

THE STRUCTURAL SAFETY ASSESSMENT OF THE OVERLOADED
MEMBERS OF HIGHWAY BRIDGES

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Abstract. A residual reliability index of load-carrying structures of existing highway bridges overloaded by unforeseen vehicular live loads is discussed. The primary and inspection structural safety margins of particular members of bridges are considered. The primary and revised survival probabilities of intact particular members are analysed. Target reliability indices of the structural members of road bridges are presented. A transformed Bayes theorem in the analysis of revised failure probabilities of overloaded members of highway bridges is recommended. The Bayesian approach is combined with the method of transformed conditional probabilities for series systems. The simple calculation of revised reliability indices of particular members is demonstrated by a numerical example.

Keywords: transformed Bayes theorem, overloaded bridge, safety margin, revised reliability index.

1. Introduction

Many highway bridges were designed and built years ago for smaller vehicular loads those that they are incorporated in existing design codes and standards. Therefore, the behaviour of structural members (girders, slabs, piers) of these bridges should be inspected and controlled more carefully. This requirement is indispensable for bridges whose structural members and bearings have been overloaded by unforeseen exceptional action effects caused by abnormal extraordinary traffic loads of heavy industrial, construction, powerhouse and way equipments (Fig. 1).



Fig. 1. Extraordinary load

For successful ordinary and scheduled maintenance of existing bridges, it is expedient to know the revised values of residual reliability indices of their overloaded members and systems. Extreme action effects and mechanical properties of materials of bridge members confirmed by quality statistical information data may be treated as an effective measure in the revision of their reliability indices. For the sake of safe motor transport, these indices for overloaded structural members of existing bridges should be defined as exactly as it is possible in bridge engineering practice. However, the semi-probabilistic limit state analysis cannot be acknowledged as an universal and reliable method in the redesign of existing structures (Ellingwood 1996; Madsen 1987; Melchers 1999).

Probabilistic models help road and bridge engineers objectively calculate load ratings of bridge structures designed not only by limit state formats, but also by allowable stress and load factor approaches (LeBeau, Wadia-Fascetti 2007). However, a probability-based design of bridge structures may be acceptable to designers only under the indispensable and easy perceptible condition that they may be translated into reality using unsophisticated but quite exact mathematical approaches. Besides, the structural safety of overloaded bridge members should be quantified by the same models as in design stages.

In spite of fairly developed up-to-date concepts of reliability, hazard and risk theories, it is very difficult to implant probability-based methods in structural design and

redesign practice due to the shortage of methodological approaches and applied mathematical models.

The intention of this paper is to introduce structural bridge engineers and researchers to the new concept of the transformed Bayes theorem in the revised reliability assessment of the structures of highway bridges subjected to abnormal action effects caused by extraordinary loads.

2. Structural safety margins of particular members

The deck system of girder highway bridges is a significant part of their superstructures directly carrying the vehicular loads. The girders of deck systems can be classified into four groups consisting of precast or cast-in-situ concrete, steel and composite (steel and concrete) bending members (Fig. 2).

The system reliability may be much higher than the girder reliability. Therefore, the difference between the reliability indices of girders and deck systems may be considered as a measure of girder bridge redundancy. However, in any case the survival or failure probability of girders should be analysed and predicted.

The resistance of structural members (girders, slabs, piers) to bending, compression, tension and torsion is represented in design practice by the resistance of their particular members (normal or oblique sections and connections). The resistance of particular members of non-deteriorating bridge structures may be treated as a stationary process. The safety margin of particular members shall be presented as:

$$Z = g(\mathbf{X}, \boldsymbol{\theta}) = \theta_R R - \theta_G S_G - \theta_{Q_s} S_{Q_s} - \theta_{Q_e} S_{Q_e}, \quad (1)$$

where $\mathbf{X}$ and $\boldsymbol{\theta}$ – the vectors of basic and additional random variables which contain the design model uncertainties associated with resistance and action effects.

The probability distribution of the resistance, R , for concrete or composite (steel and concrete) and steel particular members may be modeled by Gaussian and lognormal distributions, respectively.

The probability distributions of permanent S_G and sustained live S_{Q_s} action effects caused by self-weight and surfacing weight loads are close to a normal distribution.

The model of live loads on a highway bridge is based on heavily loaded trucks (Czarnecki, Nowak 2008). The max live load effect may be caused by one single heavy truck on the bridge or the simultaneous presence of two or more trucks on the bridge (Bhattacharya 2008). The value of this effect depends on many parameters, including the span length, girder spacing and stiffness of structural members. The intensity of abnormal extraordinary load effects also depends on these factors.

According to Caprani *et al.* (2008) and Bhattacharya (2008), the probability distribution of extreme values of traffic load effects S_{Q_e} caused by multiple trucks on girder highway bridges is close to a lognormal distribution. This distribution law for action effects of girder decks is confirmed by Liu *et al.* (2009).

The dynamic load factor $\frac{Q_{dyn}}{Q_{st}}$ for two heavily loaded trucks traveling side-by-side may be taken equal to 0.10 with the coefficients of variation of static and dynamic live loads $\delta Q_{st} = 0.10$ – 0.18 and $\delta Q_{dyn} = 0.80$ (Eamon, Nowak 2004). Then, the coefficient of variation of live loads for road bridges may be expressed as:

$$\delta Q = \left[\delta^2 Q_{st} + \left(\frac{Q_{dyn}}{Q_{st}} \delta Q_{dyn} \right)^2 + 2 \delta Q_{st} \times \frac{Q_{dyn}}{Q_{st}} \delta Q_{dyn} \right]^{\frac{1}{2}} \approx 0.25$$
– $0.30. \quad (2)$

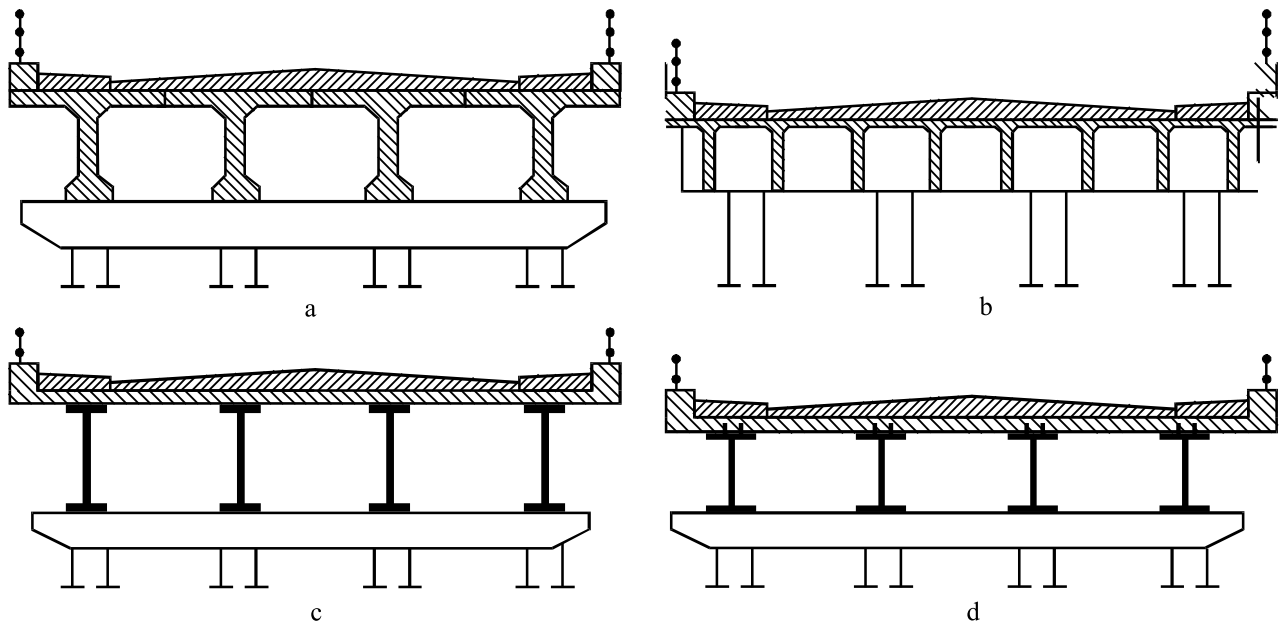


Fig. 2. Groups of the girders of deck systems: a – precast concrete; b – cast-in-situ concrete; c – steel; d – composite girder decks

Its value equal to 0.25 may be used in probability-based analysis of deck and pier structures of girder bridges (Kudzys, Kliukas 2008a).

The additional variables of a safety margin may be expressed by their means $\theta_{Rm} = \theta_{Gm} = \theta_{Qm} = 1.0-1.12$ and standard deviations $\delta\theta_R = \delta\theta_G = \delta\theta_Q = 0.05-0.14$ (Hong, Lind 1996; Stewart 2001; Vrouwenvelder 2002). The safety margin of particular members of girder span beams and slabs may be presented in the form:

$$Z_M = \theta_R M_R - \theta_G M_G - \theta_{Q_s} M_{Q_s} - \theta_{Q_e} M_{Q_e}, \quad (3)$$

where their resistance and action effect values are changed by resisting and destructive bending moments, respectively.

3. Survival probabilities of particular members

Both permanent and live loads may be treated as use-proven proof actions for structures of existing highway bridges (Hall 1998). According to this concept (Fig. 3a), the revised density function of a resistance of particular members may be presented as:

$$f_{R_r}(x) = \frac{f_R(x)F_{S_c}(x)}{\int_{-\infty}^{\infty} f_R(x)F_{S_c}(x) dx}, \quad (4)$$

where $f_R(x)$ – the density function of a member resistance R ; $F_{S_c}(x)$ – the cumulative multivariate distribution function of a conventional action effect written in the form:

$$S_c = \theta_G S_G + \theta_{Q_s} S_{Q_s} + \theta_{Q_e} S_{Q_e}. \quad (5)$$

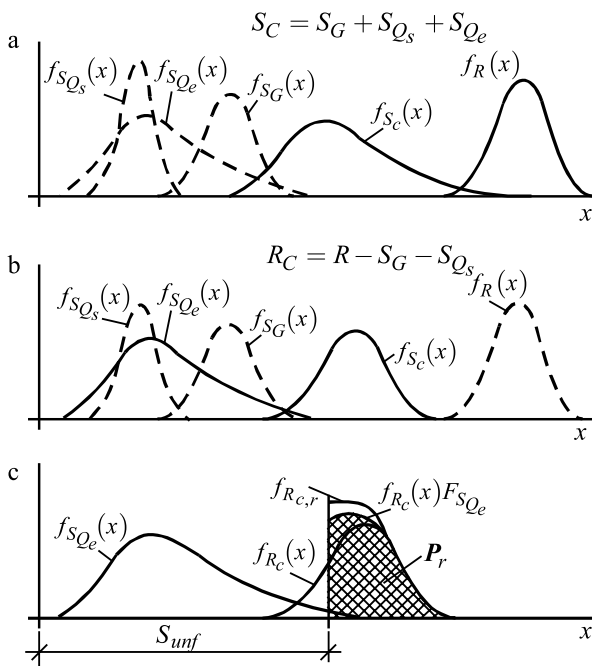


Fig. 3. Models for structural safety analysis of load-carrying members by the Hall (1998) (a) and the present author's (b, c) formulations

The design practice showed that it is better to use the revised conventional resistance, $R_{c,r}$ of particular members (Fig. 3b) the density function of which may be expressed as:

$$f_{R_{c,r}} = \frac{f_{R_c}(x)F_{S_{Q_e}}(x)}{\int_{-\infty}^{\infty} f_{R_c}(x)F_{S_{Q_e}}(x)dx}, \quad (6)$$

where the conventional resistance

$$R_c = \theta_R R - \theta_G S_G - \theta_{Q_s} S_{Q_s} \quad (7)$$

may be modeled by Gaussian or lognormal distributions.

According to Szwed *et al.* (2005), vehicular live load surcharge may be treated as an event caused by the deterministic unforeseen (usually static) action effect S_{unf} . Therefore, the revised safety margin of particular members from Eq (3) may be rewritten as:

$$ZMr = \theta_R M_R - \theta_G M_G - \theta_{Q_s} M_{Q_s} - \theta_{Q_{unf}} M_{Q_{unf}}. \quad (8)$$

When the particular member is overloaded by deterministic extraordinary action effect (Fig. 3c), the mean and variance of truncated normally distributed conventional resistance of any analysed member may be expressed as:

$$R_{c,rm} = R_{cm} + \lambda \sigma R_c, \quad (9)$$

$$\sigma^2 R_{c,r} = \sigma^2 R_c \left[1 + \lambda (\beta_{unf} - \lambda) \right], \quad (10)$$

where

$$R_{cm} = (\theta_R R)_m - (\theta_G S_G)_m - (\theta_{Q_s} S_{Q_s})_m \quad (11)$$

is the primary value of conventional member resistance

$$\beta_{unf} = \frac{\theta_{S_{unf}} S_{unf} - R_{cm}}{\sigma R_c} \quad (12)$$

is the standard normal distribution variable,

$$\lambda = \frac{\varphi(\beta_{unf})}{1 - \Phi(\beta_{unf})} \quad (13)$$

is the conventional factor, where $\varphi(\beta_{unf})$ and $\Phi(\beta_{unf})$ – the density and cumulative distribution functions of the variable β_{unf} by Eq (12). The statistics of additional variable $\theta_{S_{unf}}$ are: $\theta_{S_{unf}m} = \theta_{Q_{em}}$, $\sigma\theta_{S_{unf}} = 0$

The survival probabilities of intact particular members subjected to standard and unforeseen extreme live loads may be expressed respectively as:

$$P_S = P \{ R_c > \theta_s S_{Q_e} \} = \int_0^{\infty} f_{R_c}(x) F_{S_{Q_e}}(x) dx, \quad (14)$$

$$P_{s,r} = P \{ R_{c,r} > \theta_s S_{Q_e} \} = \int_0^{\infty} f_{R_{c,r}}(x) F_{S_{Q_e}}(x) dx, \quad (15)$$

where $f_{R_c}(x)$ and $f_{R_{c,r}}(x)$ – the density functions of primary and revised conventional member resistances R_c by

Eq (8) and $R_{c,r}$, the statistics of which are presented by Eqs (9) and (10).

The value of revised survival probability by Eq (14) helps us assess the residual reliability index of bridge members and their systems. This conventional measure of the reliability of members is their generalized reliability index as:

$$\begin{aligned}\beta &= \Phi^{-1}(\mathbf{P}_s) \text{ or } \beta = \Phi^{-1}(1 - \mathbf{P}_f), \\ \beta_r &= \Phi^{-1}(\mathbf{P}_{s,r}) \text{ or } \beta_r = \Phi^{-1}(1 - \mathbf{P}_{f,r}),\end{aligned}\quad (16)$$

where $\Phi^{-1}(\bullet)$ – the cumulative distribution function of the standard normal distribution.

According to EN 1990:2002 Eurocode: Basis of Structural Design, particular members of bridge structures may be designated in the same, higher or lower consequences class than for the entire bridge. The consequences of failure or malfunction of girder deck members may be associated with their reliability class RC2. Therefore, the target reliability index β_T of the particular and structural members of girder road bridges should be not less as 3.8.

According to AASHTO LRFD Bridge Design Specifications of 2007 (LRFDSI-4-E4) the acceptable $\beta = 3.5$ for most structural members of bridges. $\beta = 3.5$ –4.7 of girder bridge members designed by the Japanese Specification for Highway Bridges (Sugiyama, Yoshida 2008). $\beta_T = 4.0$ may be selected for bridge piers (Kudzys, Kliukas 2008b) and this value of the β_T may be selected for the structural members of girder decks of existing bridges.

4. Bayes theorem in revised reliability analysis

When additional information on overloaded existing structures is gathered, it might be applied to improve the primary their reliability indices using the Bayes theorem. According to Madsen (1987), the revised failure probability of particular members can be expressed as follows:

$$\mathbf{P}_{f,r} = \mathbf{P}\{g(\mathbf{X}, \boldsymbol{\theta}) < 0 | H\} = \frac{\mathbf{P}\{Z < 0 \cap H > 0\}}{\mathbf{P}\{H > 0\}}, \quad (17)$$

where Z – the random safety margin by Eq (1) and Eq (3); $H > 0$ – the event of visual and nondestructive inspection results showing a successful opposition of members to unforeseen extreme action effects.

The analysis of Eq (17) has disclosed that it is difficult to get the quantitative failure parameters of members revised due to some conditionalities of the event $H > 0$ and the correlation between primary Z and inspection H functions. The major disadvantage is the uncertainty of the analytical model representing stress-strain state of overloaded particular members (Ellingwood 1996; Melchers 1999). However, the revised failure probability of overloaded structural members and bearings of bridges may be defined combining the Bayesian approach with the method of transformed conditional probabilities (Kudzys, Lukoševičienė 2009).

The structural resistance R of concrete, steel and composite particular members can be based on either the yield or max strains of their reinforcing bars and profiled steels,

respectively, induced by permanent and long-term monitored live loads (Liu *et al.* 2009). Therefore, the capacity value for a member whose failure may be caused by permanent and single extraordinary live load should be significantly decreased. Thus, two safety margins of particular members should be considered as: Z by Eqs (1) or (3) and

$$H = \omega \theta_R R - \theta_G S_G - \theta_{Q_s} S_{Q_s} - S_{unf}, \quad (18)$$

as their informative or inspection safety margin, where $\omega = \frac{R_k}{R_m}$ – the ratio of characteristic and mean values of member resistance. The event of successful withstanding the overloading situation of members may be expressed as:

$$H_k = R_k - S_{Gk} - S_{Q_{sk}} - S_{unf} > 0, \quad (19)$$

when $S_{unf} \geq 1.2 S_{Q_{ek}}$ and $0.25 R_k \geq H_m$. These conditions on the min level of overloading action permits us threat S_{unf} as the informative proof load effect of considered member (Ellingwood 1996).

The statistical parameters of these safety margins are:

$$Z_m = \theta_{Rm} R_m - \theta_{Gm} S_{Gm} - \theta_{Q_{sm}} S_{Q_{sm}} - \theta_{Q_{em}} S_{Q_{em}}, \quad (20)$$

$$\begin{aligned}\sigma^2 Z &= \sigma^2 (\theta_R R) + \sigma^2 (\theta_G S_G) + \sigma^2 (\theta_{Q_s} S_{Q_s}) + \\ &\sigma^2 (\theta_{Q_e} S_{Q_e}),\end{aligned}\quad (21)$$

$$H_m = \omega \theta_{Rm} R_m - \theta_{Gm} S_{Gm} - \theta_{Q_{sm}} S_{Q_{sm}} - S_{unf} > 0, \quad (22)$$

$$\sigma^2 H = \omega^2 \sigma^2 (\theta_R R) + \sigma^2 (\theta_G S_G) + \sigma^2 (\theta_{Q_s} S_{Q_s}). \quad (23)$$

The covariance of these safety margins may be defined as:

$$\text{Cov}(Z, H) = \omega \sigma^2 (\theta_R R) + \sigma^2 (\theta_G S_G) + \sigma^2 (\theta_{Q_s} S_{Q_s}). \quad (24)$$

Then, the coefficient of correlation between safety margins Z and H is:

$$\rho_{ZH} = \frac{\text{Cov}(Z, H)}{\sqrt{\sigma^2 Z} \times \sqrt{\sigma^2 H}}. \quad (25)$$

When the probability distribution of the inspection safety margin H by Eq (18) is close to the normal distribution, the survival probability of considered particular members may be defined as:

$$\mathbf{P}\{H > 0\} = \Phi\left(\frac{H_m}{\sigma H}\right). \quad (26)$$

The bounded index of a series system Z – H may be expressed as:

$$x = \mathbf{P}\{H > 0\} \left[\frac{4.5}{(1 - 0.98 \rho_{ZH})} \right]^{\frac{1}{2}}. \quad (27)$$

The correlation factor of this system is equal to ρ_{ZH}^x , where the coefficient ρ_{ZH} is defined by Eq (25).

According to the method of transformed conditional probabilities (Kudzys, Lukoševičienė, 2009), the intersection probability of two random events $Z > 0$ and $H > 0$ is:

$$\begin{aligned} \mathbf{P}\{Z > 0 \cap H > 0\} &= \mathbf{P}\{Z > 0\} \times \mathbf{P}\{H > 0|Z > 0\} = \\ &= \mathbf{P}\{Z > 0\} \times \mathbf{P}\{H > 0\} \left[1 + \rho_{ZH}^x \left(\frac{1}{\mathbf{P}\{Z > 0\}} - 1 \right) \right] \end{aligned} \quad (28)$$

Therefore, the revised failure probability by Eq (17) may be transformed and rewritten as follows:

$$\begin{aligned} \mathbf{P}_{f,r} &= 1 - \frac{\mathbf{P}\{Z > 0 \cap H > 0\}}{\mathbf{P}\{H > 0\}} = \\ &= \frac{\mathbf{P}\{Z < 0\} \times \mathbf{P}\{H > 0\} (1 - \rho_{ZH}^x)}{\mathbf{P}\{H > 0\}} = \\ &= \mathbf{P}_f (1 - \rho_{ZH}^x) = (1 - \mathbf{P}_s) (1 - \rho_{ZH}^x), \end{aligned} \quad (29)$$

where the primary survival probability $\mathbf{P}_s$ is calculated by Eq (14). Then, the revised value of the β_T of a particular member is defined by Eq (16).

5. Numerical illustration

The β_T of overloaded but intact tee concrete beams of the existing girder highway bridge is considered. During the service of this bridge, its span beams were overloaded by the deterministic static bending moments $M_{unf} = 1920$ kNm ($= 1.5 M_{Qek}$) caused by unforeseen extraordinary traffic load.

The characteristic values and coefficients of variation of the destructive bending moments of bridge beams are:

$$M_{Gk} = 1160 \text{ kNm},$$

$$\delta M_G = 0.1;$$

$$M_{Qsk} = 424 \text{ kNm},$$

$$\delta M_{Qs} = 0.25;$$

$$M_{Qek} = 1280 \text{ kNm},$$

$$\delta M_{Qe} = 0.25.$$

The means and variances of these moments are:

$$M_{Gm} = 1160 \text{ kNm},$$

$$\sigma^2 M_G = (0.1 \times 1160)^2 = 13456 \text{ (kNm)}^2 \text{ (normal distribution);}$$

$$M_{Qsm} = 300 \text{ kNm},$$

$$\sigma^2 M_{Qs} = (0.25 \times 300)^2 = 5625 \text{ (kNm)}^2 \text{ (normal distribution);}$$

$$M_{Qem} = 880 \text{ kNm},$$

$$\sigma^2 M_{Qe} = (0.25 \times 880)^2 = 48400 \text{ (kNm)}^2 \text{ (lognormal distribution).}$$

According to Eurocode directions, the partial factors for actions $\gamma_G = \gamma_{Qs} = \gamma_{Qe} = 1.35$. Therefore, the design value of the joint bending moment is:

$$M_{Ed} = 1.35 \times (1160 + 424 + 1280) = 3866 \text{ kNm}.$$

Case 1. The statistics of the revised resisting bending moment of particular members of considered beams are:

$$M_{Rm} = 5588 \text{ kNm},$$

$$\delta M_R = 0.12,$$

$$\sigma^2 M_R = (0.12 \times 5588)^2 = 449650 \text{ (kNm)}^2,$$

$$\omega = 1 - 1.645 \times 0.12 = 0.8026,$$

$$M_{Rk} = 5588 \times 0.8026 = 4485 \text{ kNm},$$

$$\gamma_R = 1.15,$$

$$M_{Rd} = \frac{4485}{1.15} = 3900 \text{ kNm} > 3866 \text{ kNm} (= M_{Ed}). \text{ Thus,}$$

$$\omega = \frac{M_{Rk}}{M_{Rm}} = \frac{4485}{5588} = 0.8026.$$

The indispensable condition expressed by Eq (19) is satisfied because the informative safety margin

$$H = 4485 - 1160 - 424 - 1920 = 981 > 0.$$

Therefore, the β_T of beams can be analysed.

The means and standard deviations of the additional variables of beam safety margin are (Holicky, Markova 2007):

$$\theta_{Rm} = \theta_{Gm} = \theta_{Qsm} = \theta_{Qem} = 1.0,$$

$$\sigma\theta_R = \sigma\theta_G = \sigma\theta_{Qs} = \sigma\theta_{Qe} = 0.10.$$

Thus, the more exact statistics of resisting and destructive bending moments caused by permanent, sustained and extreme traffic loads may be expressed as:

$$(\theta_R M_R)_m = 5588 \text{ kNm},$$

$$\sigma^2 (\theta_R M_R) = 449650 + 5588^2 \times 0.01 = 761907 \text{ (kNm)}^2;$$

$$(\theta_G M_G)_m = 1160 \text{ kNm},$$

$$\sigma^2 (\theta_G M_G) = 13456 + 1160^2 \times 0.01 = 26910 \text{ (kNm)}^2;$$

$$(\theta_{Qs} M_{Qs})_m = 300 \text{ kNm},$$

$$\sigma^2 (\theta_{Qs} M_{Qs}) = 5625 + 300^2 \times 0.01 = 6525 \text{ (kNm)}^2;$$

$$(\theta_{Qe} M_{Qe})_m = 880 \text{ kNm},$$

$$\sigma^2 (\theta_{Qe} M_{Qe}) = 48400 + 880^2 \times 0.01 = 56144 \text{ (kNm)}^2.$$

Thus, the mean and variance of conventional resistance by Eq (6) are:

$$R_{cm} = 5588 - 1160 - 3000 = 4128 \text{ kNm},$$

$$\sigma^2 R_c = 761907 + 26910 + 6525 = 795342 \text{ (kNm)}^2.$$

The probability distributions of M_R , M_G , M_{Q_s} and M_{Q_e} are close to the normal and lognormal distributions, respectively. Therefore, the primary value of the survival probability of the beam is equal to

$$\mathbf{P}_s = \mathbf{P} \{M_{R_c} > \theta_{Q_e} M_{Q_e}\} = 0.999754.$$

It corresponds to

$$\beta = 3.48 < 3.80 (= \beta_T)$$

and shows that the load-carrying capacity of the overloaded beam is insufficient.

According to Eqs (20)–(23), the means and variances of revised safety margins of normal sections as particular members of the considered precast concrete beams may be expressed as follows:

$$Z_m = 5588 - 1160 - 300 - 880 = 3248 \text{ kNm},$$

$$\sigma^2 Z = 761907 + 26910 + 6525 + 56144 = 851486 \text{ (kNm)}^2;$$

$$H_m = 4485 - 1160 - 300 - 1920 = 1105 \text{ kNm} > 0;$$

$$\sigma^2 H = 490810 + 26910 + 6525 = 524246 \text{ (kNm)}^2.$$

According to Eq (24), the covariance of these safety margins is:

$$\text{Cov}(Z, H) = 0.8026 \times 761907 + 26910 + 6525 = 644950 \text{ (kNm)}^2.$$

Therefore, their coefficient of correlation by Eq (25) is:

$$\rho_{ZH} = \frac{644950}{(\sqrt{851486} \times \sqrt{524246})} = 0.9653.$$

According to Eq (26), the probability

$$\mathbf{P} \{H > 0\} = \Phi \left(\frac{1105}{\sqrt{524246}} \right) = 0.93648.$$

Then, according to Eq (27), the bounded index of correlation factor, ρ_{ZH}^x , of two safety margins is:

$$x = 0.93648 \times \left[\frac{4.5}{(1 - 0.98 \times 0.9653)} \right]^{\frac{1}{2}} = 0.93648 \times 9.128 = 8.548.$$

Therefore, the correlation factor

$$\rho_{ZH}^x = 0.9653^{8.548} = 0.7394.$$

According to Eq (29), the revised failure probability of the beam the analysis of which is based on the Bayesian theorem is:

$$\mathbf{P}_{f,r} = (1 - 0.999754) \times (1 - 0.7394) = 0.000064.$$

It corresponds to the survival probability $\mathbf{P}_{s,r} = 0.999936$. Thus, with the revised information data the revised reliability index $\beta_r = 3.83 > 3.80 (= \beta_T)$. It shows that the structural safety of overloaded beams is sufficient. Evidently, the considered beams are safer than it was decided by the original procedures, when the unrevised reliability index was equal to 3.48.

Case 2. The statistics of destructive bending moments of considered particular members are the same as in Case 1. However, the statistics of their revised resisting moment are:

$$M_{Rm} = 5244 \text{ kNm};$$

$$\sigma^2 M_R = (0.12 \times 5244)^2 = 449650 \text{ (kNm)}^2;$$

$$M_{Rk} = 5224 \times 0.8026 = 4209 \text{ kNm};$$

$$M_{Rd} = \frac{4209}{1.15} = 3660 \text{ kNm} < 3866 \text{ kNm} (= M_{Ed});$$

$$M_{unf} = 1920 \text{ kNm}.$$

Case 3. All statistics are from Case 2 but the deterministic bending moment $M_{unf} = 2560 \text{ kNm} (= 2M_{Qek})$.

The data on analysis of overloaded bridge beams are given in Tables 1 and 2.

Table 1. Bending moments

Case	M_{Rm} , kNm	M_{Rk} , kNm	M_{Rd} , kNm	M_{Ed} , kNm	M_{unf} , kNm	$\frac{M_{unf}}{M_{Qek}}$
1	5588	4485	3900	3866	1920	1.5
2	5244	4209	3660	3866	1920	1.5
3	5244	4209	3660	3866	2560	2.0

Table 2. Correlation parameters and reliability indices

Case	$\frac{M_{Rk}}{4H_m}$	ρ_{ZH} by Eq (25)	ρ_{ZH}^x	β by Eq (16)	β_r by Eq (16)	β_T by EN
1	1.02	0.965	0.740	3.48	3.83	3.80
2	1.27	0.961	0.734	3.29	3.64	3.80
3	5.57	0.961	0.808	3.29	3.78	3.80

The data given in Tables 1 and 2 have corroborated that the transformed Bayes theorem may be used in the revised reliability analysis of existing bridge structures if their load effect levels were sufficiently high.

6. Conclusions

The values of unforeseen exceptional vehicular forces caused by abnormal traffic loads of heavy industrial, construction, powerhouse and way equipment may be successfully applied in the structural safety assessment of members of existing highway bridges. The revised survival or failure probabilities of bridge members overloaded in the sense of abnormal moving loads may be defined fairly unsophisticatedly using the concepts of Bayesian approaches and transformed conditional probabilities.

Only the revised values of reliability indices of bridge members help engineers having min appropriate skills and experience assess bridge structural quality and allow avoid both unexpected failures and unnecessary premature repairs of bridges. These values make it possible to assess structural quality of bridge members and their systems in the present time and a near future.

The new probability-based approaches and analysis formats proposed in this paper can be used by road and bridge engineers for the perfection of the maintenance strategy of existing highway bridges during their residual service life.

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