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FINITE ELEMENT MODEL UPDATING OF SENYUVA HISTORICAL ARCH BRIDGE USING AMBIENT VIBRATION TESTS

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Abstract. This paper describes a historical arch bridge; its analytical modelling, modal testing and finite element model (FEM) updating. For this purpose, Senyuva historical arch bridge which is built in 1696 and located in Camlihemsin, Rize, Turkey, is selected as an application. 3D FEM of the bridge is modelled using ANSYS software to obtain analytical dynamic characteristics such as natural frequencies and mode shapes. Then, operational modal analysis using ambient vibrations is performed to attain dynamic characteristics experimentally. Human walking and wind are used as natural excitation. The output-only modal parameter identification is carried out using the Peak Picking method in frequency domain and the Stochastic Subspace Identification method in time domain. After, analytically and experimentally identified dynamic characteristics compared with each other and FEM of the historical bridge is updated to minimize the differences between modal parameters by changing uncertain modelling parameters such as boundary conditions. As a result, differences between the natural frequencies are reduced from 27% to 3% and good agreement is found between analytical and experimental dynamic characteristics after FEM updating.

Keywords: ambient vibration, dynamic characteristics, finite element model (FEM) updating, historical arch bridge, modal testing, operational modal analysis.

1. Introduction

Historical bridges are one of the primary engineering structures constructed by people. There are a lot of historical bridges constructed in various sizes, styles and spans all over the world. Some of them are nearly as old as a couple of thousands years. These bridges are very important part of culture heritage of countries and they should be preserved well for the next generation (Mamaghani 2006).

1266 recorded historical bridges exist in Turkey built different areas and times. They were built for different purposes such as social and economical as well as strategic aims. In spite of the fact that these bridges were designed to carry only pedestrian and horse loads, in these days as a result of modern civilization most of them are used for traffic. For conversation, restoration and reinforcement of these structures, their structural behaviours need to be known well and so dynamics characteristics of them have to be identified. Consequently, well defined numerical analysis method for these kinds of structures is needed.

It is generally expected that finite element method FEM's based on technical design data and engineering judgments can yield reliable simulation for both the static and dynamic behaviour of bridges (Bayraktar *et al.* 2007a). However, during the analytical studies, there are some uncertainties such as material properties, boundary conditions and section areas in the design. For the uncertainties, dynamic characteristics can not be estimated by the finite element (FE) analysis. So, dynamic characteristics of structures (natural frequencies, mode shapes and damping ratios) have to be determined based on experimental methods. This procedure is called as Experimental Modal Analysis.

There are two fundamentally different methods available to experimentally identify the dynamic characteristics of a structure. These are Experimental Modal Analysis (EMA) and Operational Modal Analysis (OMA). In the EMA, the structure is excited by known input force such as impulse hammer, shakers and drop weight and response of the structure is measured. In the OMA, the structure is excited by unknown input force or ambient vibrations such as human walking, vehicle traffic, wind, wave and response of the structure is measured. Some heavy forced excitations become very expensive and sometimes may cause the possible damage for the structure and so as experimental method, OMA become more attractive (Roeck *et al.* 2000). In this study, OMA procedure is used to determine the dynamic characteristics of the historical arch bridge.

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Many studies exist about the historical bridges. Bayraktar *et al.* (2007b) determined the dynamic characteristics of Historical Sinik Bridge under ambient vibrations. FEM of the bridge was updated by changing the boundary conditions. Brencich and Sabia (2008) studied on the Tanaro Bridge with 18 span masonry bridge built in 1866. The bridge was investigated both in service conditions and at different stages of its demolition. The natural frequencies, mode shapes and damping ratios were identified by dynamic tests on the bridge. Ural (2005) carried out FE analysis of Cosandere historical arch bridge. In this study, natural frequencies were calculated using *SAP200* software. Also, earthquake analysis was done using El-centro ground motion record and max principal stresses are obtained. Toker and Unay (2004) studied about mathematical modelling techniques on a prototype model of a common arch bridge under different loading conditions. Frunzio *et al.* (2001) investigated the 3D FEM analysis of a stone masonry arch bridge, involving non-linear material behaviour, in which the structural role of the spandrel walls and filling were involved. Beside these studies many studies exist about the high-cyclic behaviour of concrete bridges (Dulinskas *et al.* 2008), shape stabilization of steel suspension bridge (Juozapaitis *et al.* 2008), assessment of quality in bridges and road construction (Zavadskas *et al.* 2008; Zavadskas 2008), limit state and probabilistic analysis of bracing piers and concrete columns (Kudzys, Kliukas 2008, 2009; Mazzolani *et al.* 2009), dynamic investigation of historical heritage (Kliukas *et al.* 2008) and complex assessment methodology in bridges (Sivilevicius *et al.* 2008).

In this study, modal testing and FEM updating of a historical arch bridge are studied with detail. In the first part of the study, formulations of modal parameter extraction methods are given. Then FE analysis of the bridge to identify analytical dynamic characteristics is described. After, modal testing conducted on the bridge is described and analytically and experimentally identified dynamic characteristics compared with each other. Then, 3D FEM of the bridge is updated by changing boundary conditions.

2. Formulation

Ambient excitation does not lend itself to Frequency Response Function (FRFs) or Impulse Response Function (IRFs) calculations because the input force is not measured in an ambient vibration test. Therefore, a modal identification procedure will need to base itself on output-only data (Ren *et al.* 2004). There are several modal parameter identification techniques. These techniques are developed by improvements in computing capacity and signal processing procedures. In this study, two different methods, which are rather simple Peak Picking (PP) method in the frequency domain and the Stochastic Subspace Identification (SSI) method in the time domain, are used for modal parameter extraction.

2.1. Peak Picking method

The idea of the PP method is to perform an approximate decomposition of the system response into a set of independent, single degree of freedom (SDOF) systems, one for each mode (Brincker *et al.* 2000). The decomposition occurs by simply decomposing each of the estimated spectral density matrices, which show the singular values are the estimates of the auto spectral density of the SDOF systems, and the singular vectors are the estimates of the mode shapes.

The PP technique has some theoretical drawbacks such as (Peeters 2000):

– picking the peaks is always a subjective task;
– operational deflection shapes are obtained instead of mode shapes;
– only real modes of proportionally damped structures can be deduced by the method;
– damping estimates are unreliable.

In spite of these drawbacks, the method is often used in civil engineering practice for ambient vibration measurements due to its implementation simplicity and its speed.

The relationship between the input $x(t)$ and the output $y(t)$ can be written (Rainieri *et al.* 2007; Felber 1993):

$$\left[G_{yy}(\omega)\right] = \left[H(\omega)\right]^{*}\left[G_{xx}(\omega)\right]\left[H(\omega)\right]^{T}, \tag{1}$$

where G_{yy} – the Power Spectral Density (PSD) matrix of the output; H – the Frequency Response Function (FRF) matrix; G_{xx} – the PSD matrix of the input; * and T denote complex conjugate and transpose respectively. After some mathematical manipulations the output PSD can be reduced to a pole/residue form as follows (Rainieri *et al.* 2007; Felber 1993):

$$\left[G_{yy}(\omega)\right] = \sum_{k=1}^{m}\left(\frac{[A_k]}{j\omega-\lambda_k} + \frac{[A_k]^{*}}{j\omega-\lambda_k^{*}} + \frac{[B_k]}{-j\omega-\lambda_k} + \frac{[B_k]^{*}}{-j\omega-\lambda_k^{*}}\right), \tag{2}$$

where A_k – the k^{th} residue matrix of the output PSD. The response spectral density matrix can be written in the following final form considering a lightly damped system:

$$\left[G_{yy}(\omega)\right] = \sum_{k=Sub(\omega)}\left(\frac{d_k\psi_k\psi_k^T}{j\omega-\lambda_k} + \frac{d_k^{*}\psi_k^{*}\psi_k^T}{j\omega-\lambda_k^{*}}\right), \tag{3}$$

where d_k – a scalar constant; ψ_k – the k^{th} mode shape vector. Thus, performing the singular value decomposition of the output PSD matrix known at discrete frequencies $\omega - \omega_i$ one obtains:

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H, \tag{4}$$

where the matrix U_i – a unitary matrix holding the singular vector u_{ij}; S_i – a diagonal matrix holding the scalar singular values s_{ij}; the superscript H denotes complex conjugate and transpose. Near a peak corresponding to the k^{th} mode in the spectrum, only the k^{th} mode is dominant, and the PSD matrix approx to a rank one matrix as:

$$\hat{G}_{yy}\left(j\omega_i\right) = s_i u_{i1} u_{i_1}^H, \quad \omega_i \to \omega_k. \tag{5}$$

The first singular vector at the r^{th} resonance is an estimate of the r^{th} mode shape:

$$\hat{\phi}_r = u_{r1}. \tag{6}$$

2.2. Stochastic subspace identification method

SSI is an output-only time domain method that directly works with time data, without the need to convert them to correlations or spectra. The method is especially suitable for operational modal parameter identification, but it is an incredibly difficult procedure to explain in detail in a short way for civil engineers.

The model of vibration structures can be defined by a set of linear, constant coefficient and 2nd order differential Eq (Peeters, De Roeck 2000):

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = F(t) = Bu(t), \tag{7}$$

where M, C, K – the mass, damping and stiffness matrices; $F(t)$ – the excitation force; $U(t)$ – the displacement vector at continuous time t. Observe that the force vector $F(t)$ is factorised into a matrix B describing the inputs in space and a vector $u(t)$. Although Eq (7) represents quite closely the true behaviour of a vibrating structure, it is not directly used in SSI methods. So, the Eq of dynamic equilibrium (7) will be converted to a more suitable form: the discrete-time stochastic state-space model (Peeters, De Roeck 1999). The state-space model originates from control theory, but it also appears in mechanical/civil engineering to compute the modal parameters of a dynamic structure with a general viscous damping model (Ewins 1984). With the following definitions,

$$x(t) = \begin{pmatrix} U(t) \\ \dot{U}(t) \end{pmatrix},$$

$$A_c = \begin{pmatrix} 0 & I_{n2} \\ -M^{-1}K & -M^{-1}C \end{pmatrix}, \tag{8}$$

$$B_c = \begin{pmatrix} 0 \\ M^{-1}B \end{pmatrix}.$$

Eq (7) can be transformed into the state Eq:

$$\dot{x}(t) = A_c x(t) + B_c u(t), \tag{9}$$

where $x(t)$ – the state vector; A_c – the state matrix; B_c – the input matrix. The number of elements of the state-space vector is the number of independent variables needed to describe the state of a system.

If it is assumed that the measurements are evaluated at only one sensor locations, and that these sensors can be accelerometers, velocity or displacement transducers, the observation Eq is (Juang 1994):

$$y(t) = C_a\ddot{U}(t) + C_v\dot{U}(t) + C_d U(t), \tag{10}$$

where $y(t)$ – the outputs; C_a, C_v and C_d – the output matrices for displacement, velocity, acceleration. With the following definitions:

$$\left.\begin{aligned} C &= \left[C_d - C_a M^{-1}K \quad C_v - C_a M^{-1}C_2\right] \\ D &= C_a M^{-1} B_2 \end{aligned}\right\}. \tag{11}$$

Eq (10) can be transformed into:

$$y(t) = Cx(t) + Du(t), \tag{12}$$

where C – the output matrix; D – the direct transmission matrix. Eqs (9) and (12) constitute a continuous-time deterministic state-space model. After sampling, the state-space model looks like (Peeters 2000):

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\}, \tag{13}$$

where $x_k = (k\Delta t)$ – the discrete-time state vector; $A = \exp(A_c\Delta t)$ – the discrete state matrix; $B = \left[A - I\right]A_c^{-1}B_c$ – the discrete input matrix. If A_c is not invertible, another expression holds for B. The stochastic components (noise) are included and obtained the following discrete-time combined deterministic-stochastic state-space model:

$$\left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k \\ y_k &= Cx_k + Du_k + v_k \end{aligned}\right\}, \tag{14}$$

where w_k – the process noise due to disturbances and modelling inaccuracies; v_k – the measurement noise due to sensor inaccuracy. They are both immeasurable vector signals but assumed that they are 0 mean, white and with covariance matrices:

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix}\begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}\delta_{pq}, \tag{15}$$

where E – the expected value operator; δ_{pq} – the Kronecker delta.

The vibration information that is available in structural health monitoring is usually the responses of a structure excited by the operational inputs that are some immeasurable inputs. Due to the lack of input information it is impossible to distinguish deterministic input u_k from the noise terms w_k, v_k in Eq (14). If the deterministic input term u_k is modelled by the noise terms w_k, v_k the discrete-time purely stochastic state-space model of a vibration structure is obtained (Yu, Ren 2005):

$$\left.\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned}\right\}. \tag{16}$$

Eq (16) constitutes the basis for the time-domain system identification through operational vibration measurements. The SSI method identifies the state-space matrices based on the output only measurements and by using robust numerical techniques.

3. Finite element modelling of the bridge

A district of Rize province in the Eastern Black Sea, Çamlıhemşin is located in a valley 300 m high from the sea. It connects to the coast with a 22 km long road. Due to the landscape, the streams fall from 3000 m to the sea level within 70 km. One example is the Storm Valley which is named after the Storm Creek that runs through the district Centrum. With its length of 57 km, Storm Creek is a suitable route for rafters with fast flow and high volume. It has thus become an attraction for national and international rafters. Along its travel through tea gardens and wooded islands, unique arch bridges ornament the creek, presenting a wonderful landscape of peace and harmony. Of the well-know meadows, especially Ayder has become a tourist attraction with the architecture of the prairie homes, its thermal springs, and heliski activities underway since 2004. Some views from Çamlıhemşin, Ayder, Fırtına Valley and arch bridges on Fırtına Creek are shown Fig. 1.

In this study, Şenyuva historical arch bridge built in 1696 by native population is selected as an example. This bridge is located on Fırtına Stream in Çamlihemşin, Rize, Turkey. Bridge arch and parapets are made from cut stone. The main structural elements of bridge (stone arch, side walls and timber block) have not been repaired yet. Only, parapets have been repaired and a metal railing added in 1997. Some views of Şenyuva Bridge are shown in Fig. 2.

The bridge has a single arch. The total span of bridge is 52.4 m, the span of bridge arch is 24.8 m, the height of the arch is 12.4 m and the wide of the deck is 2.5 m. Section properties of bridge are shown in Fig. 3. The historical bridge has a stone arch; radius and thickness of which is 12.4 m and 60 cm, respectively. In addition, thickness of side walls is 50 cm and there is a timber block between the side walls, wide of which is 1.5 m. Height of the side walls at both side 9.2 m and 3.5 m, respectively. There are 60×30 cm dimensional parapets on the both side of the bridge deck.

In the analysis of historical structures, selection of the material properties accurately is very important. Because of difficulties to determine the material properties of such kind of structures, similar studies in the literature

Fig. 1. Some views of Fırtına valley

Fig. 2. Some views of Şenyuva historical arch bridge

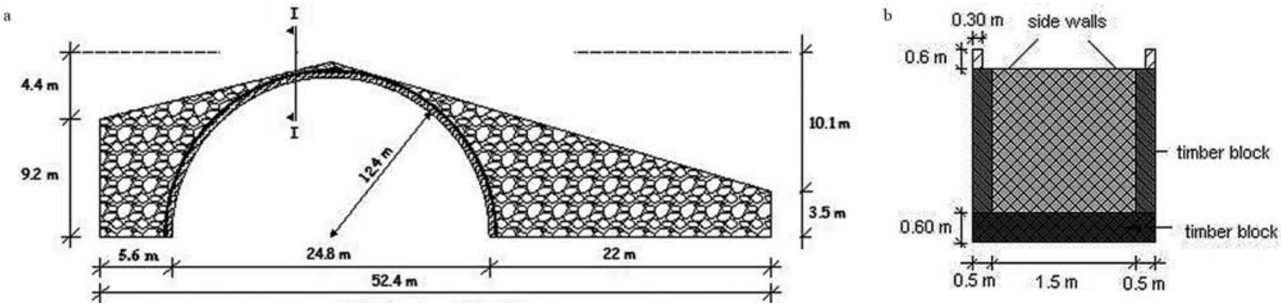

Fig. 3. Section properties of Şenyuva historical arch bridge: a – the plan view of the bridge; b – the section I-I

have been searched and material properties considered in analysis are determinated (Bayraktar *et al.* 2007b; Brencich, Sabia 2008; Toker, Unay 2004; Frunzio *et al.* 2001). The material properties considered in the analysis of the bridge are given in Table 1.

Table 1. Material properties considered in the analytical analysis

Material	Modulus of elasticity, MPa	Poisson ratio	Density, kg/m^3
Stone arches	3.0×10^3	0.25	1600
Timber block	1.5×10^3	0.05	1300
Side walls	2.5×10^3	0.20	1400

3D FEM of the bridge constituted using ANSYS software (ANSYS 2008) shown in Fig. 4. In the FEM, three dimensional solid elements have been used. As boundary conditions, all of freedoms under the bridge abutments and at the side walls are considered as fixed.

The first 5 natural frequencies are attained which range between 3–11 Hz. The first 5 vibration mode shapes of the bridge as a whole are shown in Fig. 5.

Fig. 4. FEM of Şenyuva historical arch bridge

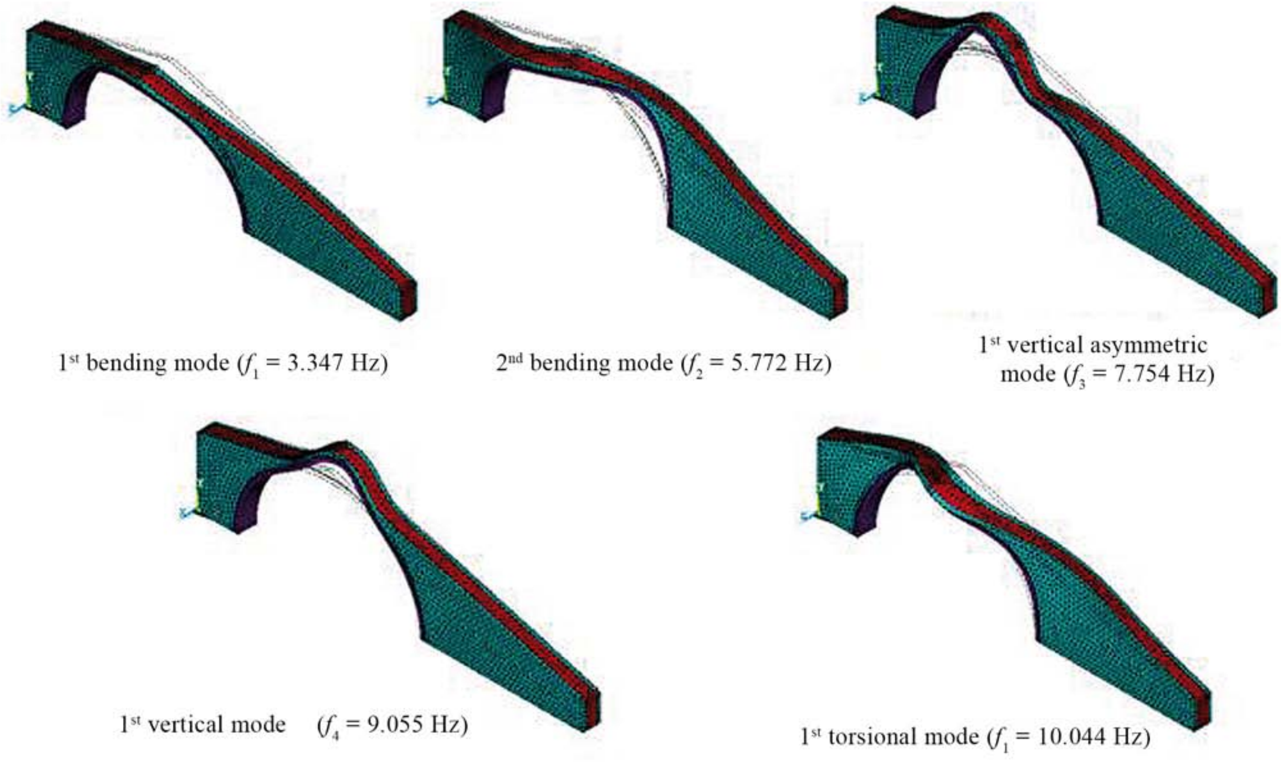

Fig. 5. Analytically identified first 5 mode shapes

4. Ambient vibration test of the bridge

Operational Modal Analysis is used to identify of the dynamic characteristics (natural frequencies, mode shapes and damping ratios) of Şenyuva historical arch bridge. The equipment used for the measurement includes 12 uniaxial accelerometers and 17-channel data acquisition system. To identify the dynamic characteristics of the bridge, structural responses at sufficient location on the deck in the vertical and horizontal directions are collected. The bridge arch span is divided into 2 pieces and accelerometers are placed at the sides and middle of the main arch span shown in Fig. 6.

The responses of bridge arch were measured by using B&K 8340 type uniaxial accelerometers. The min frequency span and sensitivity of these accelerometers are 0.1 Hz and 10 v/g, respectively. The signals were acquired in the B&K 3560 type data acquisition system and then transferred into the PULSE Lapshop software as shown in Fig. 7. For parameter estimation from the Ambient Vibration System data, the Operational Modal Analysis software (OMA) was used. The ambient vibration tests were conducted under environmental loads such as human walking and wind effects.

The Operational Modal Analysis is carried out by using the PP method in the frequency domain and (SSI) method in the time domain, and natural frequencies, mode shapes and damping ratios are determined.

Singular values of spectral density matrices, average of auto spectral density and stabilization diagram of estimated state space models of measurement setup attained from vibration signals using (PP) and (SSI) methods are shown in Fig. 8.

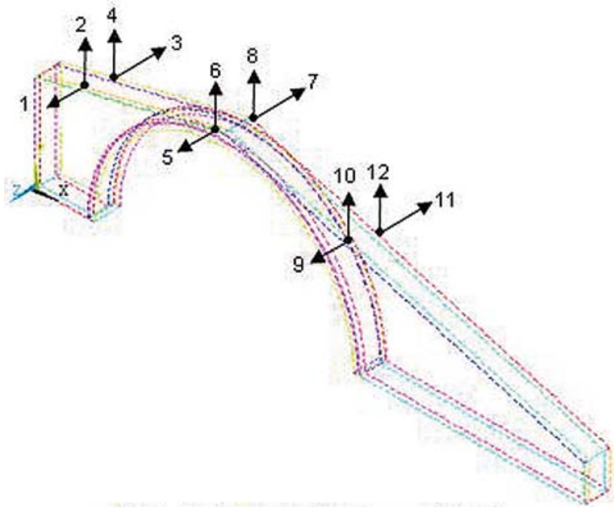

Fig. 6. Accelerometer locations on the bridge

Fig. 7. The pictures from ambient vibration test

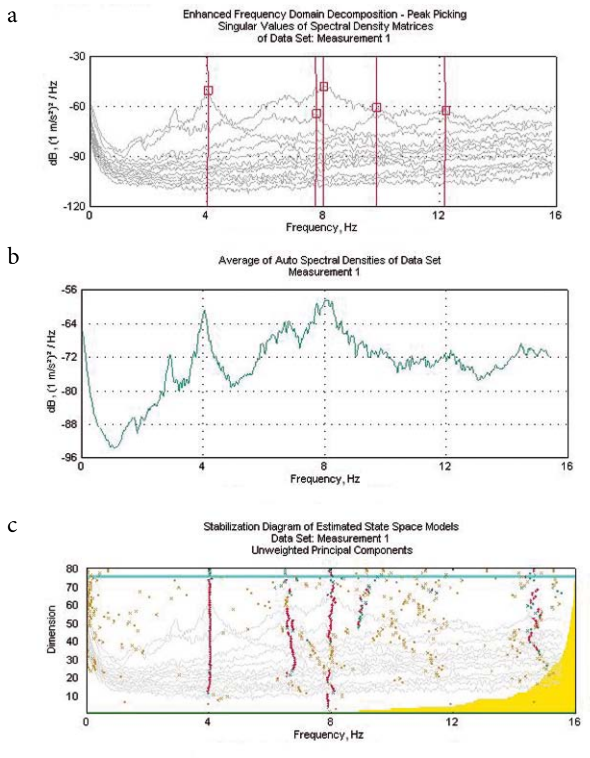

Fig. 8. Modal parameters attained using PP and SSI methods: a – singular values of spectral density matrices; b – average of auto spectral density; c – stabilization diagram of estimated state space models

The first 5 mode shapes extracted from experimental measurement are given in Fig. 9. Comparison of dynamic characteristics of Şenyuva historical arch bridge obtained from both analytical and experimental analysis is given in Table 2.

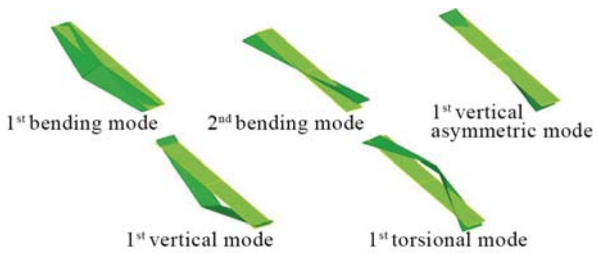

Fig. 9. Experimentally identified the 5 five mode shapes

Table 2. Comparison of analytically and experimentally identified dynamic characteristics

No.	Analytical Frequencies, Hz	Experimental		
		frequencies, Hz		Damping ratios, %
		PP	SSI	ζ
1	3.347	4.045	4.066	2.377
2	5.772	7.750	7.960	1.318
3	7.554	8.020	8.044	4.288
4	9.055	10.000	10.100	0.265
5	10.044	12.160	11.750	0.345

5. FEM updating of the bridge

It can be seen from Fig. 4 and Fig. 8 that there is a good agreement between the mode shapes. On the other hand there are some differences between natural frequencies given in Table 2. It is thought that these differences come from some uncertainties in the structural geometry such as material properties, boundary conditions and section properties. For that reasons, the FEM of the historical bridge must be updated.

Identification of material properties of historical structures using experimental methods is very difficult process. For this reason, material properties of these structures are taken from the literature. So the material properties of the arch bridge are not changed in the model updating. In this study, FEM of the bridge is updated by changing of the boundary conditions. For this purpose, stiffness on the side walls increased by the time is considered and freedoms of these regions at all directions are fixed. As it seen from Fig. 10, the stiffness is increased by the time due to piling up the earth, stone, rubbish and so on at the side walls of the bridge.

Fig. 10. Some view of stiffness at the side walls of the bridge

Frequency values obtained from experimental measurement and analytical analysis after the FEM updating are given in Table 3. When Table 3 is examined, it can be seen that there is good agreement between analytical and experimental natural frequencies after the model updating.

Table 3. Analytical and experimental frequencies after FEM updating

No.	Analytical		Experimental	
	Hz	updated, Hz	PP, Hz	SSI, Hz
1	3.347	4.070	4.045	4.066
2	5.772	7.780	7.750	7.960
3	7.554	8.020	8.020	8.044
4	9.055	10.670	10.000	10.100
5	10.044	12.090	12.160	11.750

6. Conclusions

In this paper, analytical modelling and modal testing of Şenyuva historical bridge is presented. 3D FEM of the bridge is updated to minimize the differences between analytical and experimental modal parameters by changing some uncertain modelling parameters such as boundary conditions. Comparing the results of study, the following observation can be made:

From the FEM of the bridge, the first 5 natural frequencies are attained analytically, which range between 3–11 Hz. Considering the first 5 mode shapes, these modes can be classified into bending, vertical and torsional modes.

The ambient vibration test is conducted under the natural excitations such as human walking and wind effects on the bridge for accurately extracting the dynamic characteristics using PP and SSI methods. From the ambient vibration test, the first 5 natural frequencies are attained experimentally, which range between 4–13 Hz. Considering the first 5 mode shapes, these modes can be classified into bending, vertical and torsional. Also, good agreement of identified frequencies is found between PP and SSI methods.

When the analytical and experimental results are compared with each other, it is clearly seen that there is a good agreement between mode shapes but some differences in natural frequencies. It is thought that the reasons of these differences are due to uncertainties such as material properties and boundary conditions.

To eliminate differences between both results, FEM of the bridge is updated by changing boundary conditions. After the model updating, there is a good agreement between the frequencies and mode shapes.

In this study, it is seen that modal testing and Operational Modal Analysis methods can be apply on historical bridges which have sophisticated geometric features, material types and complex dynamic behaviour.

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