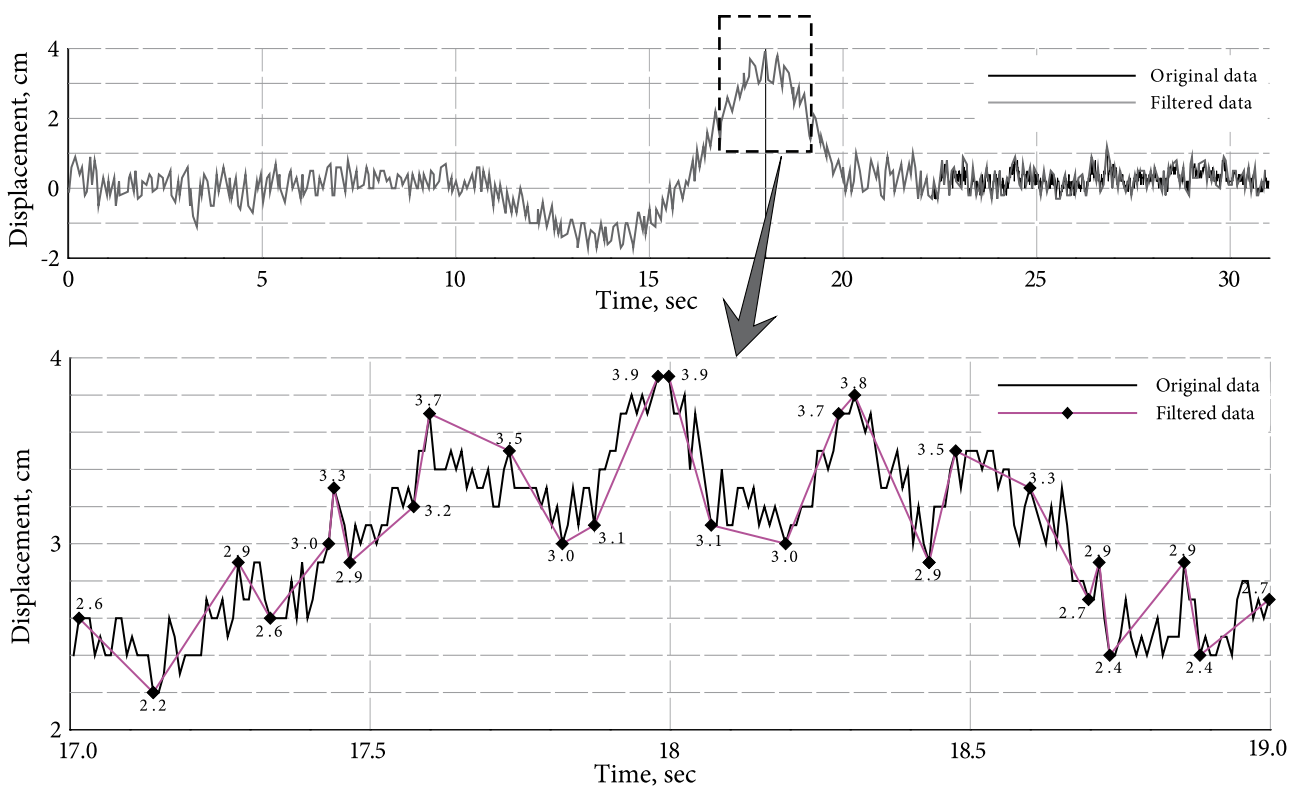


Fig. 6. Filtering method of vibration responses



the amplitude of acceleration and displacement, which were measured at the bridge under a public traffic load. These results were obtained from analysis of the data measured in the field test for each bridge as shown in Fig 7.

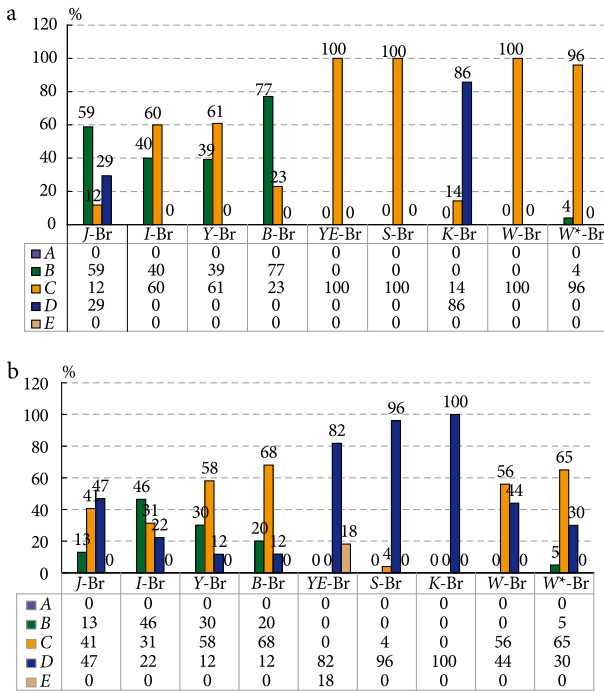


Fig. 7. Perception criteria of vibration: a – displacement; b – acceleration

The experimental results showed that the distribution of the vibration perception level slightly differed depending on the bridge considered, whereas, in general, the acceleration was better than the displacement in the results of the evaluation of the bridge’s vibration serviceability. The results of the evaluation of the max displacement and the max acceleration showed results similar to those of B (displeasure) in the cases of the B, I, W, and Y bridges. They, however, were about 1 level poorer in the acceleration evaluation criteria compared to the J, K, S, and YE bridges, the vibration problems of which were not so severe. Therefore, it would be reasonable to evaluate and determine the bridges’ vibration serviceability by measuring both their displacement amplitude and their acceleration amplitude. Moreover, care should be taken in determining the bridges’ vibration serviceability, when only the acceleration amplitude is measured without displacement.

The final evaluation of the bridge’s vibration serviceability was conducted using the Reiher-Meister’s vibration curve, as shown in Table 3, and the lower level between the vibration level of displacement and acceleration was determined as the level of vibration serviceability of the target bridge. If the vibration level of the bridge was level B or poorer, it was determined as having a problem with its vibration serviceability; and if the vibration level was level C or better, it was determined as having no problem with its vibration serviceability.

Table 3. Evaluation of serviceability of bridge

Bridge	Acceleration	Displacement	Evaluation level
B-bridge	B	B	B
I-viaduct	B	B	B
W-bridge	B	B	B
Y-bridge	B	B	B
J-bridge	D	C	C
K-bridge	D	C	C
S-bridge	D	C	C
YE-bridge	D	C	C

### 3.3. Analysis of experimental results with test vehicles

Both the vibration serviceability evaluation for a public traffic load and using the test vehicle load were conducted for W-bridge among the bridges of this study. The analysis of the measurement results for the dynamic load test using the test vehicle was conducted by weight (200, 250, and 287 kN), by velocity, and by traffic direction. The measurement results are shown in Figs 8 and 9. The case for which measurement was made for the traffic along the lane, where a gauge was installed, was indicated as “in”, and the case for the traffic along the lane opposite that where a gauge was installed was indicated as “out”. Fig. 8 shows a comparison of the data by weight and by velocity for the traffic along the lane, where a gauge was installed, and Fig. 9 shows a comparison of the data by weight and by velocity for the traffic along the lane opposite, where a gauge was installed. They include the measurement results for three groups in turn by velocity: 200, 250, and 287 kN. The perception criteria for the displacement and the acceleration were the Reiher-Meister standard for a 3.5 Hz dominant frequency of the bridge considered.

Based on the overall tendency for the displacement and acceleration in Figs 8 and 9, the displacement was almost constant, without a significant effect on the velocity variation, whereas the displacement amplitude slightly increased as the vehicle weight increased. The displacement amplitudes of bridge considered by the test vehicle that travel along the lane opposite that where a gauge was installed and of the test vehicle that ran along the lane where a gauge was installed were similar, and were different only in their absolute deflections. The acceleration amplitude slightly decreased at the 40 km/h velocity, though. Generally, there was little significant relationship between the velocity and the weight of the vehicle, whereas the amplitude varied depending on the velocity. It is thus considered reasonable to measure the values of various velocities when using the test vehicle. The normal vehicles were not controlled due to difficulties in controlling the vehicles that ran successively, while the test vehicle was running. Whether or not there were successive public vehicles during measuring the vibration response is indicated in Figs 8 and 9. The characteristics of the successively moving vehicles, generally passenger cars, were not analyzed, however, because they are not heavy vehicles that can affect the dynamic response of the bridge.

In addition, the vibration response of the heavy vehicles to public traffic, when no test vehicle was crossing the bridge, was analyzed and compared with the response of the test vehicle to determine the variations in the bridge's vibration characteristics due to the use of the test vehicle.

Figs 10 and 11 compare the vibration responses of the test vehicle with a weight of 250 kN and under a public traffic of heavy vehicles. The vibration characteristics due to public traffic were slightly lower than those of the test vehicle but generally showed similar tendencies, as shown in the Figs.

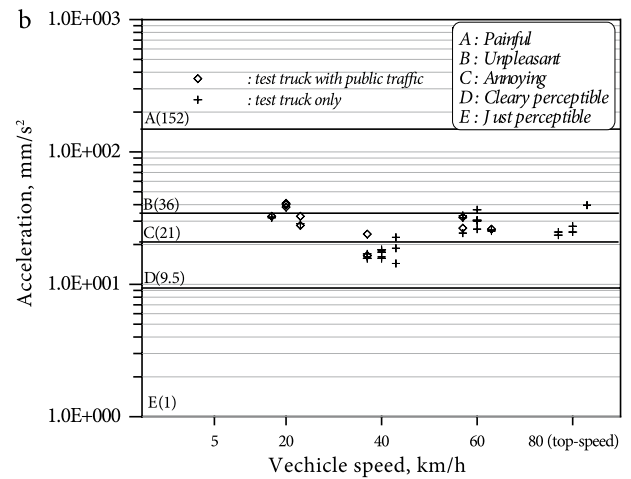
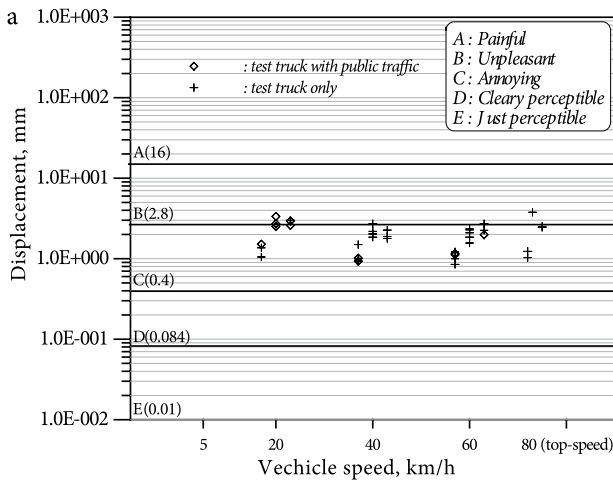


Fig. 8. Max amplitude from dynamic loading test (in-case): a – displacement; b – acceleration

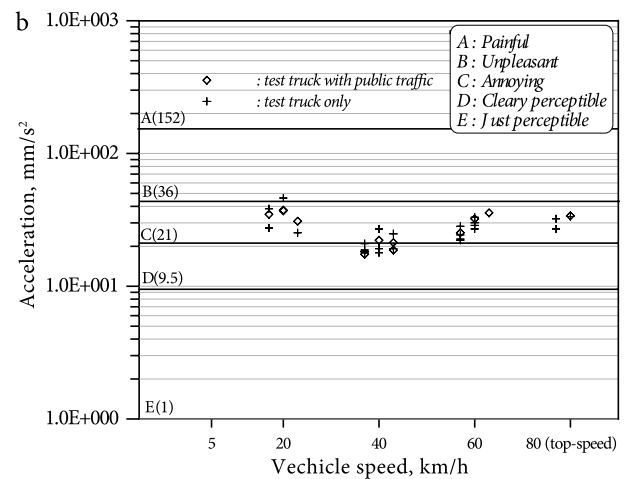
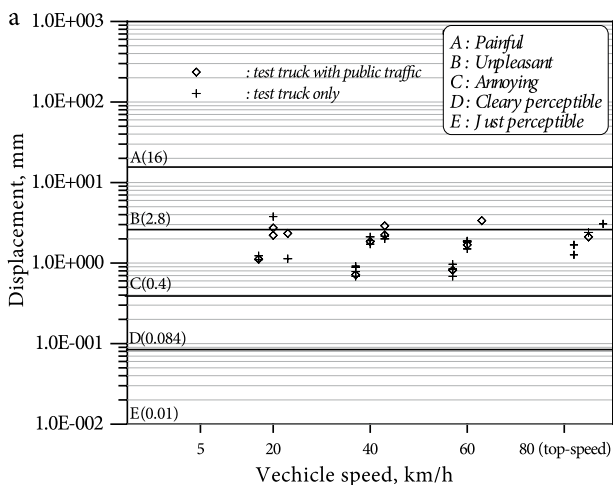


Fig. 9. Max amplitude from dynamic loading test (out-case): a – displacement; b – acceleration

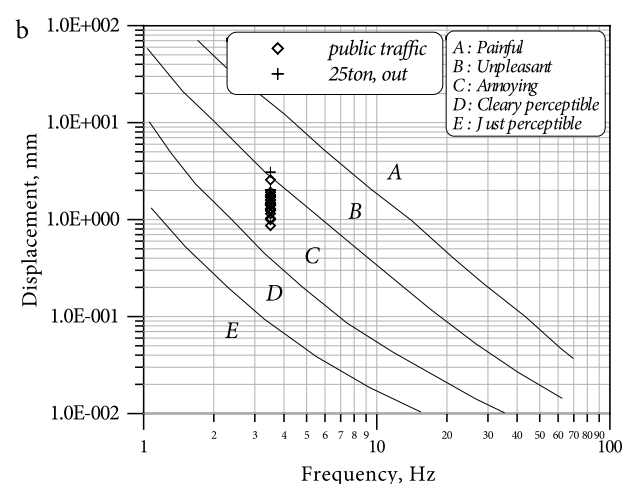
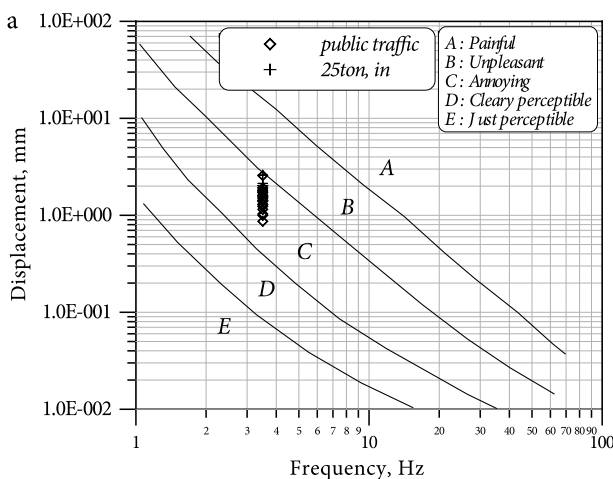


Fig. 10. Comparison with public traffic and test-vehicle (displacement): a – in; b – out

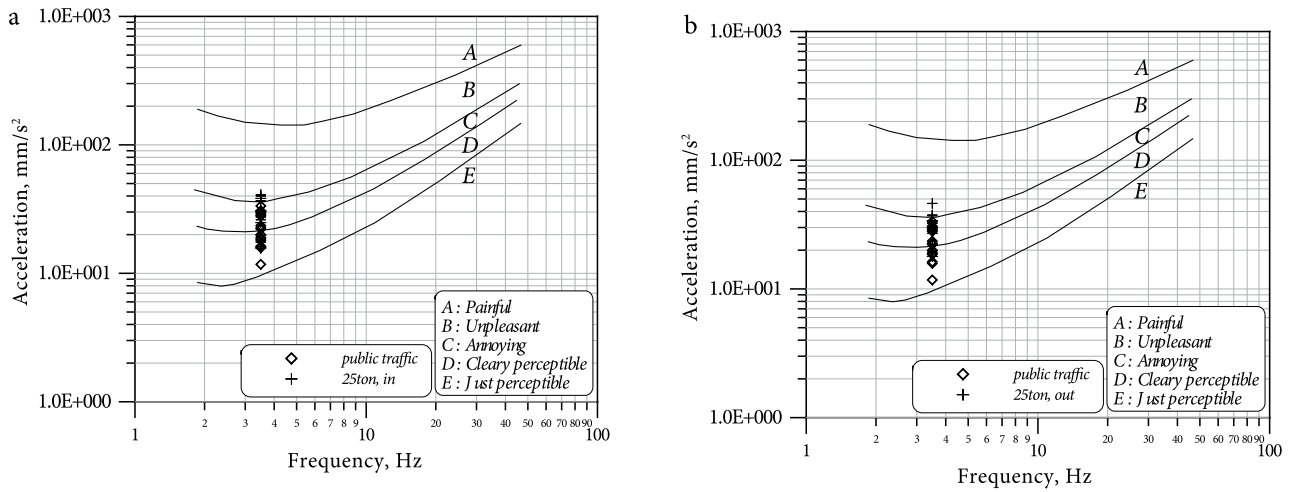


Fig. 11. Comparison with public traffic and test-vehicle (acceleration): a – in; b – out

4. Parametric study with the analytical method

A dynamic analysis was conducted and a comparison was made using the moving vehicle model for the W-bridge, which is a 5-span continuous bridge with 4 girders, by using the test vehicle to analytically determine the factor influencing the vibration serviceability of the bridge (Cheng, Y. M. and Leu, S. S. 2008; Cheng, Y. M. et al. 2009), as shown in Table 2. The W-bridge was modeled using the 3D frame element that can determine all bending, shear, and torsion (Wang et al. 1992), as shown in Fig. 12c. With respect to the moving vehicle load, as shown in Fig. 12d, the 3D 3-axle truck that was modeled to have 8 degrees of freedom was used. The road roughness of the 2 rows on both the right and left sides were assumed to be independent to allow for an analysis of that. Using the dynamic analysis model shown below, the displacement response and the acceleration response of the bridge were analyzed (Biggs 1982; Harris 1995).

4.1. Characteristics of vibrations according to vehicle velocity and surface roughness

Studies were conducted by other researchers to determine the vibration characteristics of a W-bridge and a steel plate girder bridge (Škaloud et al. 2005; Witzany et al. 2007) with respect to the velocity of the vehicle and the road roughness of the bridge, which are believed to affect bridge vibration. The velocity of the vehicle was increased by 20 km/h, from 40 to 60 km/h. In addition, the road roughness was classified into 3 levels for the analysis: very good, good, and average. The max value in displacement and acceleration increases considerably as the moving velocity increases, as shown in Fig. 13. In addition, although the vibration response showed no significant difference when the road roughness was good, the vibration characteristics increased when the surface roughness was 1 level lower (Dodds, Robson 1973). Generally, the vibration response was higher as the velocity increased and as the road roughness became poorer.

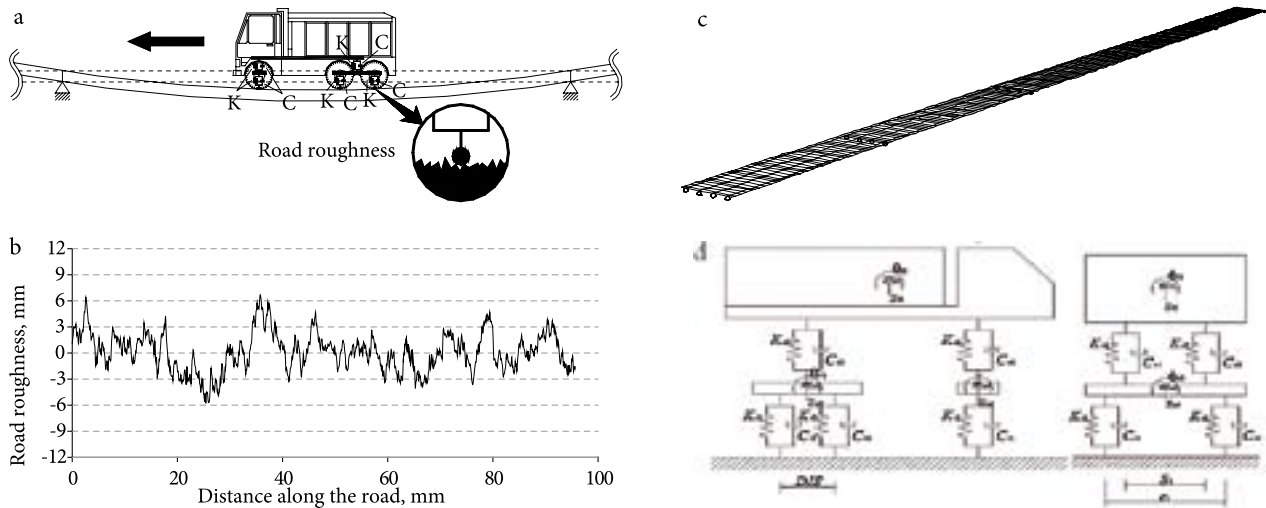


Fig. 12. Analysis model: a – modeling of vehicle–bridge interaction; b – road roughness; c – modeling of test bridge; d – modeling of test vehicle



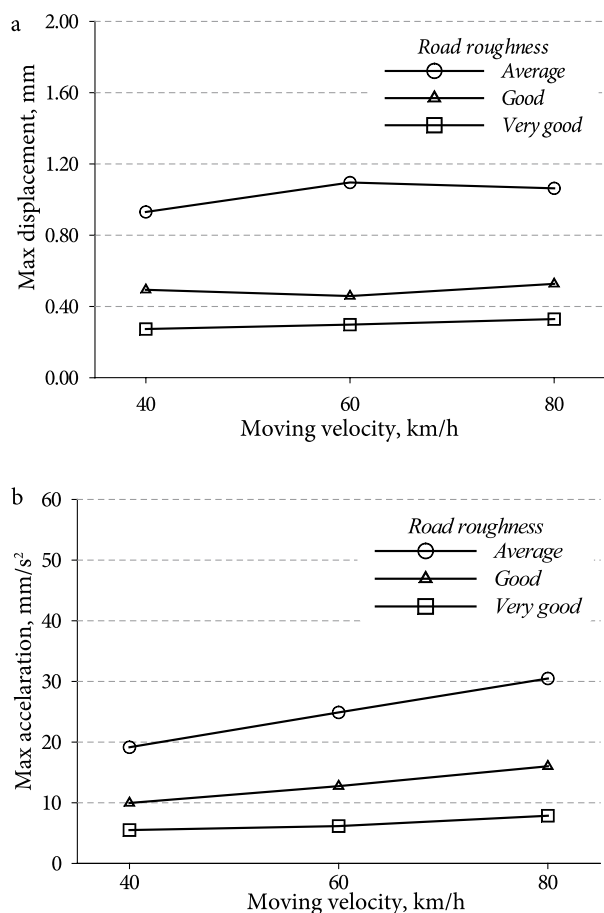


Fig. 13. Characteristics of vibration according to moving velocity and road roughness: a – displacement; b – acceleration

#### 4.2. Characteristics of vibration according to vehicle weight

The weight of the vehicle was varied from 200, 250, and 287 kN depending on the velocity to allow for an analysis of the variation of the displacement response and the acceleration response depending on the weight of the moving vehicle. The max amplitude of the displacement shows a slight reversal depending on the velocity, but gradually increased as the velocity increased, as shown in Fig. 14, and showed no significant variation depending on the weight of the vehicle, as seen in the results of the field test.

#### 5. Conclusions

The vibration serviceability of the bridge was evaluated by the field test and dynamic analysis. Based on the vibration serviceability evaluation method used in this study, its results and assessment procedure are proposed in Fig. 15.

1. It was identified that the vibration serviceability evaluation using the test vehicle and public traffic showed very similar results. According to the comparison of the vibration characteristics of the bridge at the position, where a gauge was installed and at the position, opposite that where a gauge was installed when a vehicle was crossing the bridge, the latter were slightly

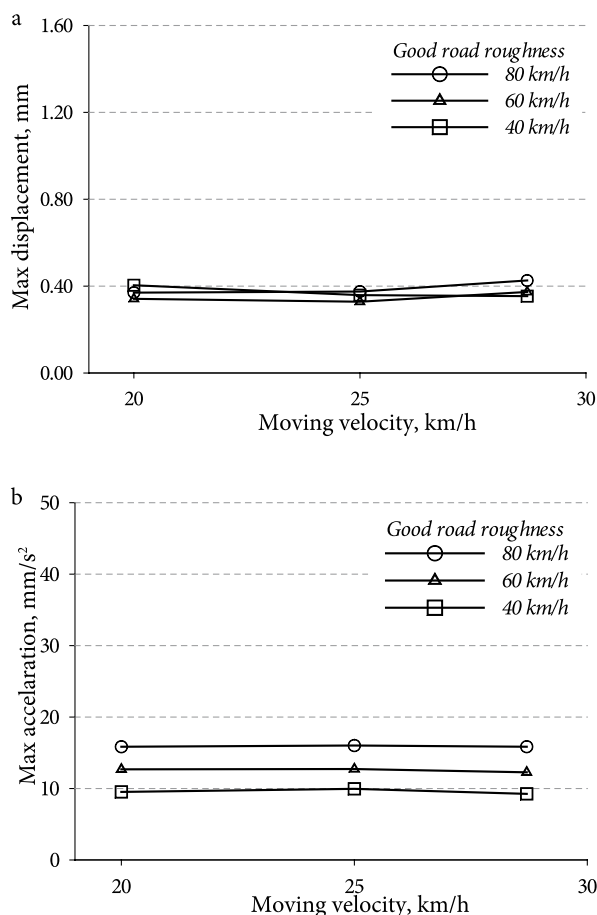


Fig. 14. Characteristics of vibration according to vehicle weight: a – displacement; b – acceleration

more significant, but both showed similar tendencies on the whole.

2. All together, the bridge's vibration serviceability variation showed no specific relationship with the velocity of the test vehicle, but it is considered reasonable to analyze, as much as possible, the bridge's vibration characteristics for various velocities. Especially, according to the analysis results, the amplitude variation depending on the velocity was small, when the surface roughness was good, but the amplitude increased with the velocity, when the surface roughness was average. This means that a running test is necessary, especially under various velocities, when the surface roughness of the bridge considered is poor.
3. The deflection amplitude or the acceleration amplitude varied depending on the total weight of the test vehicle, whereas the vibration amplitude generally showed no significant variation within the range of 200–287 kN, the total weight of the dump truck that was used as the test vehicle in the safety evaluation. This is supported by the analytical results.
4. According to the results of the vibration response analysis depending on the measurement position on the bridge, i.e. for the vibration response of the lane of the test vehicle that crossed the bridge and for the vibra-

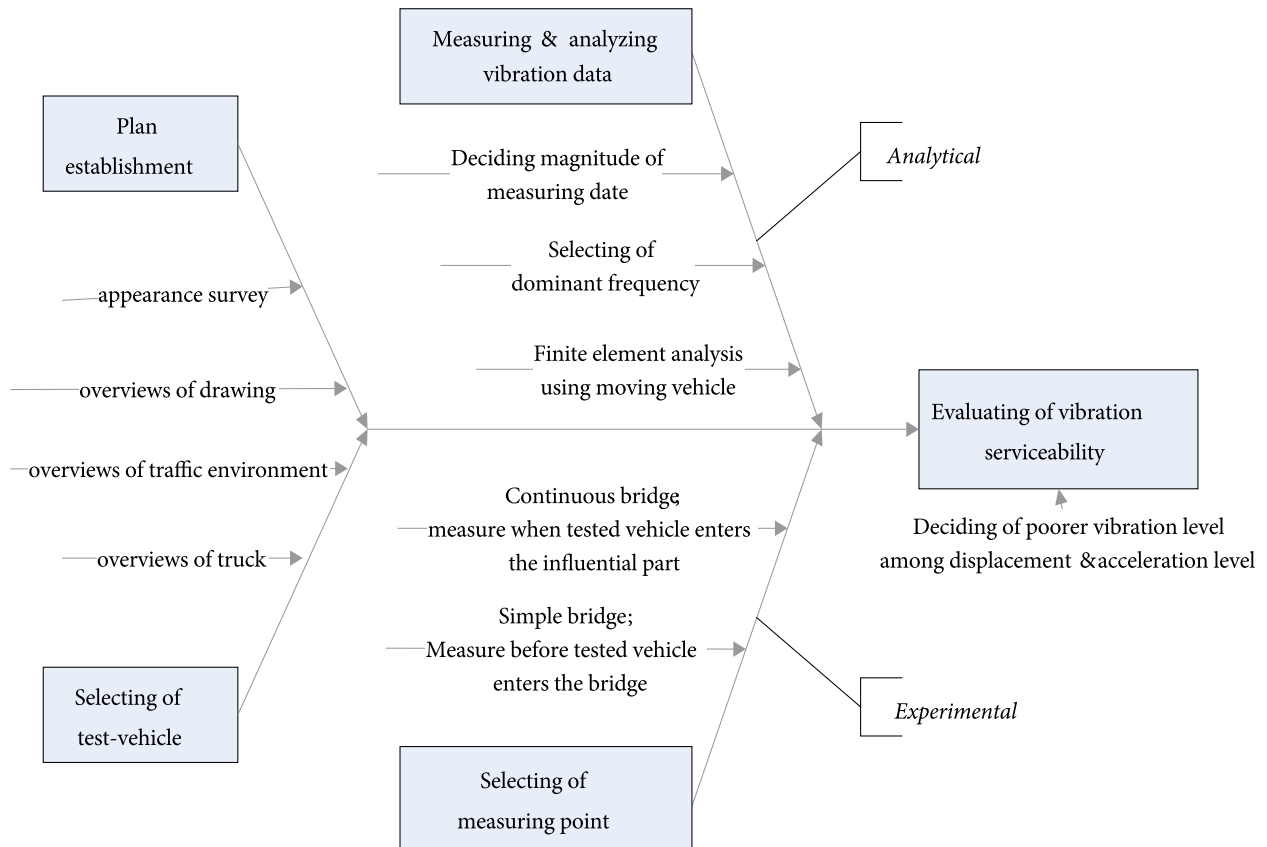


Fig. 15. Procedure for evaluating serviceability of bridges

tion response of the lane opposite which the test vehicle crossed, the vibration characteristics did not significantly vary depending on the measurement position, when the bridge surface roughness was good. The vibration response variation depending on the measurement position would not be significant because the road roughness of bridges in actual use generally is good. Thus, the Reiher-Meister curve suggested in this study is suitable to be considered for vibration serviceability criteria that can be expected in normal highways.

- The procedure of the serviceability assessment using analytical and experimental method is shown in Fig. 15: first of all, select the measuring points and analyze the vibration data measured with analytical method. Furthermore, the vibration serviceability problem in the bridge system using finite element method with the subspace iteration method is analyzed for comparing the experimental results. It is apparent that there is good agreement between the data. Thus an alternative way of solving the serviceability problems is judged accurately for assessing the vibration serviceability in the bridge. It is expected that this paper will be instrumental to practical use for effective assessment.

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