THE BALTIC JOURNAL
OF ROAD AND BRIDGE ENGINEERING

2011
6(1): 23–29

INVESTIGATION OF A NEW TRANSITION CURVE

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Abstract. This paper describes derivation of the curvature change for the transition curve in a 2nd degree increasing concave formulation. The investigation of the usability of the developed transition curve in new designs and rehabilitation of existing roads are also included in this study. New transition curves are compared with the most used transition curves (clothoid, sinusoidal and bloss) in highways, and it has been targeted to indicate whether it is the most economic and comfortable one or not.

Keywords: highway, transition curve, curvature, superelevation, clothoid, bloss, sinusoidal.

1. Introduction

Rapid developments in vehicle technology have resulted in the production of high-speed vehicles. Parallel to those rapid developments, the increase in the design speed limit in basic highway projects became inevitable. Design of the transition horizontal geometry is important for high-speed road projects. Curves are the route elements for the reformation of existing roads or new constructed highways with respect to new project speeds. Crossing from alignment to a circle with *R* radius causes inconvenience especially on roads where the vehicles are driven with high-speed. Since the vehicle in the alignment is not affected by any force, it begins to be affected by the centrifugal force when it enters the curve. The centrifugal force affects the equilibrium of the vehicle and disturbs the traveler more. To decline the effect of the centrifugal force, reducing the speed *V* or enlarging the radius *R* can be a solution. However, the most effective way of declining the centrifugal force is to give the slope in width superelevation to the curve in the road. However, this slope can not be given suddenly. This slope should start from a certain part of the road and increase slowly until the required value is reached.

When these solutions are investigated one by one, it can be said that some disadvantages given below are caused:

– decreasing the velocity *V* while entering the curve not only contradicts the targets of the modern road routes but also increases the number of accidents occuring at the curves because of the driver faults;
– increasing the curve radius decreases the centrifugal force but cannot avoid the sudden occurrence of this force;
– giving a slope in width superelevation to the curve has specific limits which causes other disadvantages when exceeded.

According to Gintalas *et al.* (2008) most of the accidents on the roads occur on the horizontal curve and especially the accidents occur at the beginning and at the end of the horizontal curve. The transition zones that are used to pass from alignment to curve are regions which are used to change the speed of the drivers. Drivers, to decrease speed when approaching the curve and to increase speed when leaving the curve, use these regions (Dell'Acqua, Russo 2010). Thus, placing a curve which has a curvature that gradually changes between a curve and an alignment so as to avoid the occurrence of centrifugal force can also be thought as another important solution (Pirti 2009).

Using a transition curve with a gradually changing curvature instead of a circular curve with a constant curvature allows the high speed vehicles to enter the curves more safe without decreasing their velocities. This also avoids the shock effect of the centrifugal force and therefore the centrifugal force exists gradually. The most important property of the transition curves is the curvature change which directly depends on the curvature length *l*.

There are many researches conducted on the transition curves. The most experienced transition curves among these recommended curves in the open literature are clothoid, lemniscates, cubic parabola, cubic spiral sinusoidal, bloss and 2*R* radius circles (Can *et al.* 2005; Pirti 2009; Walton, Meek 2005).

Clothoid is often prefered for highways and railway route design (Meek, Walton 2004a; 2004b).

ISSN 1822-427X print / ISSN 1822-4288 online
doi: 10.3846/bjrbe.2011.04
http://www.bjrbe.vgtu.lt

On the other hand, according to Crews (2009) clothoid curves are applicable for transitions with tangential deflections ranging from 0 to 90 degrees. It is also recorded that, these clothoid curves provides a coherent course of curvature. Since the tabulation and stake out of these clothoid curves are much harder when compared to others, these curves are not common as others.

Instead of a clothoid, a parabola of 5th degree is recommended by Dr Ing Bloss to be used while obtaining a transition. This parabola provides a smaller shift-P with a longer transition and a larger spiral extension (*K*), besides having the advantage of vis-à-vis the clothoid. In case of increased stretch speed, these properties are important factors in reconstruction of the track. From a load dynamic point of view, the Land Development Civil, Survey Professionals (Crews 2009) reported that this parabola of 5th degree is more favorable in case of superelevation ramp.

This study describes derivation of the curvature change for the transition curve in a 2nd degree increasing concave formulation. The investigation of the usability of the developed transition curve in new designs and rehabilitation of existing roads has been also included in this study.

On the other hand, the obtained new transition curves are compared with the most used transition curves (clothoid, sinusoidal and bloss) in highways with regard to curvature change, angle of deviation, superelevation, lateral acceleration and total curve length. The most convenient curve is determined as a result of this comparison and it has been targeted to indicate whether it is the most economic and comfortable one or not.

2. Investigation of a new transition curve

In this study, curvature change is taken as a function from 2nd degree. According to this function, two situations are emerging. At the first situation (G1), curvature changes upward the concave transition curve. At the second situation (G2), curvature changes downward the concave transition curve. Therefore, in this work, for every two situations the research is made and the Eqs of curves are given.

2.1. Determination of curvature change of a curve having a 2nd degree increasing concave formulation

The curvature function of the new curve is suggested to be a 2nd degree parabola, which is given in Eq (1) (Kuloğlu 1988):

$$k = Al^2 + Bl + C, \tag{1}$$

where k – the curvature of any point on the transition curve; A, B and C – coefficients, l – transition curve length, m.

The curvature change diagram of the curve is shown in Fig. 1.

A, B and C coefficients in Eq (1) are obtained with respect to the following boundary conditions:

1st boundary condition – the radius at junction of alignment and the transition curve is infinite, where $l = 0$, $k \neq 0$;

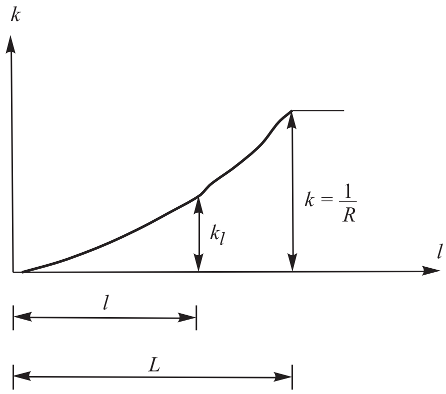

Fig. 1. Curvature change from 2nd degrees (up the right concave)

2nd boundary condition: the derivative of the Eq (1) can be different from 0, $l = 0$ for $k' \neq 0$, or the derivative of the Eq (1) can be chosen as 0 (specific case), $l = 0$ for $k' = 0$;

3rd boundary condition – the radius on the junction point of the circle arc and the transition curve is to be the intended value, R. In this case $I = L$ for $k = \frac{1}{R}$.

From 1st boundary condition $C = 0$, and from 2nd boundary condition first case $k' = 2Al + B$ and $l = 0$ for $k' = B$.

From 3rd boundary condition

$$k = AL^2 + BL + C \text{ and } \frac{1}{R} = AL^2 + BL + C. \tag{2}$$

If values obtained from the 1st and 2nd a boundary conditions are used in Eq (2), following derivations can be obtained:

$$\frac{1}{R} = AL^2 + k'L, \tag{3}$$

and

$$A = \frac{1}{RL^2} - \frac{k'}{L}. \tag{4}$$

If A, B, C coefficients obtained depending on the boundary conditions are used in Eq (1), the function of the curvature of any point on the transition curve can be obtained as Eq (5):

$$k = \left(\frac{1}{RL^2} - \frac{k'}{L}\right)l^2 + k'l. \tag{5}$$

This function can be written for the 2nd boundary condition second case (specific case):

$$k_1 = \frac{l^2}{RL^2}. \tag{6}$$

This curve which is determined by Eq (6) will be named as transition curve 1 (G1) in this study.

2.2. Determination of curvature change of a transition curve having a 2nd degree downward concave curvature change

The curvature function of the new curve is suggested to be a 2nd degree parabola (Kuloğlu 1988):

$$l = Ak^2 + Bk + C. \quad (7)$$

The curvature change diagram of the curve is shown in Fig. 2.

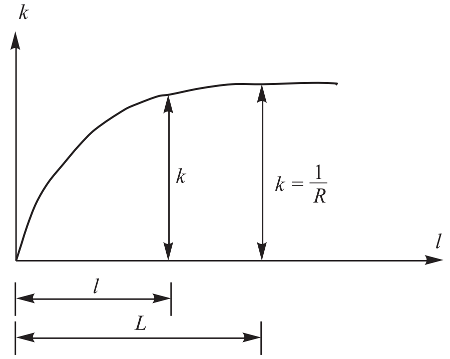

Fig. 2. Curvature change from 2nd degrees (downward concave)

A, B and C coefficients in Eq (7) are obtained with respect to the following boundary conditions:

1st boundary condition $k = 0$ for $l = 0$;

2nd boundary condition $k = 0$ for $l' \neq 0$;

3rd boundary condition $k = \frac{1}{R}$ for $l = L$.

Using these boundary conditions A, B and C in Eq (7) can be written as, $C = 0$, $B = 0$ and $A = LR^2$.

If these obtained coefficients are substituted into Eq (7), curvature function at any point can be obtained as

$$k = \frac{1}{R}\sqrt{\frac{l}{L}}. \quad (8)$$

This curve which is determined by Eq (8) will be named as transition curve 2 (G2) in this study.

3. Determination of other elements of G1 and G2 transition curves

Fig. 3 presents elements of general curve.

3.1. The tangential angle at any point of transition curves

The value of the tangential angle of a curve is equal to the area of the curvature diagram. The mathematical expression is given in the Eq (9):

$$\tau = \int_0^1 k\, dl. \quad (9)$$

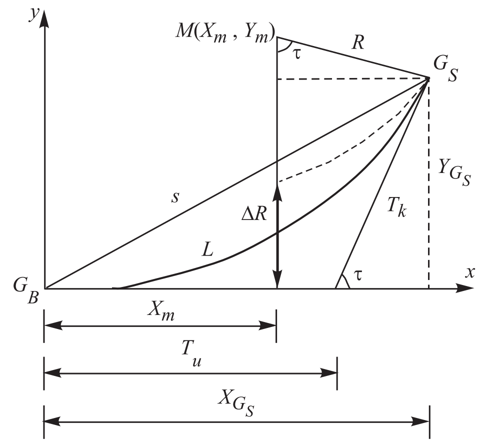

Fig. 3. Elements of general curve: R – radius; X_m, Y_m – coordinate of circle centre; L – transition curve length; G_B – transition curve starting; G_S – transition curve finishing; X_{GS}, Y_{GS} – cartesian coordinate of point at junction of the transition curve and circular arc; T_k – short tangent; T_u – long tangent; ΔR – curve shift; τ – tangential angle

The tangential angle at any point of a transition curve by substituing the values of k in Eqs (5) and (8) can be determined as:

for G1
$$\tau = \frac{l^3}{3RL^2} \quad (10)$$

and

for G2
$$\tau = \frac{2l^{\frac{3}{2}}}{3RL^{\frac{1}{2}}}. \quad (11)$$

The value of the tangential angle at junction of the transition curve and circular arc can be found by using $l = L$. The tangential angle at any point of a transition curve can be determined as:

for G1
$$\tau = \frac{L}{3R} \quad (12)$$

and

for G2
$$\tau = \frac{2L}{3R}. \quad (13)$$

3.2. Determining Cartesian coordinates (X, Y)

The Cartesian coordinates (X, Y) of the transition curves are calculated via using Fresnel integral (Jeffrey, Dai 2008). The cosines of the tangent of a planar curve are calculated as follows:

$$\cos\tau = \frac{dx}{dl}, \quad \sin\tau = \frac{dy}{dl}; \quad (14)$$

$$X = \int_0^1 \cos\tau\, dl, \quad Y = \sin\tau\, dl. \quad (15)$$

These integrals are known as the Fresnel Integral (Jeffrey, Dai 2008). The Fresnel Integral is solved by series expansion of the exponential function of sinτ and cosτ. The relations are obtained when the value of the tangential angel obtained from Eq (14) is substituted into the integrals:

for G1 $$X = \int_0^1 \cos\left(\frac{l^3}{3RL^2}\right) dl,\ Y = \int_0^1 \sin\left(\frac{l^3}{3RL^2}\right) dl \qquad (16)$$

and

for G2 $$X = \int_0^1 \cos\left(\frac{2l^{\frac{3}{2}}}{3RL^{\frac{1}{2}}}\right) dl,\ Y = \int_0^1 \sin\left(\frac{2l^{\frac{3}{2}}}{3RL^{\frac{1}{2}}}\right) dl. \qquad (17)$$

The Cartesian coordinates of any point on the transition curve can be derivated by Mac Laurin series expansion and integration of these angle functions as:

for G1 $$X = 1 - \frac{l^7}{126A^6} + \frac{l^{13}}{25272A^{12}},$$
$$Y = \frac{l^4}{12A^3} - \frac{l^{10}}{1620A^9} + \frac{l^{16}}{466560A^{15}} \qquad (18)$$

and

for G2 $$X = l - \frac{l^4}{4R^2L} + \frac{l^7}{852R^4L^2},$$
$$Y = \frac{l^2}{3RL^{\frac{1}{2}}} - \frac{4l^5}{405R^3L^{\frac{3}{2}}} + \frac{l^8}{9720R^5L^{\frac{5}{2}}}. \qquad (19)$$

3.3. The change of the superelevation

The change of the superelevation of the curve variation depends on the change of the curvature (Fig. 4). The change of the superelevation of G1 and G2 is given in Fig. 3. The superelevation value at the initial point of the transition curve is 0. Since radius is infinitive at this point $\frac{1}{R} = 0$. The superelevation reaches a max at the junction of the curve and the circular arc.

The theoretical superelevation at any point of the curve is calculated from the Eq (20):

$$d = 0.00786\frac{V^2}{R_l}. \qquad (20)$$

$\frac{1}{R_l}$ is the curvature at any point. Therefore, superelevation formulations are obtained by substituiting Eqs (6) and (8) for G1 and G2 into Eq (20):

for G1 $$d_1 = d\frac{l^2}{L^2} \qquad (21)$$

and

for G2 $$d_1 = d\frac{\sqrt{l}}{\sqrt{L}}. \qquad (22)$$

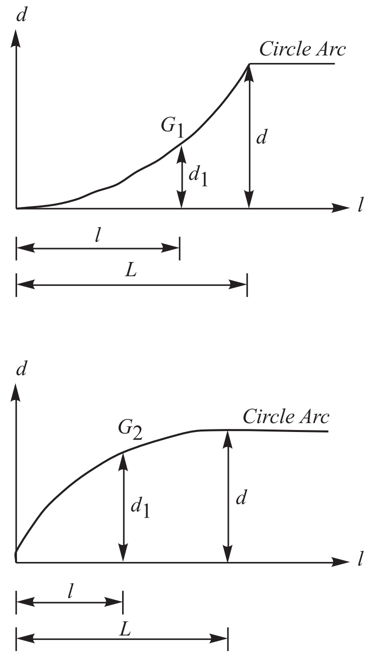

Fig. 4. The change of the superelevation

3.4. Change of the lateral acceleration

The lateral acceleration changes are also obtained by the same way following the superelevation changes. The lateral acceleration at a curve can be calculated by Eq (23):

$$a = \frac{V^2}{12.96R} - 0.0981d. \qquad (23)$$

If Eq (23) is written for G1 and G2 and the values of superelevation substituted into these Eqs are obtained:

for G1 $$a_1 = \left(\frac{V^2}{12.96R} - 0.0981d\right)\frac{l^2}{L^2} \qquad (24)$$

and

for G2 $$a_1 = \left(\frac{V^2}{12.96R} - 0.0981d\right)\frac{\sqrt{l}}{\sqrt{L}}. \qquad (25).$$

3.5. Shift of the transition curves

Shift of the transition curves in the all transition curves is given as follow, $\Delta R = Y_{G_S} - R(1-\cos\tau)$. According to this Eq, ordinate of point at junction of the transition curve and circular arc (Y_{G_S}) and tangential angle (τ) in the same point is different for every transition curve. Shift of G1 and G2:

for G1 $$\Delta R = \frac{L^2}{36R} - \frac{L^4}{9720R^3} + \frac{L^6}{4199040R^5} \qquad (26)$$

and

for G2 $$\Delta R = \frac{L^2}{9R} - \frac{L^4}{243R^3} + \frac{L^6}{16402.5R^5}. \qquad (27)$$

4. Comparison of transition curves 1 and 2 with clothoid, sinusoidal and bloss curves

Comparing the transition curve derived in this study with the other curves in point of curvature change, angle of deviation, lateral acceleration, superelevation and total curve length, the most convenient and economic curve is chosen. L and R are both assumed to be 300 m and V is chosen as 90 km/h so as to draw the curves.

Comparison parameters are shown in the Tables 1 and 2. Figs 5–9 give the drawings by the calculation according Eqs in these Tables.

Table 1. Comparison parameters 1

Transition curves	Change curvature	Tangential angle, rad	Cartesian coordinates
G1	$k_1=\frac{l^2}{RL^2}$	$\tau=\frac{L}{3R}$	$X=1-\frac{l^7}{126A^6}+\frac{l^{13}}{25272A^{12}}$ $Y=\frac{l^4}{12A^3}-\frac{l^{10}}{1620A^9}+\frac{l^{16}}{466560A^{15}}$
G2	$k=\frac{1}{R}\sqrt{\frac{l}{L}}$	$\tau=\frac{2L}{3R}$	$X=l-\frac{l^4}{4R^2L}+\frac{l^7}{852R^4L^2}$ $Y=\frac{l^2}{3RL^{\frac{1}{2}}}-\frac{4l^5}{405R^3L^{\frac{3}{2}}}+\frac{l^8}{9720R^5L^{\frac{5}{2}}}$
Clothoid	$k=\frac{l}{RL}$	$\tau=\frac{L}{2R}$	$X=l-\frac{l^5}{40R^2L^2}+\frac{l^9}{3456R^4L^4}$ $Y=\frac{l^3}{6RL}-\frac{l^7}{336R^3L^3}+\frac{l^{11}}{42240R^5L^5}$
Sinusoidal	$k=\frac{1}{R}\left[\frac{l}{L}-\frac{1}{2\pi}\sin\left(\frac{2\pi l}{L}\right)\right]$	$\tau=\frac{200L}{2\pi R}$	$X=L-0.02190112582400869\frac{L^3}{R^2}$ $Y=L\left[0.1413363707560822\frac{L}{R}-0.0026731818162654\frac{L^3}{R^3}\right]$
Bloss	$k=\frac{1}{R}\left(\frac{3l^2}{L^2}-\frac{2l^3}{L^3}\right)$	$\theta=\frac{l^3}{RL^2}-\frac{l^4}{2RL^3}$	$X=L-\frac{L^3}{43.8261R^2}+\frac{L^5}{3696.63R^4}$ $Y=\frac{3L^2}{20R}-\frac{L^4}{363.175R^3}$

Table 2. Comparison parameters 2

Transition curves	Superelevation	Lateral acceleration	Shift of the transition curves
G1	$d_1=d\frac{l^2}{L^2}$	$a_1=\left(\frac{V^2}{12.96R}-0.0981d\right)\frac{l^2}{L^2}$	$\Delta R=\frac{L^2}{36R}-\frac{L^4}{9720R^3}+\frac{L^6}{4199040R^5}-..$
G2	$d_1=d\frac{\sqrt{l}}{\sqrt{L}}$	$a_1=\left(\frac{V^2}{12.96R}-0.0981d\right)\frac{\sqrt{l}}{\sqrt{L}}$	$\Delta R=\frac{L^2}{9R}-\frac{L^4}{243R^3}+\frac{L^6}{16402.5R^5}-..$
Clothoid	$d_1=d\frac{l}{L}$	$a_1=\left(\frac{V^2}{12.96R}-0.0981d\right)\frac{l}{L}$	$\Delta R=\frac{L^2}{24R}-\frac{L^4}{9688R^3}+\frac{L^6}{506880R^5}-..$

Continued

Continued Table 2

Transition curves	Superelevation	Lateral acceleration	Shift of the transition curves
Sinusoidal	$d_1 = d\left[\frac{l}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi l}{L}\right)\right]$	$a_1 = \left(\frac{V^2}{12.96R} - 0.0981d\right)\left[\frac{l}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi l}{L}\right)\right]$	$\Delta R = X - R\sin\Delta_s$
Bloss	$d_1 = d\left[\frac{3l^2}{L^2} - \frac{2l^3}{L^3}\right]$	$a_1 = \left(\frac{V^2}{12.96R} - 0.0981d\right)\left(\frac{3l^2}{L^2} - \frac{2l^3}{L^3}\right)$	$\Delta R = Y - R(1-\cos\Delta_s) = \frac{L^2}{40R} - \frac{L^4}{6696.58R^3}$

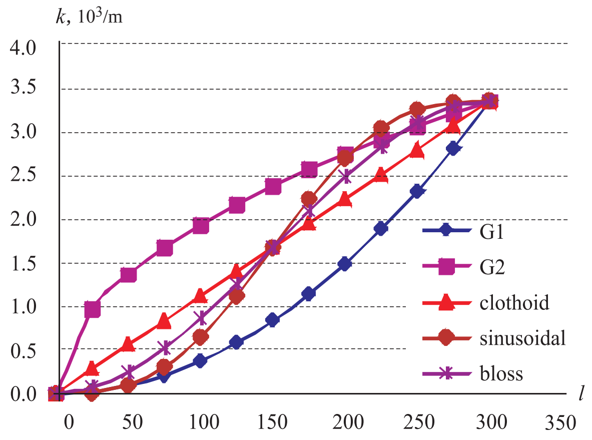

Fig. 5. Curvature changes

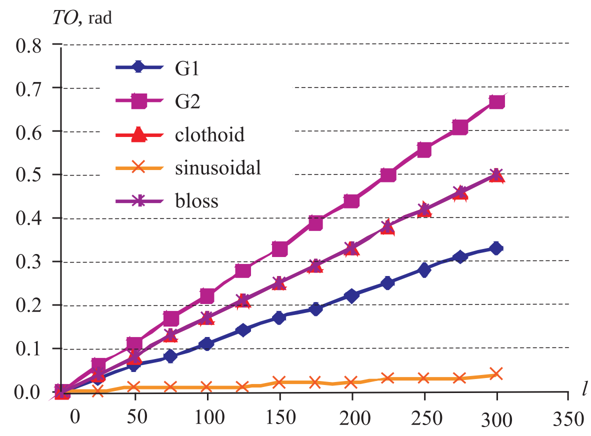

Fig. 6. Tangential angle changes

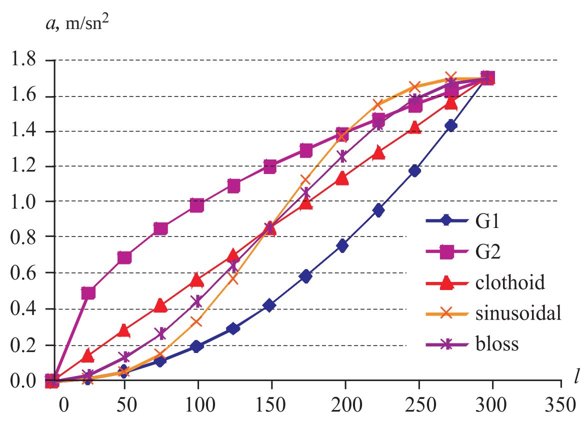

Fig. 7. Lateral acceleration changes

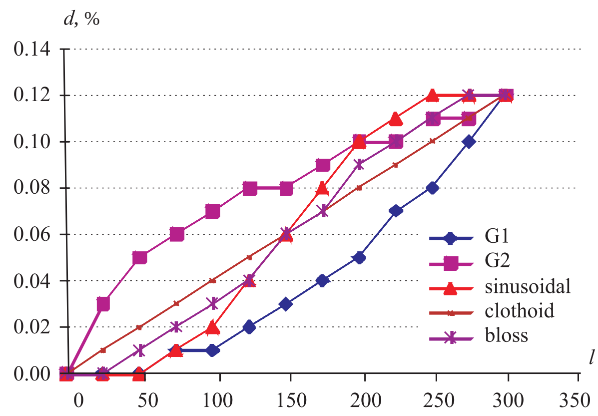

Fig. 8. Superelevation changes

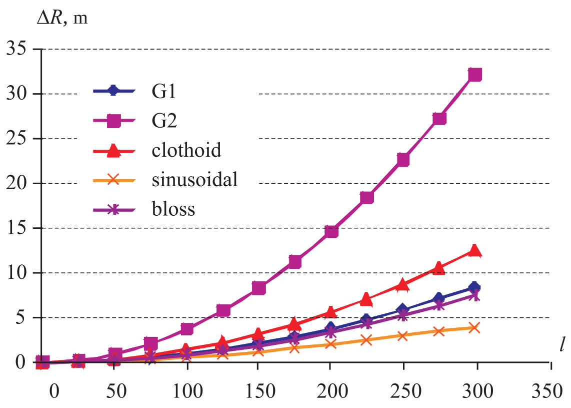

Fig. 9. Shift changes

While the deviation angle of sinusoidal curve and G1 is the lowest, it has the highest values for G2. The clothoid and bloss curves have almost the same deviation angle.

Lateral acceleration is max at the finishing and zero at the beginning of all the transition curves whichever transition curve is used. If the lateral acceleration occurs gradually, less discomfort for the passengers crossing from the alignment to the curve is provided. This can be achived by G1 as seen in Fig. 7. In this Fig while a lateral acceleration of 0.42 m/s^2 is effective for G1 at a point of L/2 distance from the beginning of the curve, a lateral acceleration of 1.20 m/s^2 is effective for G2 and the other transition curves have 0.85 m/sn^2 acceleration angle at the same point. It is seen that G1 has lower values of acceleration angle for the other distances.

The curvature in G1 is less than other curves. Besides, at the beginning of the transition curve, there is a transduction between the curvature and alignment.

If all the transition curves are examined in terms of superelevation, superelevation is same at the beginning and finishing points. In other points, superelevation changes in accordance with curvature change.

Since circular arcs are used at the existing routes, curve shift of the transition curves are crucial in point of economy. Therefore, having low curve shift both G1 and bloss curves gain the advantage over the other transition curves (G2, clothoid and sinusoidal).

5. Conclusions

In this article, two new transition curves under the label G1 and G2 have been developed. These transition curves have been compared with the other curves used in highway (clothoid, sinusoidal and bloss) in terms of curvature change, shift of transition curve, lateral acceleration, tangential angle and superelevation. When the total transition length is considered, it has been observed that G1 has less curvature change than the other curves. Also, the rate of shift and lateral acceleration for G1, clothoid, bloss and sinusoidal is less in L/2 length. As for 3L/4, the rate of shift and lateral acceleration for G1 has been observed less than the other curves. Therefore, G1 is likely to be an economical solution for refining the geometric standards of the present highway. Additionally, it will cause fewer disturbances for the passengers while passing from alignment to curve and vice versa, which has been regarded as a significant feature distinguishing G1 from the other transition curves.

Finally, G1 and sinusoidal curve's having less tangential angle and G1 requiring less superelevation, though nonlinear, compared with the other curves makes it superior to the other curves. Overall, it has been observed that G1 is more convenient than the other transition curves.

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Received 23 March 2009; accepted 20 December 2010