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MODELLING OF THE INVERSE CREEP OF ROAD BITUMEN MODIFIED WITH SBS COPOLYMER

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Abstract. Polymer modified road bitumens, being examples of viscoelastic materials, are very interesting with respect to rheology. The phenomenon of inverse creep (retarded strain recovery) observed in samples of asphalt binders modified by the addition of SBS copolymer was analyzed in the present paper. Laboratory tests of strain recovery were conducted on samples of the three selected asphalt binders: the 50/70 penetration grade base bitumen and binders modified by the addition of 4% and 8% of the SBS copolymer. The extended procedure for the determination of elastic recovery was used as the research method. The results of the experiment were approximated using three linear viscoelastic rheological models: Burgers, Dual Kelvin+Newton and Dual Kelvin+Maxwell. When analyzing the results of modelling carried out by using the Burgers model it has been found that they were not satisfactory. Much greater compatibility of modelling and experiment results ($R^2 > 0.99$) was achieved by using models containing the dual Kelvin element.

Keywords: bitumen, copolymer, rheology, viscoelasticity, relaxation, strain recovery, inverse creep.

1. Introduction

Since the 1980s polymer modified bitumens have been widely used in road construction as a binder in asphalt mixes. Much research was devoted to demonstrating the beneficial effects of polymer modification on the properties of bitumen and asphalt mixtures (Ho, Zanzotto 2005; Radziszewski 2007; Scholten *et al.* 2010; Yildirim 2007). Improving the elastic properties of bitumen by the polymer addition is particularly important. It should also be noted that the polymer modified binders, being examples of materials exhibiting viscoelastic properties, are very interesting with respect to rheology. Linear viscoelastic rheological models, which are obtained through the serial or parallel connection of Hookean elastic elements (springs) and Newtonian viscous parts (dashpots), are usually used to describe the phenomena occurring in such materials. Different systems of two-, three-, and four-parameter models have been clearly described in the literature (Barnes *et al.* 1989; Derski, Ziemba 1968). In the present study, the phenomenon of retarded strain recovery, also called an inverse creep (Bodnar *et al.* 2006), observed during the tests performed on samples of asphalt binders, has been analyzed. A mathematical description of this phenomenon was made using three viscoelastic models: Burgers, Dual Kelvin+Newton and Dual Kelvin+Maxwell.

The paper presents the theory that a very accurate mathematical description of inverse creep phenomenon of polymer modified bitumen using linear viscoelastic models containing the dual Kelvin element is possible. The objective of the research was modelling the inverse creep using three selected rheological models, on the basis of the results of the experiment carried out on samples of asphalt binders, using a specially developed laboratory methods, evaluation of the modelling results, as well as verification of the formulated theory.

2. Experimental

2.1. Materials

Three asphalt binders were selected for this study: a base bitumen 50/70 penetration grade and two modified binders prepared by blending the base bitumen with the linear styrene-butadiene-styrene (SBS) copolymer, 4% and 8% by weight (marked 50/70 + 4% SBS and 50/70 + 8% SBS, respectively). The base bitumen has been produced from Russian (Ural) crude oil. The specimens of polymer modified binders were mixed at 180 °C (50/70 + 4% SBS) or 190 °C (50/70 + 8% SBS) at a speed of 120 rpm during 2 h. Basic parameters of the tested binders are presented in Table 1.

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Table 1. Basic properties of the investigated asphalt binders

Properties	Bitumen		
	50/70	50/70 + 4% SBS	50/70 + 8% SBS
Penetration at 25 °C, dmm	68.4 ± 1.4	54.4 ± 1.1	43.4 ± 1.2
Softening Point (R&B), °C	44.9 ± 1.0	53.0 ± 1.1	97.6 ± 1.6
Fraass Breaking Point, °C	–7.5 ± 1.8	–9.3 ± 2.3	–16.0 ± 2.5
Viscoelasticity Interval (R&B – Fraass BP), °C	52.4 ± 2.0	62.3 ± 2.6	113.6 ± 3.0

SBS thermoplastic elastomer, Kraton D-1101CM was used to modify the bitumen properties. According to the manufacturer this is a linear block copolymer based on polystyrene (its content is 31 ± 1% m), and polybutadiene. It contains anti-oxidant and is supplied as milled powder. The typical properties of Kraton D-1101CM are: elongation at break – 880%, tensile strength – 33 MPa, 300% modulus – 2.9 MPa, specific gravity – 940 kg/m^3.

2.2. Test method

In this study, the modified elastic recovery test was conducted using a standard ductilometer. Test specimens were prepared according to standard *EN 13398:2010 "Bitumen and Bituminous Binders – Determination of the Elastic Recovery of Modified Bitumen"*. However, the test method was extended when compared to the standard procedure described in *EN 13398:2010*. The test conditions were as follows:

- temperature of the tests: 5, 15, 25, and 35 °C maintained with an accuracy of ±0.5 °C;
- stretching speed: 50 mm/min;
- max elongation: 200 ± 0.5 mm; it corresponds to the max value of strain $\varepsilon_{max} = 6.667$ (666.7%);
- the specimens were cut in half after the following periods of time: 0 s (immediately after stretching), 1800 s (only at 15 °C), 3600 s and 7200 s (only at 15 °C);
- strain recovery was measured at the time t: 2, 5, 10, 15, 20, 25, 30, 40, 50, 60, 75, 90, 105, 120, 135, 150, 165 and 180 min after cutting the samples. In order to determine reliable values of strain recovery of the tested binders, the measuring equipment was expanded by adding a specially designed instrument. Stretched samples (after being cut in half) were placed on the 10 mm thick glass with the calibrated scale. Brass pins attached to the glass plate allowed for accurate placement of the specimens in positions corresponding to the elongation of 200 mm.

The strain recovery tests have been conducted on the specimens cut 30, 60 and 120 min after stretching in order to investigate the relationship between the time of relaxation t_r (at constant strain – elongation of 200 mm) and strain recovery after the specimens were cut in half. Fig. 1 shows the main stages of the procedure described above.

Some parts of the intended research have not been successful. Specimens of 50/70 penetration grade bitumen, tested at 5 °C broke before they reached the elongation of 200 mm and those examined at 35 °C were too soft to test. Moreover, samples of the binder 50/70 + 8% SBS tested at 35 °C have been broken before the end of the specified time of relaxation t_r = 3600 s.

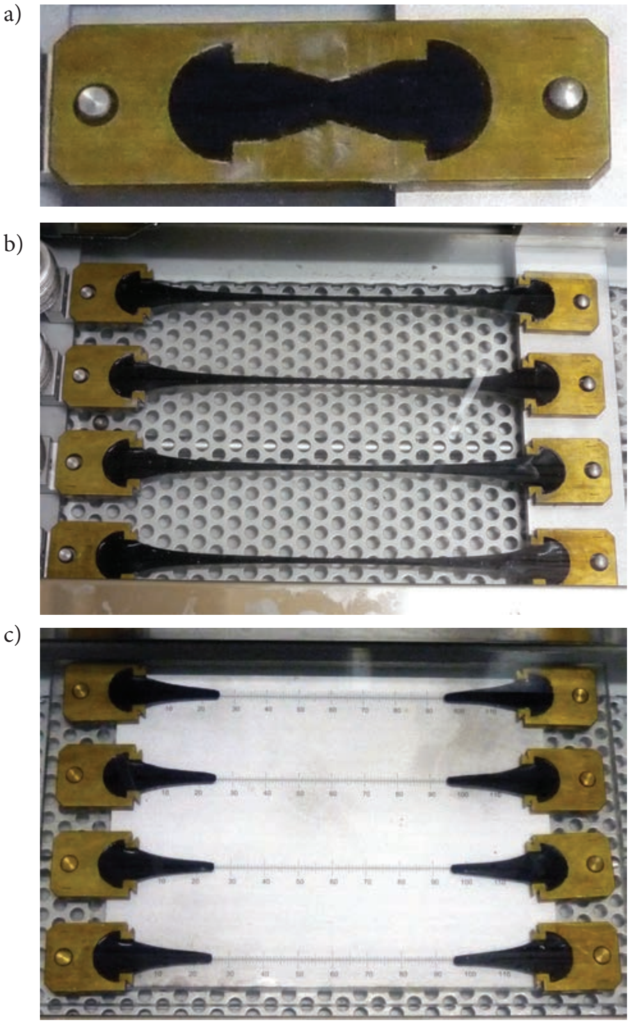

Fig. 1. Stages of procedure for determining the strain recovery: a – sample prepared for testing conditioned in a water bath at a fixed test temperature; b – samples stretched at a constant speed of 50 mm/min until the elongation of 200 mm; maintaining a constant strain in order to achieve the effect of relaxation; c – measuring the strain recovery at specified time intervals by using a calibrated glass plate

3. Test results and modelling

In the process of modelling, the Burgers rheological model (Fig. 2a) was used as the basic one. Researchers have shown that it simulates the behaviour of bitumens quite well (Cebon 2000). The following processes occurring in the case of bitumen and asphalt mixtures can be simulated by using the Burgers model: an immediate strain (elastic), creep, strain recovery: immediate (elastic) and retarded, permanent strain and relaxation at constant strain. An example of practical application of the Burgers model to describe the viscoelastic behaviour of asphalt concrete has been presented in the paper (Laurinavičius *et al.* 2006). The papers (Grabowski *et al.* 2002; Grabowski, Slowik 2003) demonstrate that the use of simpler viscoelastic models, i.e. two-element – the Maxwell and Kelvin models, as well as three-element – the Jeffreys model do not provide satisfactory results of modelling. The exception is the three-element standard model (consisting of Hooke and Kelvin elements connected in series), which can be used when the test material shows no permanent strain. The differential equation for the Burgers model can be written as follows (Skrzypek 1986):

$$\sigma+\left(\frac{\eta_0}{E_0}+\frac{\eta_0}{E_1}+\frac{\eta_1}{E_1}\right)\dot{\sigma}+\frac{\eta_0\,\eta_1}{E_0\,E_1}\ddot{\sigma}=\eta_0\,\dot{\varepsilon}+\frac{\eta_0\,\eta_1}{E_1}\ddot{\varepsilon}, \quad (1)$$

where σ – stress, Pa; ε – strain; E_0, E_1, η_0, η_1 – moduli of elasticity, Pa, and dynamic viscosity, Pas, respectively – according to the diagram shown in Fig. 2a. The total strain in the model at time t is a sum of strain generated in each of the three elements connected in series: Hooke (ε_H), Newton (ε_N) and Kelvin (ε_K):

$$\varepsilon(t)=\varepsilon_H(t)+\varepsilon_N(t)+\varepsilon_K(t). \quad (2)$$

Since: $\varepsilon_H=\frac{\sigma}{E_0}$, $\varepsilon_N(t)=\frac{\sigma t}{\eta_0}$ and $\varepsilon_K(t)=\frac{\sigma}{E_1}\left[1-\exp\left(-\frac{E_1}{\eta_1}t\right)\right]$, then:

$$\varepsilon(t)=\frac{\sigma}{E_0}+\frac{\sigma t}{\eta_0}+\frac{\sigma}{E_1}\left[1-\exp\left(-\frac{E_1}{\eta_1}t\right)\right]. \quad (3)$$

In this paper the author focuses on the problem of mathematical description of the inverse creep of asphalt binders which begins at the moment of unloading (cutting in half) of the samples ($t = 0$, $\sigma = 0$). Strain at time t from the moment of cutting the specimen can be described by the equation:

$$\varepsilon(t)=\frac{\sigma_0}{E_0}+\frac{\sigma_0}{E_1}\exp\left(-\frac{t}{\tau_1}\right)+\varepsilon_\infty, \quad (4)$$

where σ_0 – value of stress just before cutting the sample; τ_1 – retardation time $\left(\tau_1=\frac{\eta_1}{E_1}\right)$; ε_∞ – permanent strain (generated in Newtonian dashpot of viscosity η_0), calculated at $t\to\infty$.

Since there was no possibility of determining the values of σ_0, the Eq (4) allowed for the calculation of the following parameters of the Burgers model: $\frac{\sigma_0}{E_0}$, $\frac{\sigma_0}{E_1}$, τ_1 and ε_∞. The modelling was carried out employing the least squares of deviation and using the *Nonlinear Least Squares Curve Fitter* software. The results of strain determination in samples ε against time t (19 pairs of results for each approximated curve) were the data used for modelling. The parameters calculated using the Burgers model have been presented in Table 2. The effects of modelling have been shown graphically in Figs 3 and 4. The experiment results have been presented as points and the modelling curves as

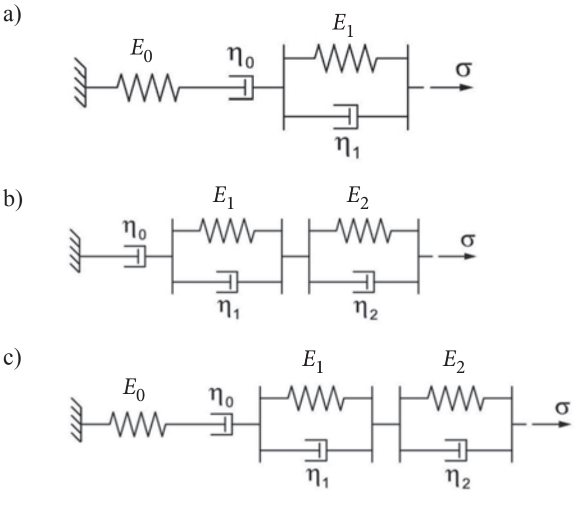

Fig. 2. Schematic representation of linear viscoelastic rheological models applied in the mathematical description of inverse creep of the tested asphalt binders: a – Burgers; b – Dual Kelvin+Newton; c – Dual Kelvin+Maxwell

Table 2. Results of modelling of the inverse creep of the tested asphalt binders – the Burgers model parameters

SBS copolymer content, %	T, °C	t_r, s	$\frac{\sigma_0}{E_0}$	$\frac{\sigma_0}{E_1}$	τ_1, s	ε_∞	R^2
0	15	0	0.201	0.398	3565	6.068	0.972
0	15	1800	0.117	0.415	6459	6.134	0.988
0	15	3600	0.052	1.050	37905	5.565	0.997
0	15	7200	-0.008	0.543	18080	6.132	0.996
0	25	0	0.148	0.754	6372	5.764	0.988
0	25	3600	0.063	0.480	4148	6.124	0.989
4	5	0	2.414	2.748	1205	1.504	0.974
4	5	3600	0.864	3.237	2097	2.565	0.988
4	15	0	3.103	2.142	1020	1.422	0.952
4	15	1800	0.943	3.325	1736	2.399	0.982
4	15	3600	0.590	3.335	2361	2.742	0.989
4	15	7200	0.494	3.282	2861	2.890	0.989
4	25	0	3.023	2.723	1208	0.920	0.966
4	25	3600	1.407	3.905	1490	1.355	0.980
4	35	0	3.842	2.317	614	0.508	0.966
4	35	3600	2.195	3.126	746	1.346	0.970
8	5	0	3.089	2.398	1202	1.179	0.964
8	5	3600	1.327	3.890	1882	1.450	0.984
8	15	0	4.165	2.426	573	0.076	0.977
8	15	1800	3.074	3.436	651	0.156	0.978
8	15	3600	2.617	3.698	882	0.352	0.975
8	15	7200	2.424	3.816	987	0.427	0.975
8	25	0	5.629	1.037	139	0.001	0.999
8	25	3600	5.731	0.851	552	0.084	0.962
8	35	0	6.498	0.168	301	0.001	0.990

lines. All the tests of strain recovery were carried out on six specimens. An average value of uncertainty (confidence interval), calculated in accordance with the procedure described in (Slowik 2010), was about 1% – at 95% confidence level.

Since it was found that the results of modelling carried out by using the Burgers model are not fully satisfactory, both qualitatively (shape of the model curves compared to the experiment results) and quantitatively (values of the coefficient of determination R^2), more complicated models developed by serial connection of two Kelvin elements with Newtonian dashpot (Dual Kelvin+Newton – Fig. 2b) or with the Maxwell element (Dual Kelvin+Maxwell – Fig. 2c), described also by (Cebon 2000) as a general viscoelastic material, were used. For these models, the Eq (4) takes the following forms, respectively:

$$\varepsilon(t)=\frac{\sigma_0}{E_1}\exp\left(-\frac{t}{\tau_1}\right)+\frac{\sigma_0}{E_2}\exp\left(-\frac{t}{\tau_2}\right)+\varepsilon_\infty \qquad (5)$$

or

$$\varepsilon(t)=\frac{\sigma_0}{E_0}+\frac{\sigma_0}{E_1}\exp\left(-\frac{t}{\tau_1}\right)+\frac{\sigma_0}{E_2}\exp\left(-\frac{t}{\tau_2}\right)+\varepsilon_\infty, \qquad (6)$$

where $\tau_1=\frac{\eta_1}{E_1}$; $\tau_2=\frac{\eta_2}{E_2}$; $E_0, E_1, E_2, \eta_1, \eta_2$ – moduli of elasticity, Pa, and dynamic viscosity, Pas, respectively – according to the diagrams shown in Figs 2b and 2c.

Calculated values of the Dual Kelvin+Newton model parameters are presented in Table 3. The effects of modelling have been shown graphically in Figs 5 and 6. The corresponding results achieved by using the Dual Kelvin+Maxwell model have been presented in Table 4, as well as in Figs 7 and 8.

4. Discussion

Simulation of the inverse creep of asphalt binders was possible due to modification and a significant extension of the standard method for determination of elastic recovery, described in *EN 13398:2010*. In particular, the measurement of the length of the samples cut in half using a calibrated glass plate yielded a very positive result contributing to the improvement of readings of the results. It also allowed us to obtain measurement reliability and uncertainty at an acceptable level. There are two important elements of the study, i.e. a significant lengthening of the observation time up to 180 min in the case of strain recovery (according to *EN 13398:2010* the measurement is finished after 30 min

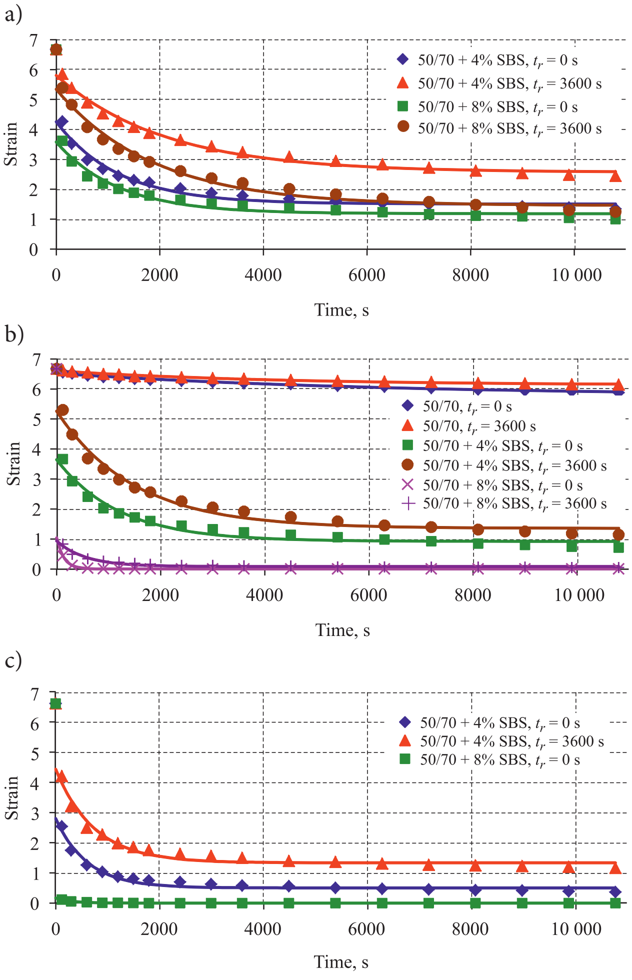

Fig. 3. Inverse creep curves of asphalt binders tested at: a – 5 °C; b – 25 °C; c – 35 °C; the Burgers model

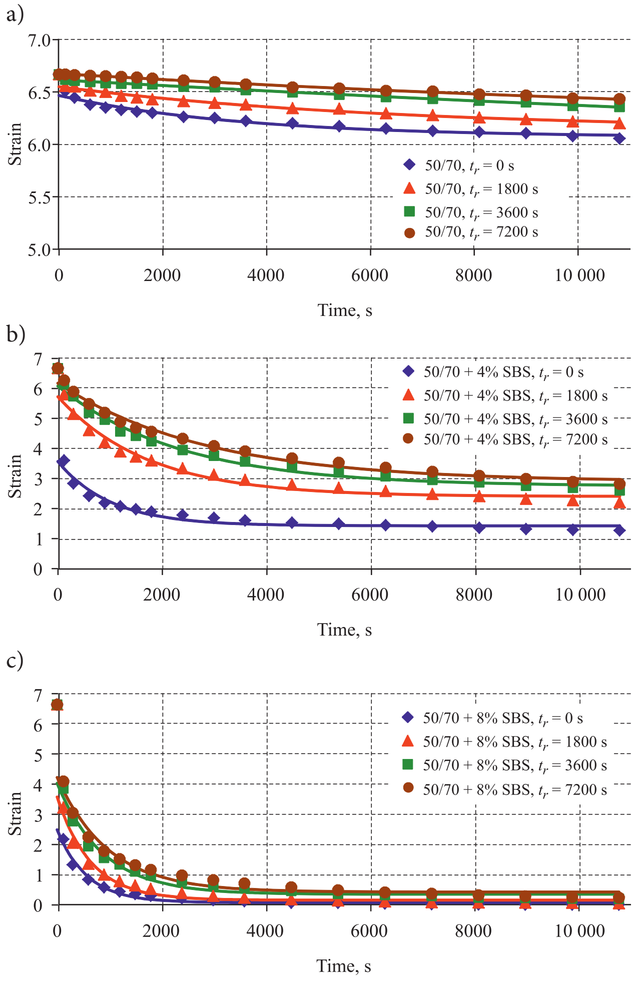

Fig. 4. Inverse creep curves of asphalt binders tested at 15 °C: a – 50/70; b – 50/70 + 4% SBS; c – 50/70 + 8% SBS; the Burgers model

Table 3. Results of modelling of the inverse creep of the tested asphalt binders – parameters of the Dual Kelvin+Newton model

SBS copolymer content, %	T, °C	t_r, s	$\frac{\sigma_0}{E_1}$	τ_1, s	$\frac{\sigma_0}{E_2}$	τ_2, s	ε_∞	R^2
0	15	0	0.409	6600	0.262	174	5.989	0.997
0	15	1800	0.437	7742	0.129	91	6.100	0.992
0	15	3600	1.251	46563	0.054	59	5.362	0.998
0	15	7200	0.435	12317	−0,021	656	6.252	0.997
0	25	0	1.026	13830	0.213	350	5.411	0.998
0	25	3600	0.579	12206	0.179	807	5.900	0.998
4	5	0	2.131	1831	3.098	97	1.428	0.998
4	5	3600	2.742	3131	1.476	258	2.397	0.998
4	15	0	1.464	1966	3.863	84	1.333	0.998
4	15	1800	2.513	3130	1.920	293	2.188	0.999
4	15	3600	2.709	4307	1.477	487	2.421	0.999
4	15	7200	2.887	5601	1.334	565	2.407	1.000
4	25	0	2.094	1939	3.737	86	0.827	0.996
4	25	3600	2.900	2567	2.547	221	1.172	0.998
4	35	0	1.521	1047	4.680	66	0.464	0.998
4	35	3600	1.864	1470	3.524	124	1.261	0.998
8	5	0	1.830	1953	3.735	81	1.096	0.997
8	5	3600	3.137	3089	2.227	240	1.222	0.997
8	15	0	1.659	891	4.964	61	0.043	0.999
8	15	1800	2.280	1052	4.280	90	0.102	0.999
8	15	3600	2.500	1503	3.885	117	0.262	0.998
8	15	7200	2.646	1680	3.673	128	0.320	0.998
8	25	0	0.484	210	6.183	33	0.000	1.000
8	25	3600	0.535	981	6.063	40	0.069	1.000
8	35	0	0.109	441	6.558	23	0.000	1.000

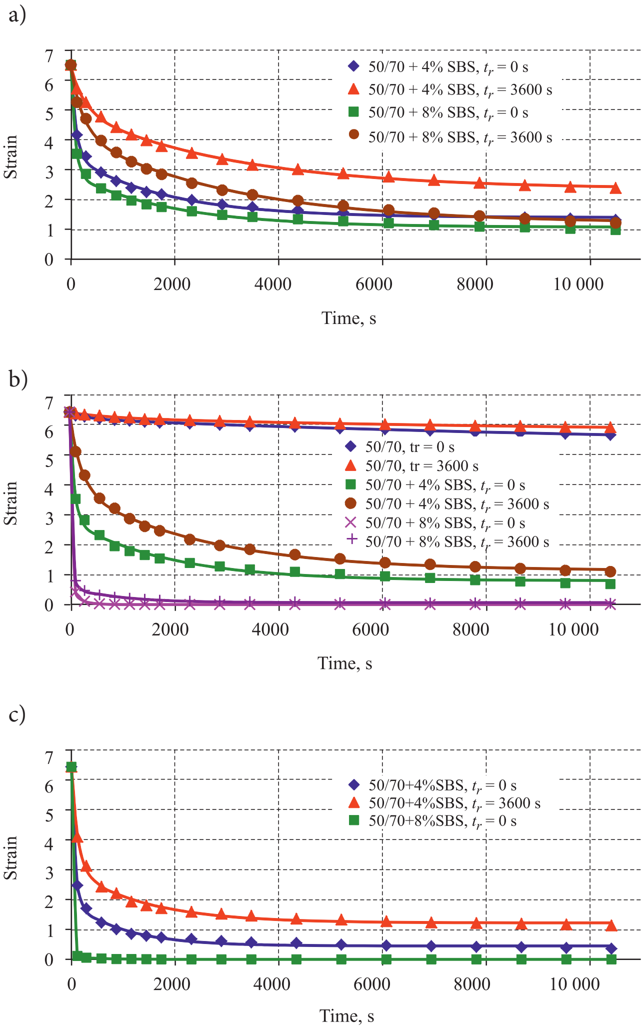

Fig. 5. Inverse creep curves of asphalt binders tested at: a – 5 °C; b – 25 °C; c – 35 °C; the Dual Kelvin+Newton model

since cutting the samples) and an attempt to analyze the impact of the time of relaxation on the inverse creep curves of the tested asphalt binders. This phenomenon was studied at length in the case of the tests conducted at a temperature of 15 °C (time of relaxation t_r was equal to 0, 1800, 3600 and 7200 s). In other cases (tests carried out at 5, 25 and 35 °C), only the behaviour of samples cut immediately after stretching (t_r = 0 s) and samples whose constant level of strain has been maintained for a period t_r = 3600 s were compared.

Specimens of asphalt binders used for testing were selected in such a way as to obtain large differences in their viscoelastic properties. Thus, a sample of the bitumen 50/70 modified with the addition of 8% of SBS copolymer can be regarded as an example of highly modified bitumen which under certain conditions shows a complete strain recovery (no permanent strain). A sample of the bitumen 50/70 with the addition of 4% of SBS copolymer can be considered an example of medium-modified bitumen, which shows a permanent strain. A sample of the 50/70 base bitumen has been used in the study as a reference – an example of unmodified bitumen which shows unfavourable elastic properties and a considerable degree of permanent deformation. The procedure for modelling of the observed inverse creep started with the use of the Burgers model (Fig. 2a) – a very popular and often described in the literature. This model has been chosen due to the fact that the viscous and elastic elements were connected in such a way that the model can be used to describe all the phenomena observed during testing the asphalt binders, i.e. immediate strain, creep, strain recovery – both immediate and retarded, permanent strain, and also relaxation at constant strain. Upon analyzing the results of modelling carried out by using the Burgers model it can be noted that despite the relatively high values of the coefficient of determination R^2 (from 0.952 to 0.999), the modelling curves differ significantly from the results of the experiment. The most significant incompatibilities have been found in the case of the immediate strain recovery and permanent strain values. For a number of specimens, it was observed that the values of immediate strain recovery, calculated using the Burgers model are larger than the corresponding results achieved 2 min after cutting the samples. A similar problem occurred in the

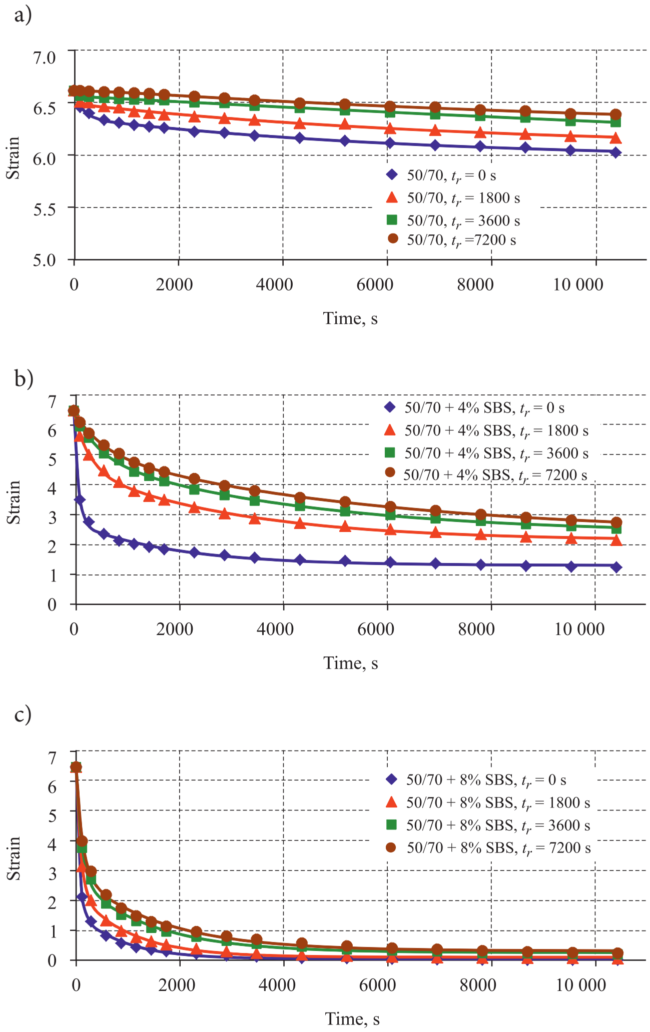

Fig. 6. Inverse creep curves of asphalt binders tested at 15°C: a – 50/70; b – 50/70 + 4% SBS; c – 50/70 + 8% SBS; the Dual Kelvin+Newton model

Table 4. Results of modelling of the inverse creep of the tested asphalt binders – parameters of the Dual Kelvin+Maxwell model

SBS copolymer content, %	T, °C	t_r, s	$\frac{\sigma_0}{E_0}$	$\frac{\sigma_0}{E_1}$	τ_1, s	$\frac{\sigma_0}{E_2}$	τ_2, s	ε_∞	R^2
0	15	0	0.102	0.441	8973	0.191	368	5.932	0.998
0	15	1800	0.087	1.264	44966	0.112	983	5.203	0.997
0	15	3600	0.019	1.263	47077	0.034	76	5.350	0.997
0	15	7200	0.002	0.434	12316	−0.022	624	6.253	0.997
0	25	0	0.076	1.492	25259	0.196	713	4.903	0.999
0	25	3600	0.025	0.713	18898	0.194	1074	5.734	0.998
4	5	0	1.711	1.663	2685	1.938	302	1.355	0.999
4	5	3600	0.502	2.321	4522	1.618	645	2.226	1.000
4	15	0	2.188	1.181	2994	2.033	225	1.265	0.999
4	15	1800	0.406	2.256	3908	1.907	480	2.097	0.999
4	15	3600	0.254	2.507	5584	1.650	749	2.256	0.999
4	15	7200	0.140	2.844	6349	1.384	708	2.298	1.000
4	25	0	2.431	1.467	4481	2.170	399	0.600	0.999
4	25	3600	0.701	2.494	3310	2.393	416	1.079	0.999
4	35	0	3.305	0.718	3133	2.274	281	0.370	0.999
4	35	3600	1.407	1.257	2479	2.806	280	1.196	0.998
8	5	0	2.428	1.368	3563	1.904	321	0.966	0.999
8	5	3600	0.800	2.638	4729	2.244	588	0.985	1.000
8	15	0	3.474	0.951	1549	2.228	218	0.013	0.999
8	15	1800	2.010	1.621	1475	2.961	208	0.074	0.999
8	15	3600	1.655	1.853	2167	2.953	274	0.205	1.000
8	15	7200	1.608	1.878	2681	2.956	337	0.226	1.000
8	25	0	5.034	0.375	229	1.257	68	0.000	1.000
8	25	3600	5.300	0.361	1523	0.946	156	0.059	0.998
8	35	0	6.413	0.089	496	0.165	95	0.000	0.998

case of permanent deformation, where the calculated values ε_∞ (when $t \to \infty$) were higher than the results obtained 180 min after cutting the samples. The above-mentioned incompatibilities, which can be clearly seen in Figs 3a, 3b and 4b, are hard to accept. The discussion presented above leads to the conclusion that the Burgers model is not useful for quantitative description of the inverse creep of the analyzed asphalt binders. It was therefore decided to use other, more complicated viscoelastic models. Applying the generalized Kelvin or Maxwell models has been taken into consideration. However, in the analyzed case, they are not useful due to the limitations of their construction (the first one shows no permanent deformation, and the second one does not show the complete strain recovery). Thus, two models constructed through modification of the Burgers one, has been chosen. The first one has been made by replacing the free Hookean spring (modulus of elasticity E_0) in the Burgers model with a Kelvin element (of the parameters E_2 and η_2). Thus, a model consisting of two Kelvin elements and a Newtonian dashpot connected in series has been developed. This model does not describe the immediate strain recovery. However, due to the use of the dual Kelvin element (two retardation times), it effectively describes the large curvature observed when analyzing the strain recovery against time relationship, resulting in high-speed inverse creep in the initial phase of the test – just after cutting the samples. As an effect, high values of the coefficient of determination R^2 (from 0.992 to 1.000) have been achieved. The third model was obtained by serial connection of previously described one with the additional Hookean spring (modulus of elasticity E_0). The result is a model consisting of two Kelvin and one Maxwell elements connected in series (Fig. 2c). The results achieved by using this model show the best compatibility with the results of the experiment, as evidenced by the highest value of the coefficient of determination R^2 (from 0.997 to 1.000). Furthermore, a description of the relaxation effect of reducing the value of immediate strain recovery, as well as increase of the permanent strain value in the case of polymer modified asphalt binders can be regarded as correct.

It can be said that those two models containing dual Kelvin element yielded acceptable results. The main difference is due to their behaviour just after cutting the samples. The model which contains a free Hookean spring

(Fig. 2c) describes the phenomenon of immediate strain recovery. On the other hand, there is no such possibility in the case of the model without that element (Fig. 2b). Due to the test method used, it was not possible to accurately measure the length of the samples right after unloading. The first reliable results have been obtained 2 min after cutting the samples. Therefore, it is difficult to say which of the two models simulate, in a manner closer to reality, the course of the observed inverse creep of samples previously subjected to tension.

The author has observed a different nature of the strain changes with regard to time in the latest studies of copolymer SBS modified binders, carried out using Bending Beam Rheometer (BBR) at the following temperatures: –40 °C, –32 °C, –24 °C and –16 °C. On the basis of those test results, a very high value of the immediate deflection recovery of the tested beams (measured already 0.5 s after unloading the beam) was discovered. It can be assumed that the use of the Dual Kelvin+Maxwell model in this case should be the most appropriate. On the other hand, the mathematical description proposed by using the Dual Kelvin+Newton model, could differ significantly from the experiment results.

One of the practical implications resulting from the application of mathematical methods for describing the inverse creep of asphalt binders specimens (the Dual Kelvin+Maxwell model in particular) discussed in the present paper is a possibility of calculating the values of two important properties, which would be practically impossible to determine following the laboratory test method used in the study. These are: immediate strain recovery $\left(\frac{\sigma_0}{E_0}\right)$ and permanent strain ε_∞. Based on these parameters, we can precisely determine the coefficient of retardation α_s (Judycki 1989), which shall take the following, modified form (symbols as in the previous formulas):

$$\alpha_s = \left(1 - \frac{\sigma_0}{E_0(\varepsilon_{\max} - \varepsilon_\infty)}\right)100, \%. \tag{7}$$

The coefficient of retardation α_s is a measure of strain recovery rate after cutting the samples, and may be a useful parameter to assess the effectiveness of bitumen modification by polymer addition. The author intends to develop this problem further in the publications to follow.

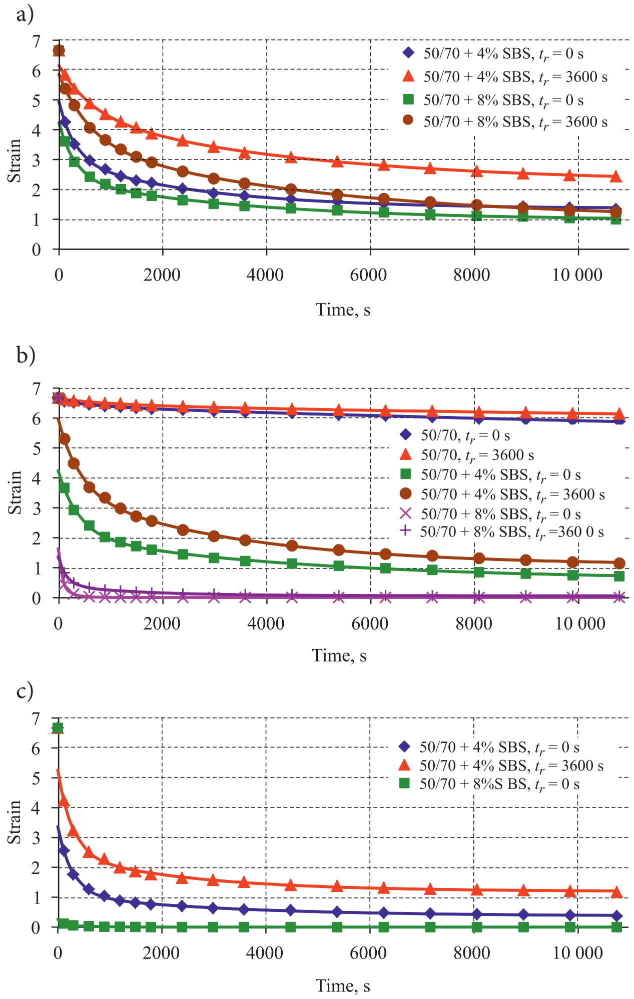

Fig. 7. Inverse creep curves of asphalt binders tested at: a – 5 °C; b – 25 °C; c – 35 °C; the Dual Kelvin+Maxwell model

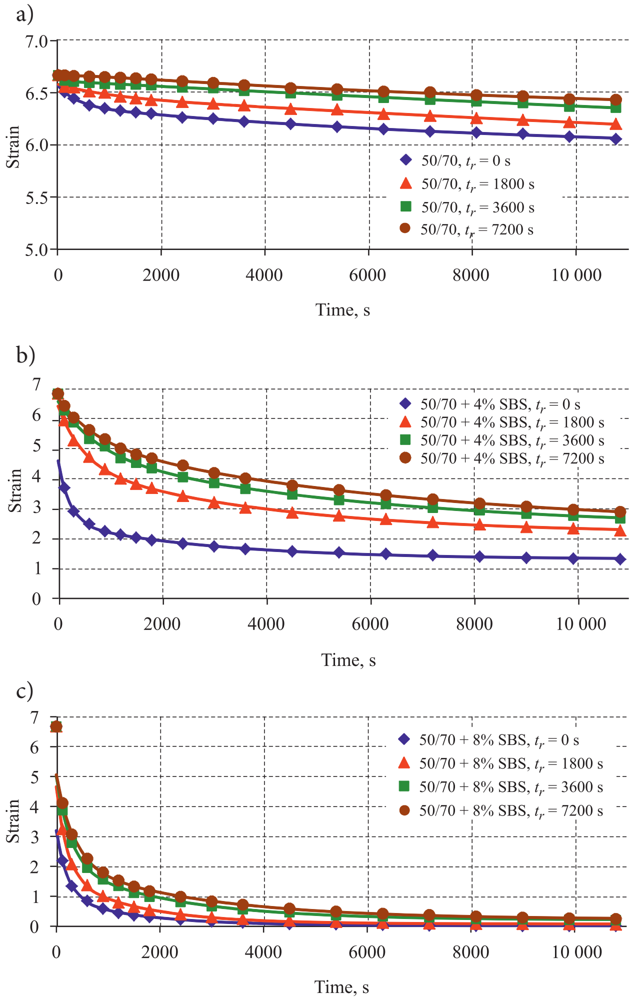

Fig. 8. Inverse creep curves of asphalt binders tested at 15 °C: a – 50/70; b – 50/70 + 4% SBS; c – 50/70 + 8% SBS; the Dual Kelvin+Maxwell model

5. Conclusions

The Burgers model which is regarded as a very useful model describing rheological phenomena occurring in the case of bitumen and asphalt mixes does not give fully satisfactory results of modelling of the inverse creep of the investigated asphalt binders.

Much better compatibility of the modelling results in comparison with the results of the experiment ($R^2 > 0.99$) was obtained by using models consisting of the dual Kelvin element connected in series with Newton or Maxwell elements. Applying the Dual Kelvin+Maxwell allows for the calculation of two important values, i.e. immediate strain recovery and permanent strain of the tested binders, which are difficult to measure by the laboratory method.

The results of modelling carried out in accordance with the procedure presented in the paper can be used in practice to determine the coefficient of retardation in order to assess the efficiency of bitumen modification with polymers.

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