FINITE ELEMENT MESH SIZE EFFECT ON DEFORMATION PREDICTIONS OF REINFORCED CONCRETE BRIDGE GIRDER

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Abstract. Present research was dedicated to investigation of finite element size effect on deformation predictions of reinforced concrete bending members. Experimental beams have been modelled by commercial finite element software *ATENA*, using two main approaches for simulating tension-stiffening: stress-crack width (fracture mechanics approach) and average stress-average strain relationships. The latter approach uses the ultimate strain adjusted according to the finite element size. It was shown that the modelled post-cracking behaviour of the beams is dependent on the finite element mesh size. To reduce this effect, a simple formula has been proposed for adjusting the length of the descending branch of the constitutive relationship. Post-cracking behaviour of a reinforced concrete bridge girder has been investigated assuming different finite element mesh sizes. The analysis has shown that the proposed technique allows reducing the dependence of calculation results on the finite element size.

Keywords: reinforced concrete (RC), finite element size, deformations, tension-stiffening, crack band model.

1. Introduction

Deformational behaviour of cracked reinforced concrete (RC) members is a complex process including a wide range of effects, such as, different strength and deformation proper-ties of steel and concrete, concrete cracking, tension-softening and tension-stiffening, bond slip between reinforcement and concrete, etc. Rather than attempting to provide a complete mechanical description of the behaviour of concrete, reinforcement and their interaction, physical models are aimed at which are as simple as possible and reflect the main influences governing the response of structural concrete.

Different methods have been utilised to study the response of structural components. The use of numerical methods to study these components has also been used. Unfortunately, early attempts to accomplish this were computationally ineffective. In recent years, however, the use of finite element (FE) analysis has increased due to progressing knowledge and capabilities of computer software and hardware (Gribniak *et al.* 2006, 2008; Kaklauskas *et al.* 2008). Numerical techniques have been intensively progressing for last decades and commercial FE softwares (*MSC MARC, ABAQUS, DIANA, SBETA, ATENA*, etc.) now offer a useful tool for analysis of RC structures (Gribniak *et al.* 2006, 2007; Mang *et al.* 2009).

Present study was aimed at investigation of FE mesh size effect on deformation predictions of RC bridge girder. Based on two main approaches of tension-stiffening, analysis of RC elements has been performed using FE package *ATENA*.

2. Approaches in tension-stiffening

In the early development of the theory of RC, deformation problems were simply ignored. First attempts to assess deflections of flexural RC members were based on classical principles of strength of materials. However, elastic calculations may significantly underestimate deflections of cracked members. On the other hand, disregard of the tensile concrete may lead to a significant overestimation of deflections, particularly for lightly reinforced members. The intact concrete between cracks carries tensile force due to the bond between the steel and concrete. The average tensile stress in the concrete can be a significant fraction of the tensile strength of concrete. This effect is called by tension-stiffening and is often accounted for in design by an empirical adjustment to the stiffness of the fully cracked cross-section.

Many theoretical models of RC in tension have been proposed to predict cracking and deformations of RC members. Generally, these models may be separated into four main approaches (Gribniak 2009):

– *Semi-empirical*: the earliest approaches were developed based on the analysis of test data. Such simplified calculation techniques are broadly presented in the design codes;
– *Stress transfer*: these approaches aim at modelling bond between concrete and reinforcement steel;
– *Fracture mechanics*: such approaches use the fracture mechanics principles to predict cracking behaviour of plain and reinforced concrete elements;
– *Average stress-average strain*: simple approaches, extensively used in numerical analyses, based on smeared crack model.

Present study employs the latter two approaches.

2.1. Fracture mechanics

Initiated in 1960th by Kaplan (1961), the study of fracture mechanics has progressed by the turn of the century. Kesler *et al*. (1972) showed that the linear-elastic fracture mechanic model of sharp cracks was inadequate for concrete structures. Inspired by the softening and plastic models of the fracture process zone (FPZ) initiated in the works of Barenblatt (1962) and Dugdale (1960), Hillerborg *et al*. (1976) have proposed the first nonlinear theory of fracture mechanics for concrete. In order to illustrate the size dependence in a simple and dimensionless way, Hillerborg introduced the concept of a characteristic length, l_{ch}, as a unique material property:

$$l_{ch} = \frac{G_F E_c}{f_{ct}^2},\qquad(1)$$

where G_F – the fracture energy, defined as the energy required forming a complete crack; E_c and f_{ct} – the deformation modulus and tensile strength of concrete.

The fictitious crack model (Hillerborg *et al*. 1976) is a suitable and simple model for FPZ, which may be viewed as a specialisation of other more general approaches (Elices *et al*. 2002). For example, Broberg (1999) for materials that fail by crack growth and coalescence depicts the appearance of FPZ in a cross-section normal to the crack edge. He proposes to describe FPZ, in general, by decomposing it into cells. The behaviour of the single sell is defined by relationships between its boundary forces and displacements. This is very similar to the definition of FE in computations, and when these cells are assumed to be cubic (or prismatic) and to lie along the crack path, the resulting model is very similar to the smeared crack approach used for concrete, and, more specifically to the Bažant's crack band approach (Bažant, Planas 1998). The latter model was found to be in good agreement with the basic fracture data (Bažant, Oh 1983), and has been recognised convenient for programming. It is nowadays the crack band model used in industry and commercial FE codes: *DIANA* (Rots 1988), *SBETA* (Cervenka, Pukl 1994; Cervenka *et al*. 1998) and *ATENA* (Cervenka *et al*. 2002).

Fig. 1a presents FE model of a homogeneous plain concrete element subjected to a uniaxial tension. In this model the deformational behaviour can be determined using the softening curve shown in Fig. 1b. The area under

the softening curve is defined as the fracture energy G_F. Though a simple softening curve is shown in Fig. 1, a variety of advanced constitutive models have been proposed. Some of them were reviewed by the authors (Gribniak 2009; Kaklauskas 2001, 2004). It should be pointed out that if a crack is assumed to occur in a single element, then obtained relationship becomes a function of FE length h.
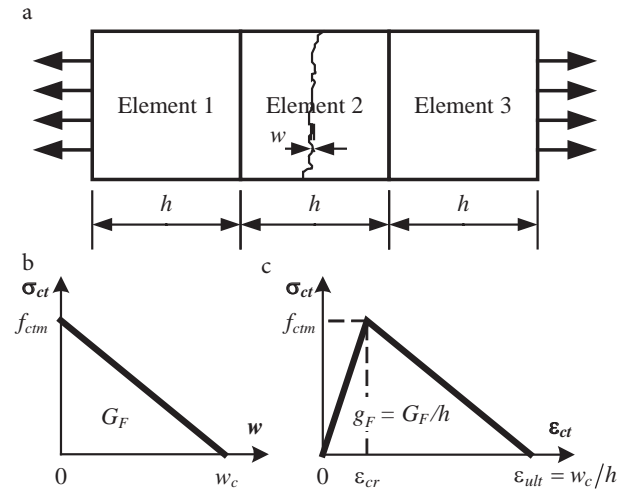

Fig. 1. Concrete element subjected to tension (a); stress-crack width model (b) and average stress-strain model (c) (Gribniak *et al*. 2007)

2.2. Average stress-average strain

This simple approach, extensively applied in numerical analyses, is based on use of average stress-strain relationship. The approach introduced by Rashid (1968) is based on smeared crack model, i.e. the cracks are smeared out in the continuous fashion and the cracked concrete is assumed to remain a continuum. The concrete becomes orthotropic with one of the material axes being oriented along the direction of cracking.

Differently from the discrete crack model tracing individual cracks, smeared crack model deals with average strains and stresses. This model can handle single, multiple and distributed cracks in a unified manner. Thus, it can be used for both, plain and RC structures (Cervenka 1995). In FE analysis, smeared crack model has proven to be more flexible and more computationally effective concerning the discrete crack model since no topological constraints exist.

Most of the continuum-based FE methods incorporate tension-stiffening by the constitutive law of tensile concrete (Barros *et al*. 2001; Ebead, Marzouk 2005; Kaklauskas *et al*. 2007; Lin, Scordelis 1975; Prakhya, Morley 1990; Suidan, Schnobrich 1973). In present research, behaviour of RC member is modelled assuming a uniform tension-stiffening relationship over the whole tension area of concrete. Stress in the concrete is taken as the combined stress due to tension-stiffening and tension-softening, collectively called the tension-stiffening. Based on the above

approach, a number of stress-strain relationships for cracked tensile concrete have been proposed. Kaklauskas (2001, 2004) and Bischoff (2001) have carried out a comprehensive review of the relationships.

3. FE size effect on post-cracking behaviour

It is obviously that fracture energy in the real structure must be constant for any piece of material. In FE analysis above assumption requares that the strains in each FE should constant over a bandwidth, δ_e:

$$G_F = \delta_e \times g_F = \delta_e \times \int \sigma d\varepsilon = const, \qquad (2)$$

where σ and ε – the stress and displacement across the crack initiation zone; g_F – the work dissipated for the creation of a unit area of fully developed crack. In above equation, the bandwidth may be assumed equal to the FE size, h (Fig. 1). For an obtained δ_e the total energy dissipation in the element can be defined:

$$G_{F,e} = \delta_e \times G_F. \qquad (3)$$

However, if the bandwidth is estimated incorrectly, obtained $G_{F,e}$ is also inaccurate as well as the resulting load-displacement predictions (Gribniak et al. 2007). Various possibilities defining δ_e exist.

In crack band model (used in ATENA) the ratio of the FE size, h, to the characteristic length, l_{ch}, called the scaling factor, is used to adjust the average slope of post-peak softening curve (Fig. 2). The user specifies a constitutive relationship for the basic case $\dfrac{h}{l_{ch}} = 1$. As far as possible, the size of FE is assuming equal to l_{ch}. Vořechovský (2007) has assumed l_{ch} equal to 8 cm, whereas, Reinhardt (1996) has proposed to take it from 10 to 50 cm. If, for computational effectiveness, the mesh size needs to be larger, the post-peak portion of the assumed tension-stiffening relationship is scaled horizontally as shown in Fig. 2. The FE size effect on post-cracking behaviour of RC members has been discussed in more details by Gribniak et al. (2007).
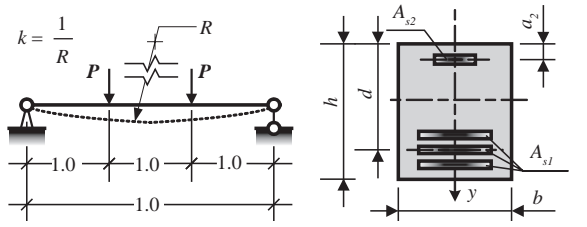
4. Numerical experiment using FE software ATENA

In order to study FE mesh size effect on deformation predictions of RC elements, a numerical experiment has been performed. The data of three RC beams tested by the au-

thors has been employed. All specimens were of rectangular section with nominal length 3280 mm (span 3000 mm). Concrete mix proportion was selected to assure C35/45 class and was taken to be uniform for all experimental specimens. Main parameters of the beams are given in Table 1.

Three layers of the tensile reinforcement were placed in the first two beams, whereas the third beam had two layers. The effective depth d (Table 1) is taken in regard to the centroid of the reinforcement area. It should be noted that such reinforcing scheme is characteristic to the RC bridge girders. Only half of the beam was modelled due to symmetry conditions. As shown in Fig. 3, the beams were modelled taking five different FE mesh sizes, h: 8.5; 15; 30; 50 and 80 mm.

Table 1. Main characteristics of the test specimen

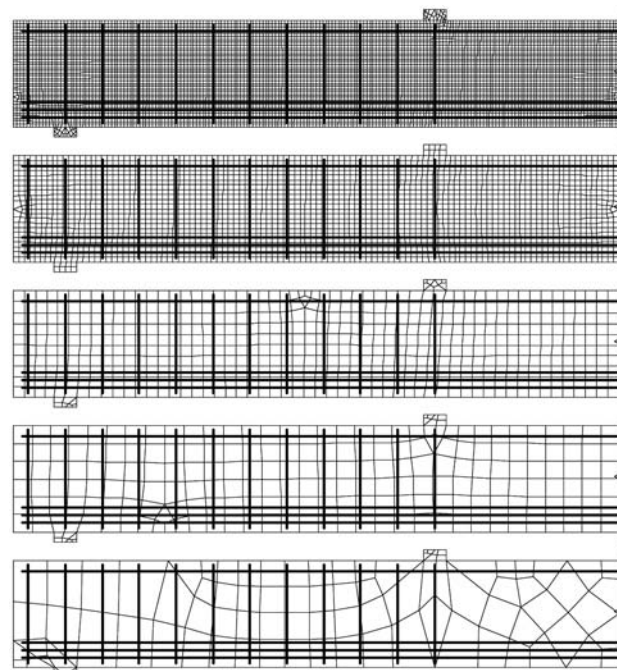| Beam	h	b	d	a_2	A_{s1}	A_{s2}	f_{cm}	f_{ctm}	p
	mm				mm²		MPa		%
S1-1	299	282	248	25	696		49.7	3.53	1.00
S2-1	301	279	254	30	430	57.3	49.4	3.52	0.61
S3-1-1	305	280	271	39	229		47.8	3.39	0.30

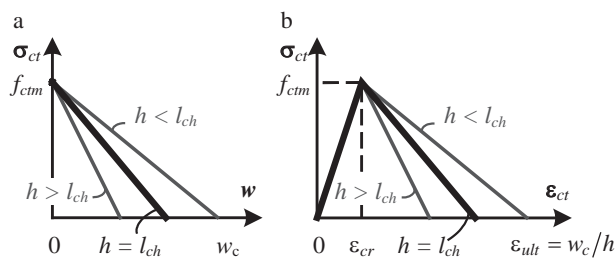**Fig. 3.** FE models of beam *S2-1*

Fig. 2. Scaled constitutive relationships (Gribniak et al. 2007)

FE model of the specimen was considered in a plane stress state with non-linear constitutive laws for concrete and reinforcement. An isoparametric quadrilateral finite element with four integration points was used.

Each analysis was performed using two approach of tension-stiffening modelling based on: fracture mechanics and average stress-average strain. In the first approach, the linear crack opening law has been used (Fig. 1b). Since G_F was not measured experimentally, a default value offered by *ATENA* was applied (Cervenka *et al.* 2003):

$$G_F = 2.5 \times 10^{-5} \times f_{ctm}, \qquad (4)$$

were f_{ctm} – the tensile strength of concrete, MPa (Table 1).

In the second approach, a linear tension-stiffening model shown in Fig. 1c has been employed. According to Eq (4) and (5), both approaches were related by this relationship:

$$\varepsilon = \frac{w}{\delta}; \ \varepsilon_{ult} = \frac{w_c}{h} = \frac{2G_F}{f_{ctm}h} = \frac{5 \times 10^{-5}}{h}. \qquad (5)$$

Fig. 4a shows curvatures predicted by two tension-stiffening approaches:

- fracture energy based on a linear stress-crack width relationship (Fig. 1b). The fracture energy was obtained by Eq (4);
- using a linear average stress-average strain relationship (Fig. 1c). The ultimate strain was obtained by Eq (5).

5. Accuracy analysis

Accuracy analysis has been performed to evaluate agreement of the measured and theoretical curvatures calculated for different FE mesh sizes (Fig. 3). The analysis is based on the statistical procedure proposed by the first author (Gribniak 2009) and described below.

5.1 Sliced data transformation

The beams possessed reinforcement ratio p ranging from 0.3 to 1.0% (Table 1) and, therefore, had different ultimate strength. To ensure even contribution of each test specimen and consistency of the statistical analysis, a procedure called the sliced data transformation was employed. It is based on following steps:

Step 1. Curvatures were calculated by both techniques using different FE mesh sizes.

Step 2. The transformation is made by introducing 11 levels of loading intensity $\bar{M}$ taken in relative terms between the cracking and ultimate bending moment:

$$\bar{M} = \frac{M - M_{cr}}{M'_{ult} - M_{cr}}; \quad \bar{M} = \{0; 0.1; \dots 0.9; 1\}, \qquad (6)$$

where M'_{ult} – the reference ultimate bending moment calculated for each member under assumption of yielding strength 500 MPa for tensile reinforcement; M_{cr} – the reference cracking moment according *CEB-FIB Model Code 1990: Design Code (CEB_FIP Model Code 1990)*:

$$M_{cr} = \frac{I_{el} f_{ct}}{y_t}; \ f_{ct} = 0.3\sqrt[3]{f_{cm}^2}, \qquad (7)$$

where I_{el} – the moment of inertia of the uncracked section; y_t is the distance of the extremely tensile layer from the top edge of the section; f_{cm} – the compressive cylinder strength of concrete (Table 1). According to Eq (6), $\bar{M} = 0$ and $\bar{M} = 1$ correspond to reference cracking and failure of the RC element, respectively.

Step 3. As shown in Fig. 4, the experimental and calculated diagrams were sliced at the above loading levels (shown by horizontal lines). The target points were derived by means of linear interpolation.

Step 4. Accuracy of the predictions was estimated by means of a relative error Δ calculated at each level $\bar{M}$ for each of the test beams according *CEB_FIP Model Code 1990*:

$$\Delta = \frac{k_{calc}}{k_{obs}}, \qquad (8)$$

were k_{calc} and k_{obs} are the mid-point curvatures interpolated at the level $\bar{M}$ from calculated and original test data, respectively.

5.2. Analysis of the results

The error Δ is considered as a random variable; therefore, methods of statistics can be used for accuracy assessment of deflection prediction. Statistics estimating the central tendency and variability serve to measure precision of the predictions. The central tendency can be regarded as a consistency parameter of a calculation method. The postulate of minimum variance was used to evaluate accuracy of a model. The type of distribution of a random variable is very important for making sound conclusions of the statistical analysis. To check whether the probability distribution is normal, a normality test, proposed by Durbin (1961), was performed.

The analysis has shown that the distribution of probability of the relative error Δ is normal. Thus, the central tendency and the variability are reflected by the expectation μ_Δ and the variance σ_Δ^2. These parameters can be estimated by the sample mean m_Δ and the standard deviation s_Δ.

Above statistics are given in Table 2. Each statistics was derived from a sample which contains 33 data points. Two main statistical conclusions about accuracy of curvature predictions can be drawn: 1) it is independent from type of tension-stiffening approach used in the FE analysis; 2) FE mesh size effect on the accuracy is significant. The obtained results are illustrated by Fig. 4a.

The observed mesh-dependence allows taking into account the structural size effect, i.e. the increasing of member strength with decreasing of its size (Cervenka, Pukl 1994). This can be performed when the numerical models of the structural elements, having different geometrical sizes, will contain the same number of FE. However, above effect may attribute some extra stiffness to RC member in case of fine FE meshes.

To reduce mesh-dependence, it is proposed to introduce scaling factor $\delta_{(1)\to(2)}$:
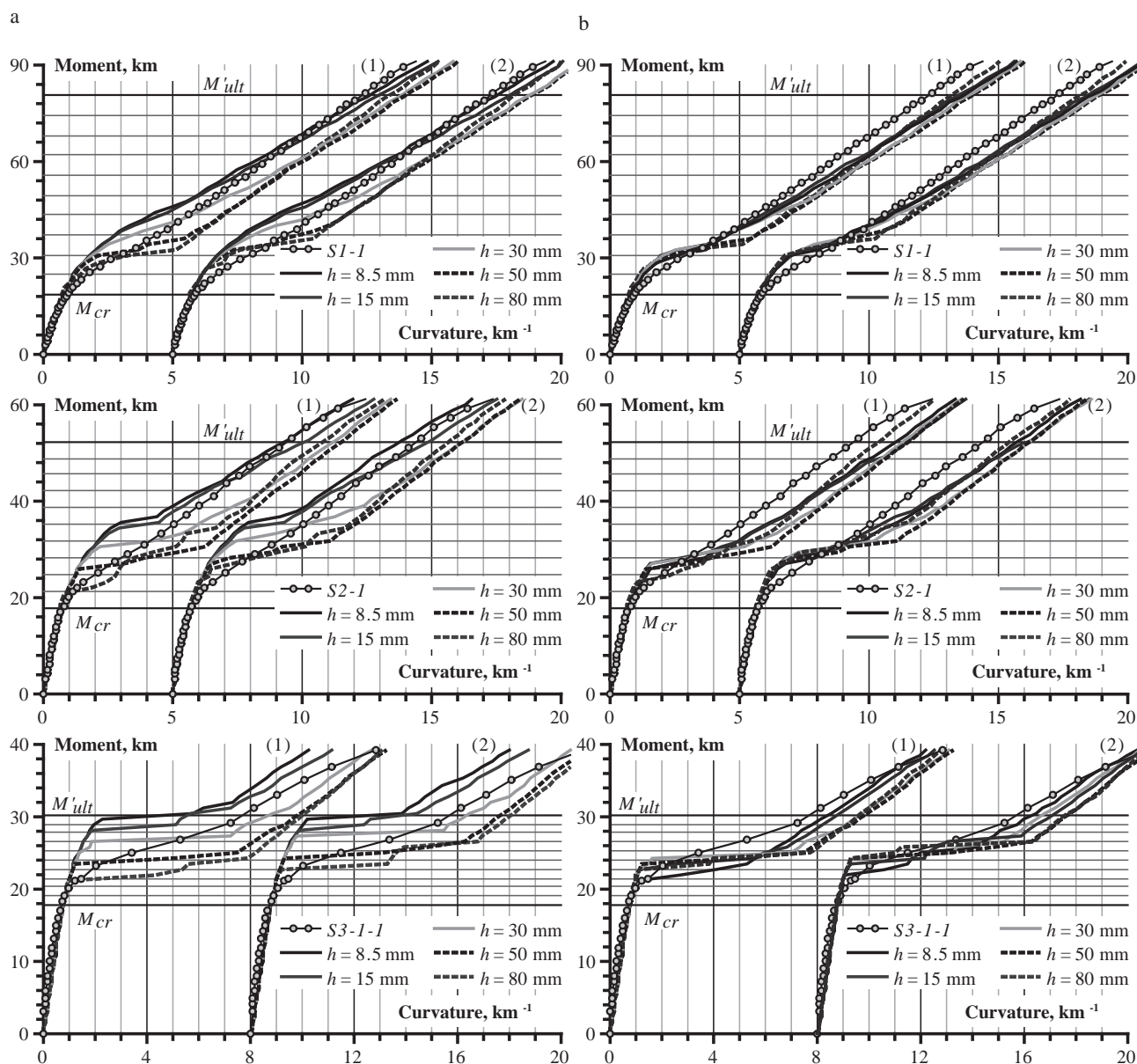
a   b

Fig. 4. Moment-curvature diagrams of experimental beams calculated using stress-crack width (1) and average stress-average strain (2) relationships

$$\delta_{(1)\to(2)} = \sqrt{\frac{h(2)}{h(1)}}, \qquad (9)$$

where $h(1)$ and $h(2)$ – the reference and the actual sizes of the FE mesh, respectively.

This factor reduces the deformation energy, dissipated in the FE after cracking. Scaling is performed by adjusting the descending branch of the constitutive relationship with change of FE mesh size. The scaling factor is multiplied by G_F (in the first approach) or by ε_{ult} (in the second approach).

Analysis of the results, listed in Table 2, indicates that the best accuracy of the predictions has been reached when 50 mm meshing was used. Therefore, for further analysis, this FE size was taken as the reference size.

The moment-curvature relationships calculated after scaling are shown in Fig. 4b. The figure clearly demonstrates that the applied technique reduces mesh-dependence. In support to this, a statistical analysis has shown that mesh-dependence has become insignificant (compare the statistics presented in Tables 2 and 3).

Table 2. Basic statistics (sample mean and standard deviation) derived for both tension-stiffening approaches

Approach	Fracture mechanics (1)					Average stress-strain (2)				
h, mm	8.5	15	30	50	80	8.5	15	30	50	80
m_Δ	0.74	0.78	0.93	1.16	1.33	0.73	0.78	0.89	1.08	1.13
s_Δ	0.24	0.23	0.27	0.31	0.46	0.23	0.23	0.27	0.29	0.30

Table 3. Basic statistics derived for both tension-stiffening approaches after scaling

Approach	Fracture mechanics (1)					Average stress-strain (2)				
h, mm	8.5	15	30	50	80	8.5	15	30	50	80
m_Δ	1.16	1.07	1.09	1.16	1.13	1.05	1.00	1.03	1.08	1.00
s_Δ	0.30	0.25	0.29	0.31	0.30	0.18	0.19	0.26	0.29	0.27

6. Modelling of RC bridge girder

Post-cracking deformational behaviour of a typical three-span RC framed bridge, subjected to uniformly distributed load q has been investigated. Main geometrical parameters of the overpass are shown in Fig. 5. All material parameters were assumed the same as for beam *S2-1* (Table 1).

As shown in Fig. 6, the bridge was modelled taking four different mesh sizes, h: 50; 100; 200 and 400 mm. Each FE simulation was performed using two concepts of tension-stiffening based on: fracture mechanics and average stress-average strain.

First, mid-span deflections (section C-C in Fig. 5) were calculated applying the reference (no scaling) tension-stiffening models, i.e. G_F was calculated by Eq (4) (in the first approach) and ultimate strain by Eq (5) (in the second approach). The predicted load-deflection diagrams are given in Fig. 7a. Similar extent of mesh-dependence to that obtained in the previous analysis (Fig. 4a) can be stated.

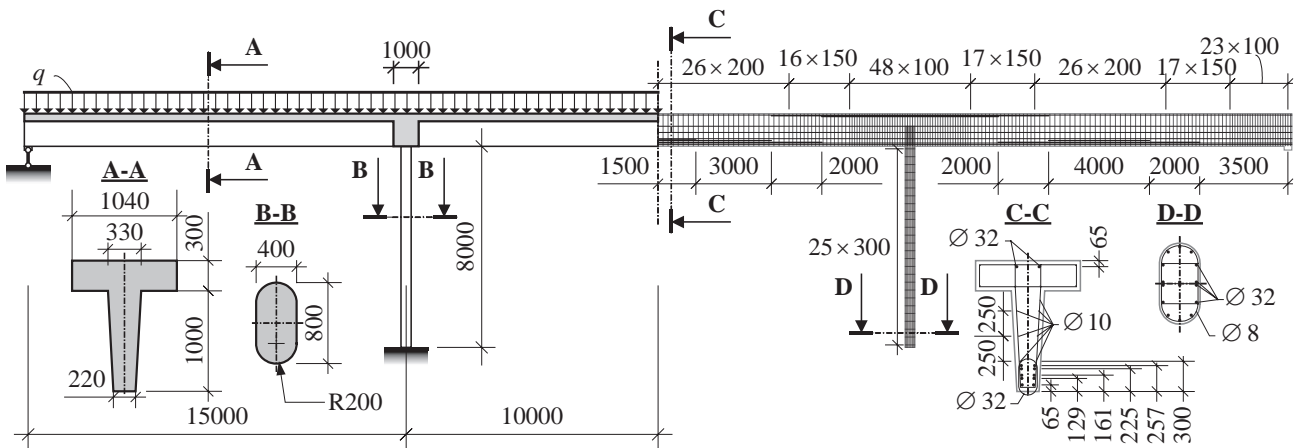Second, the analysis was executed applying scaled tension-stiffening relationships. The reference mesh size

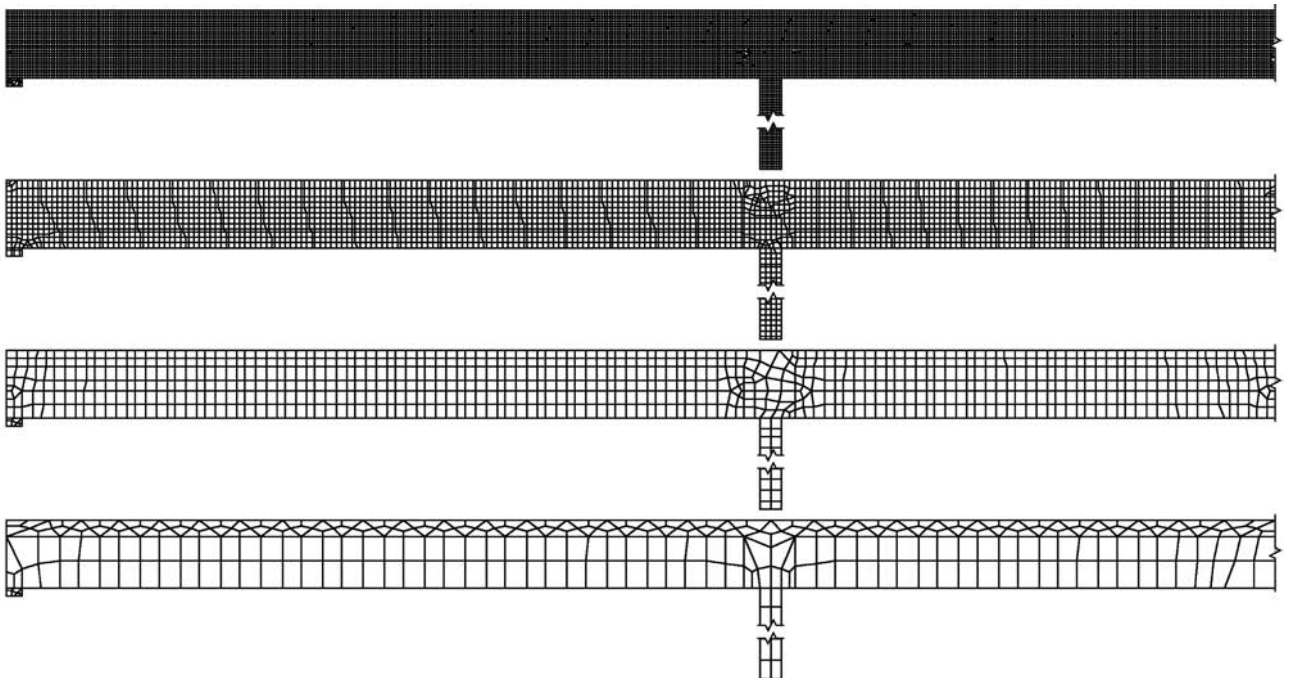**Fig. 5.** Loading and reinforcing scheme of the modelled overpass

Fig. 6. FE models of the overpass
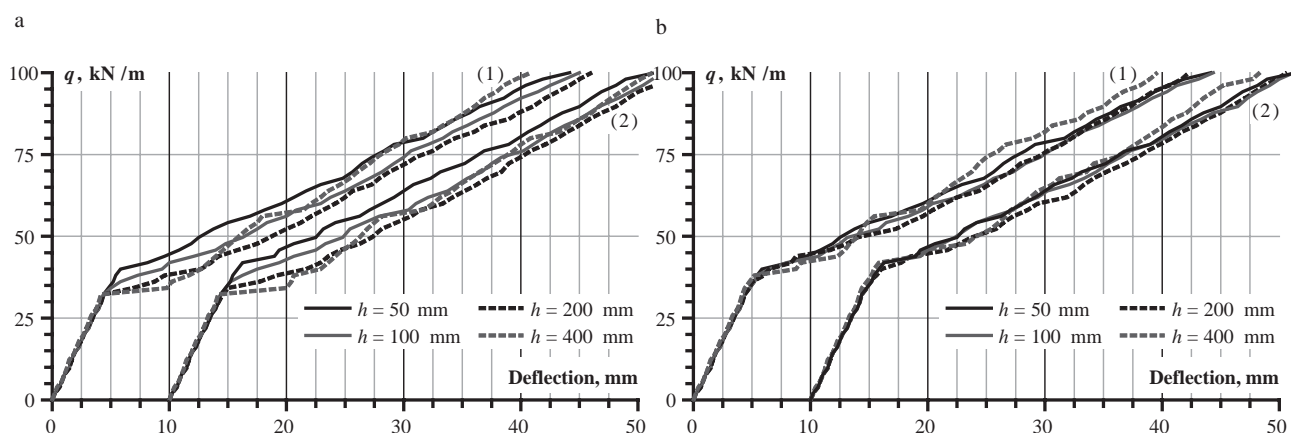
a

b

Fig. 7. Mid-span deflection of RC bridge girder calculated using reference (a) and scaled (b) tension-stiffening relationships

equal to 50 mm was used. The calculated deflections are presented in Fig. 7b. As in the previous analysis, the applied scaling technique was capable reducing mesh-dependence. However, though scaling reduces mesh-dependence, it does not eliminate the effect completely. Other factors

having influence on the above effect might be the cracking pattern, numerical peculiarities of the solution procedure and etc.

Finally, cracking behaviour of the overpass was analysed. Fig. 8 presents the crack pattern modelled using
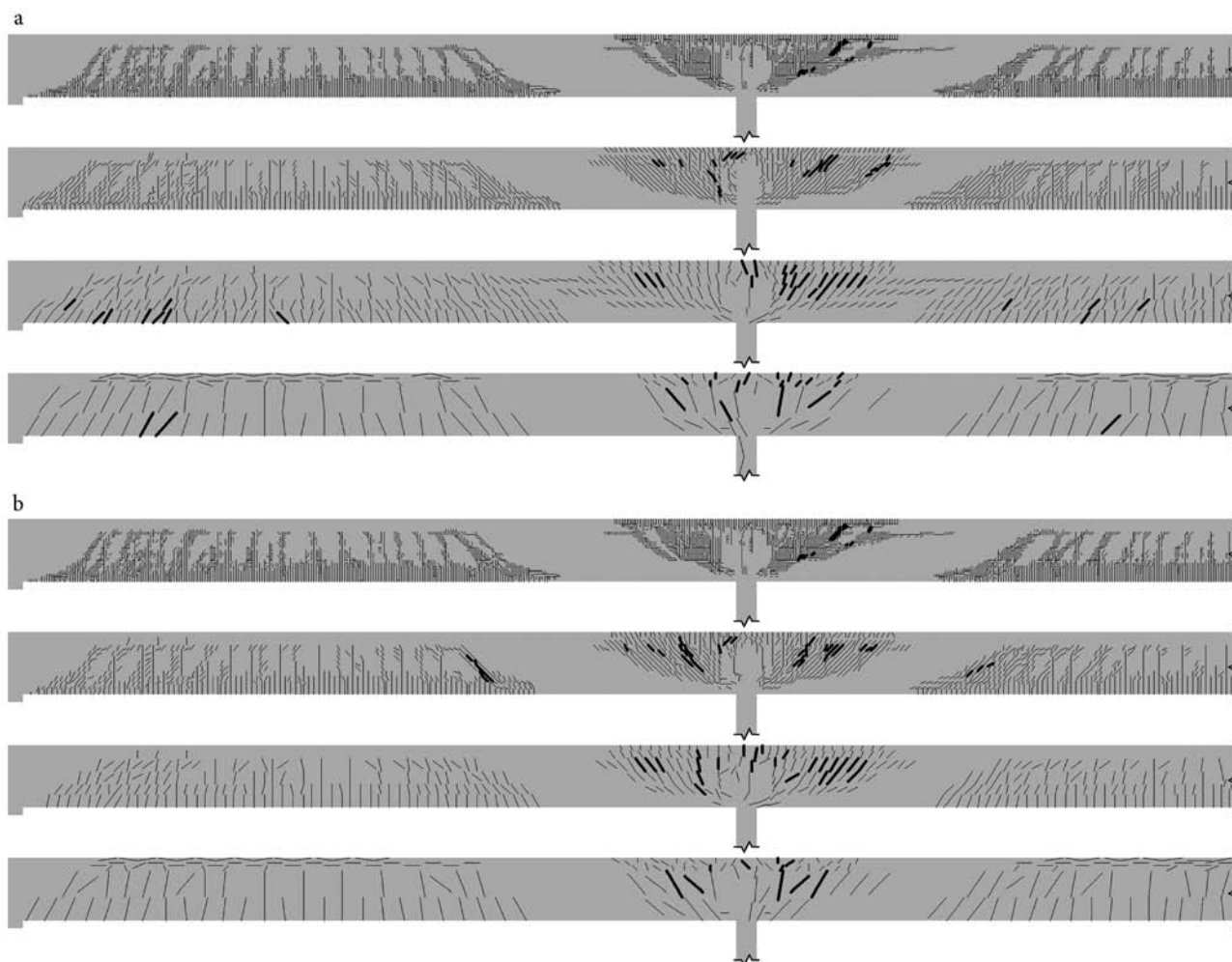
a

b

Fig. 8. Crack pattern of RC bridge girder simulated using fracture mechanics approach with different FE mesh size

fracture mechanics approach. The figure clearly demonstrates that the finest meshing most realistically predicts the crack pattern. It can be observed that in the reference model, the cracking pattern and the maximal crack widths were affected by change of FE mesh size (see Fig. 8a), whereas it was far less sensitive after scaling (see Fig. 8b).

7. Concluding remarks

The FE size effect in curvature analysis has been investigated. Numerical modelling has been performed using two approaches of tension-stiffening based on: fracture mechanics and average stress-average strain. Based on the obtained results, a statistical analysis of accuracy of the predictions has been carried out. Two main conclusions were made. First, the accuracy was independent from the approach of tension-stiffening. Second, the accuracy was strongly mesh-dependent, which was found to be statistically significant.

A scaling technique has been proposed to reduce mesh-dependence. A simple formula has been proposed for adjusting the length of the descending branch of the tension-stiffening relationship with change of mesh size. It was shown that the proposed technique was capable reducing the mesh-dependence. Though the technique reduces dependence of calculation results on mesh size, it does not eliminate the effect completely. Other factors affecting the calculation results might be the cracking pattern, numerical peculiarities of the solution procedure, local effects due to discrete location of the reinforcement bars and etc.

Dependence of mesh size effect on deformations and cracking has been investigated for a typical three-span RC bridge girder. It was shown that the finest meshing most realistically predicts the crack pattern. In the reference model, the cracking pattern and the max crack widths were affected by change of FE mesh size, whereas it was far less sensitive after scaling. The proposed technique is recommended in cases of rough meshing to avoid extra stiffness.

Acknowledgement

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