



## OPTIMIZATION OF BRIDGE TRUSSES HEIGHT AND BARS CROSS-SECTIONS

Stanislovas Kalanta<sup>1</sup>, Juozas Atkočiūnas<sup>2</sup>, Tomas Ulitinas<sup>3</sup>✉, Andrius Grigusevičius<sup>4</sup>

Dept of Structural Mechanics, Vilnius Gediminas Technical University,  
Saulėtekio al. 11, 10223 Vilnius, Lithuania

E-mails: <sup>1</sup>stanislovas.kalanta@vgtu.lt; <sup>2</sup>juozas.atkociunas@vgtu.lt; <sup>3</sup>tomas.ulitinas@vgtu.lt;

<sup>4</sup>andrius.grigusevicius@vgtu.lt

**Abstract.** The problems of optimal design of truss-type structures, aimed at determining the minimal volume (weight) of the structure, while optimizing the bar cross-sections and the truss height, are considered. The considered problem is treated as a nonlinear problem of discrete optimization. In addition to the internal forces of tension or compression, the elements of the truss can have the bending moments. The cross-sections of the bars are designed of the rolled steel profiles. The mathematical models of the problem are developed, taking into account stiffness and stability requirements to structures. Nonlinear discrete optimization problems, formulated in this paper, are solved by the iterative method using the mathematical programming environment MATLAB. The buckling ratios of the bars under compression are adjusted in each iteration. The requirements of cross-section assortment (discretion) are secured using the method of branch and bound.

**Keywords:** elastic truss bar structures, discrete optimization, finite element method, nonlinear optimization problem.

### 1. Introduction

The paper considers the problem of weight (volume) optimization of elastic trusses by optimizing the cross-sections of their elements and the height of the structure. Truss bars can have not only the internal forces of tension or compression but also the bending moments. The simplest problems of truss optimization were formulated in (Haug, Arora 1980; Maciulevicius 1964, 1966; Majid 1974; Rao 2009) but the form of the bar cross-section was not defined in these works. In (Maciulevicius 1964, 1966) the cross-sections and the layout of the bars are optimized, when the coordinates of the nodes are known, and the assumptions are made that the radii of the cross-sections and slenderness ratios of the bars do not vary. Other works (Goremikins, Serdjus 2010; Kalanta *et al.* 2009) describe the optimization problems where the top chord of the truss is subjected to distributed loading and, therefore, the truss bars have the bending moments. The design parameters are the areas of the bars' cross-sections and the height of the truss. The optimization parameters are defined by performing the design of various fixed-height trusses, using computer-aided design programs. The rectangular cross-sections are designed, when the height-width relationship is fixed and only one load case is evaluated (Goremikins, Serdjus 2010), or when the cross-sections are chosen from steel profiles' assortments and up to three load cases are

considered (Kalanta *et al.* 2009). In both cases, the optimal truss height was determined without the direct computation of mathematical programming problems, though, in (Kalanta *et al.* 2009) the mathematical models for the height optimization problem of the truss with horizontal and parabolic bottom chord were offered.

The aim of the present work is to develop the mathematical models and solution algorithms for the optimization problems of the elastic truss height and bar cross-section design. The mathematical models of the problems are formulated and solved as nonlinear discrete mathematical programming problems. According *STR2.05.08:2005 "Design of Steel Structures"* the finite element method is applied, taking into account the strength and stability requirements (Atkočiūnas *et al.* 2011; Kalanta 1995). The cross-sections of the bars are designed with standard steel profiles.

Various specific algorithms for solving nonlinear structural optimization problems, such as genetic (Hayalioglu 2000; Hayalioglu, Degertekin 2004; Zheng *et al.* 2006), discrete optimization (Gutkowski 1997) and other algorithms (Karkauskas 2004; Manickarajah *et al.* 2000; Yuge *et al.* 1999; Feng *et al.* 2006), have been developed in recent years. In this paper, the solutions of the optimization problems are made by using the mathematical programming environment MATLAB. An iterative method

with the buckling ratios of bars under compression adjusted in each iteration is used. The requirements of cross-section discretisation are secured, using the method of branch and bound.

## 2. Discrete model and main dependencies of the structure

### 2.1. Discrete model

The equilibrium finite elements of four types are created (Kalanta 2007) for modelling any bar structure. However, in this paper, the trusses are modelled by using the following two types of elements  $k = 1, 2, \dots, r$  (Kalanta 1995, 2007):

- 1) the element under tension or compression (Fig. 1a subjected to the action of the axial force  $N_k(x) = N_k$ ;
- 2) the element under bending and tension or compression with three nodes (Fig. 1b), in which  $N_k(x) = N_k$ , and the bending moments are described by the parabolic function as follows:

$$M_k(x) = \left(1 - \frac{3x}{l_k} + \frac{2x^2}{l_k^2}\right)M_{k1} + \left(\frac{4x}{l_k} - \frac{4x^2}{l_k^2}\right)M_{k2} + \left(-\frac{x}{l_k} + \frac{2x^2}{l_k^2}\right)M_{k3} = \sum_{j=1}^3 H_{kj}(x)M_{kj}; \quad (1)$$

where  $H_{kj}(x)$  – the form function of the bending moments;  $M_{kj}$  – the bending moment of the  $j^{\text{th}}$  node in the element;  $x$  – the coordinate of the section.

The internal forces of any  $k^{\text{th}}$  element are described by such an interpolation function

$$S_k(x) = [H_k(x)]S_k, \quad (2)$$

where  $S_k$  – the vector of the internal forces of the element nodes;  $[H_k(x)]$  – the interpolation matrix of the internal forces composed of the form functions of the internal forces  $H_{kj}(x)$ .

### 2.2. Main dependencies

The main dependencies of the structure are represented by the equilibrium and geometric equations and strength and stability conditions.

#### 2.2.1. Equilibrium equations

The equilibrium equations make two groups:

- the equilibrium equations of nodes, which relate the element's nodal internal and external forces, acting in the nodes;
- the equilibrium equations of the bending elements, which relate the element's nodal internal forces and the external force acting on the element. These equations are formed only for the elements subjected to distributed loads, i.e. for the element of the

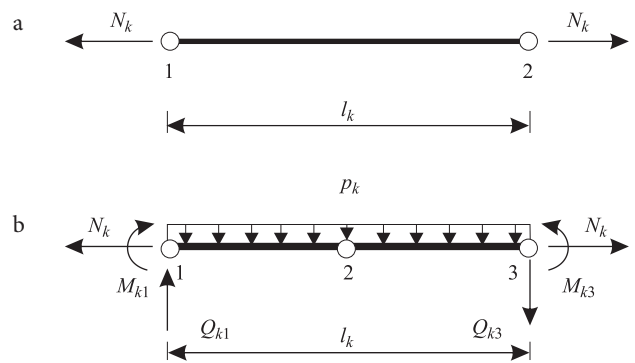


Fig. 1. Finite elements: a – the element under tension or compression; b – the second-order element or the element under bending and tension or compression

second type. The equilibrium of the internal and external forces of these elements is described by the equation (Kalanta 2007):

$$\frac{4}{l_k^2}(-M_{k1} + 2M_{k2} - M_{k3}) = p_k, \quad k = 1, 2, \dots, r,$$

where  $p_k$  – the intensity of the element's distributed loading. This equation is derived from the differential equilibrium equation of the bending element

$$-\frac{d^2 M_k(x)}{dx^2} = p_k,$$

considering the bending moment function (1).

Then system of equilibrium equations for all finite elements and nodes are:

$$[A]S = F, \quad (3)$$

where  $S = [S_1, S_2, \dots, S_r]$  denotes the vector of the internal forces of the structure, composed of the elements' vector of the internal forces  $S_k$ ;  $F$  – the vector of the external forces.

The matrix  $[A]$  of the equilibrium equations' ratios can also be formed using the equilibrium equations of the finite elements

$$P_k = [A_k]S_k, \quad k = 1, 2, \dots, r,$$

where the relation between the elements' reactions  $P_k$  (Fig. 2) and the internal forces  $S_k$  (Fig. 1) is described. With reference to the principal of virtual displacements (Kalanta 1995) the formula for calculating the matrix of the equilibrium equations' ratios is derived as follows:

$$[A] = [C]^T [\bar{A}],$$

where  $[\bar{A}] = \text{diag}[A_k]$  – a quasi-diagonal matrix, diagonally filled with the blocks of matrices  $[A_k]$ ,  $k = 1, 2, \dots, r$   $[C]$  – the matrix of the compatibility equations  $\bar{u} = [C]u$  of the displacements of the elements' nodes  $\bar{u} = [u_1, u_2, \dots, u_r]^T$  and the global displacements  $u$  of the structure's nodes.

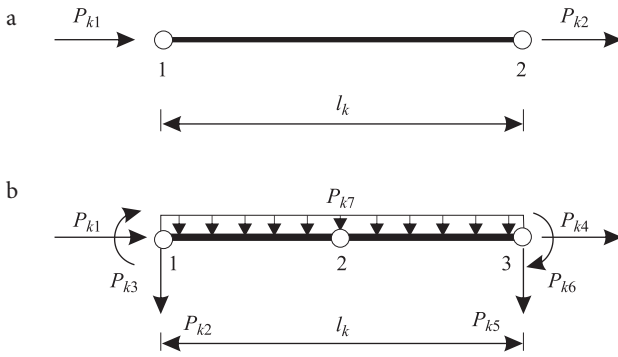


Fig. 2. Finite element reactions of positive directions

The matrices of equilibrium equations for the elements considered in this paper (Fig. 1) are as follows (Kalanta 2007):

$$\mathbf{P}_k = \begin{bmatrix} P_{k1} \\ P_{k2} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} N_k,$$

$$[\mathbf{P}_k] = \begin{bmatrix} P_{k1} \\ P_{k2} \\ P_{k3} \\ P_{k4} \\ P_{k5} \\ P_{k6} \\ P_{k7} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ \frac{3}{l_k} & \frac{-4}{l_k} & \frac{1}{l_k} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{l_k} & \frac{-4}{l_k} & \frac{3}{l_k} & 0 \\ \frac{1}{l_k} & \frac{-4}{l_k} & \frac{1}{l_k} & 0 \\ 0 & 0 & -1 & 0 \\ \frac{-4}{l_k^2} & \frac{8}{l_k^2} & \frac{-4}{l_k^2} & 0 \end{bmatrix} \begin{bmatrix} M_{k1} \\ M_{k2} \\ M_{k3} \\ N_k \end{bmatrix}.$$

## 2.2.2. Geometrical equations

Geometrical equations

$$\Theta = [A]^T \mathbf{u} \quad (4)$$

are derived by applying the principle of virtual forces. These equations relate the strains of the bars  $\Theta$  to the displacements of the nodes  $\mathbf{u}$ . Physical equations  $\Theta = [D]\mathbf{S}$  can be inserted into the equations' system (4) and the geometric equation takes the form:

$$[D]\mathbf{S} - [A]^T \mathbf{u} = \mathbf{0}, \quad (5)$$

where  $[D] = \text{diag}[D_k]$  and

$$[D_k] = \int_0^{l_k} [H_k(x)]^T [d_k] [H_k(x)] dx \quad (6)$$

flexibility matrix of the  $k^{\text{th}}$  element;  $[d_k]$  – flexibility matrix of the infinitesimal element. The ratios of matrix  $[D_k]$  are calculated by the formula  $d_{ij} = d_k \int_0^{l_k} H_{ki}(x) H_{kj}(x) dx$ ,

where  $d_k = \frac{1}{EA_k}$  for tension or compression, while  $d_k = \frac{1}{EI_k}$  for bending;  $I_k$  – the inertia moment of cross-section,  $E$  – the elasticity modulus. For the element under tension or compression  $[D_k] = \frac{l_k}{EA_k}$ , while for the element under bending and tension or compression

$$[D_k] = \begin{bmatrix} [D_{k1}] & & & \\ & [D_{k2}] & & \\ & & & \\ & & & \end{bmatrix} = \frac{l_k}{30EI_k} \begin{bmatrix} 4 & 2 & -1 & 0 \\ 2 & 16 & 2 & 0 \\ -1 & 2 & 4 & 0 \\ 0 & 0 & 0 & \frac{30I_k}{A_k} \end{bmatrix}.$$

## 2.2.3. Strength and stability conditions

Strength condition of the  $j^{\text{th}}$  section of the element under bending and tension or compression is described by the inequality:

$$\sigma_{j,\max} = \frac{|N_j|}{A_j} + \frac{|M_j|}{W_j} \leq R, \quad (7)$$

where  $A_j$ ,  $W_j$  are the cross-section area and the resistance moment;  $R$  – the design strength of the material (taking into account the reliability factor). Multiplying it by  $A_j$ , the following strength condition is obtained:

$$|N_j| + c_j |M_j| \leq RA_j = N_{0j}$$

or

$$\begin{aligned} N_j + c_j M_j - RA_j &\leq 0, & -N_j + c_j M_j - RA_j &\leq 0, \\ N_j - c_j M_j - RA_j &\leq 0, & -N_j - c_j M_j - RA_j &\leq 0, \end{aligned} \quad (8)$$

where  $c_j = \frac{A_j}{W_j}$ ,  $N_{0j} = RA_j$  – the design axial force,

determining the load-bearing capacity of the elastic cross-section. These strength conditions are verified at all finite element nodes and in the critical sections of the elements subjected to distributed loading and where the bending moment is the highest. The coordinate of the critical section

$$x = \frac{(3M_{k1} - 4M_{k2} + M_{k3})l_k}{(4M_{k1} - 8M_{k2} + 4M_{k3})}$$

is determined from the condition  $Q = \frac{dM_k(x)}{dx} = 0$ , while the bending moment of the critical section is

$$M_{kx} = b_1 M_{k1} + b_2 4M_{k2} + b_3 M_{k3},$$

where  $b_1 = 1 - \frac{3x}{l_k} + \frac{2x^2}{l_k^2}$ ,  $b_2 = \frac{4x}{l_k} - \frac{4x^2}{l_k^2}$ ,  $b_3 = -\frac{x}{l_k} + \frac{2x^2}{l_k^2}$ .

The strength conditions of all elements of the structure are described by the inequality:

$$[\Phi_s]S - [G_s]A_0 \leq 0; \quad (9)$$

where  $A_0$  – the vector of the cross-section areas (design variables). Non-zero elements of the matrix  $[G_s]$  are equal to the products  $RA_j$ .

Furthermore, the bars under compression have to satisfy the stability inequality:

$$\frac{|N_j|}{\phi_j A_j} \leq R_{st} \text{ or } -N_j \leq \phi_j A_j R_{st}. \quad (10)$$

The buckling ratio  $\phi_j$  of the bars under central and non-central compression is determined according to the design code *STR2.05.08:2005*. The stability conditions of all elements under compression are described by a similar inequality:

$$[\Phi_p]S - [G_p]A_0 \leq 0. \quad (11)$$

Non-zero elements of the matrix  $[G_p]$  are equal to the products  $\phi_j A_j R_{st}$ .

By combining the conditions (9) and (11) the following inequality, describing the strength and stability conditions of the structure, is derived:

$$[\Phi]S - [G]A_0 \leq 0. \quad (12)$$

### 2.2.4. Stiffness conditions

$$[E]u \leq u^+ \quad (13)$$

are described by the constraints of the structure's displacements of the truss nodes  $u_i \leq u_i^+$ , where  $u_i^+$  is the maximal value of the  $i^{\text{th}}$  admissible displacement.

## 3. Mathematical model and solution algorithm of optimization problem

### 3.1. Mathematical models

A bar structure subjected to the load cases  $v = 1, 2, \dots, p$  is considered. Bars are designed based on the set  $\Pi$  of steel rolled profiles. Let us describe the cross-section areas of the bars by the vector  $A_0$ , whereas the loading, internal forces and displacements of the  $v^{\text{th}}$  load case are described by the vectors  $F_v, S_v, u_v$ . It is clear that all these vectors must satisfy the equilibrium and geometric equations, as well as strength, stability and stiffness conditions of the structure, likewise the requirements of the profiles' assortment and structural requirements. Thus, based on the described dependencies, the minimal volume (weight) design problem of the elastic structure is described by a mathematical model as follows:

$$\min f = L(h)^T A_0, \quad (14)$$

when

$$\begin{aligned} [A(h)]S_v &= F_v, \\ [D(A_0, h)]S_v - [A(h)]^T u_v &= 0, \\ [\Phi(A_0)]S_v - [G(\phi)]A_0 &\leq 0, \\ [E]u_v &\leq u^+, A_0 \geq A_0^-, A_0 \in \Pi, \\ v &= 1, 2, \dots, p. \end{aligned} \quad (15)$$

The main unknowns in this model are the vectors  $A_0, h, u_v, S_v, k = 1, 2, \dots, r$  where  $h$  is the vector of the truss height parameters  $h_j$ ;  $A_0 \geq A_0^-$  are structural constraints of cross-sections. Since the geometry of trusses is defined only partially, the vector of the bars' lengths  $L(h)$ , static equations' matrix  $[A(h)]$  and flexibility matrix of geometric equations  $[D(A_0, h)]$  are also unknown. The submatrix  $[\Phi_s(A_0)]$  of the matrix  $[\Phi(A_0)]$  depends on the areas of the bars' cross-sections because it involves the ratios  $c_j = \frac{A_j}{W_j}$ , while the submatrix  $[G_p(\phi)]$  of the matrix

$[G(\phi)]$  depends on the buckling ratios  $\phi$ , the values of which are the functions of the elements' slenderness (for the elements under bending, these are also the functions of the internal forces). Thus, the objective function, equilibrium, geometric equations, as well as strength and stability conditions in the problem (14)–(15), are nonlinear. The cross-section areas  $A_j$  should be taken from the set  $\Pi$  of cross-sections, i.e. from the profiles' assortment. Therefore, a mathematical model of the optimization problem is the problem of nonlinear discrete programming. The height parameters  $h_j$  of the structure and cross-sectional areas of the bars  $A_j$ , which would correspond to the minimal volume (weight) of the structure and satisfy the requirements of strength, stiffness and stability, should be found.

By eliminating the internal forces

$$S_v = [D(A_0, h)]^{-1} [A(h)]^T u_v \text{ and geometric equations,}$$

tions, the model can be rearranged to make the following optimization problem:

$$\min f = L(h)^T A_0, \quad (16)$$

when

$$\begin{aligned} [A(h)][D(A_0, h)]^{-1}[A(h)]^T u_v &= F_v, \\ [\Phi(A_0)][D(A_0, h)]^{-1}[A(h)]^T u_v - [G(\phi)]A_0 &\leq 0, \\ [E]u_v &\leq u^+, A_0 \geq A_0^-, A_0 \in \Pi, \\ v &= 1, 2, \dots, p, \end{aligned} \quad (17)$$

where  $[A(h)][D(A_0, h)]^{-1}[A(h)]^T = [K(A_0, h)]$  is the global stiffness matrix of the structure.

**3.2. Solution algorithm**

The number of the unknowns in the mathematical model (16)–(17) is smaller than that found in the problem (14)–(15). However, the degree of nonlinearity of the constraints (17) is higher, therefore, the solution of the problem (16)–(17) is more complicated. For this reason, the optimal solution should be sought applying the mathematical model (14)–(15).

These functions are used for the cross-section inertia moment  $I$  and the factor  $c$

$$I = a_1 A^{b_1}, \quad c = a_2 A^{b_2}, \quad (18)$$

describing their relation with the cross-section area  $A$ , where  $a_1, b_1, a_2, b_2$  are the ratios, depending on the type of the profile. The values of these ratios can be determined by using the regression, i.e. by applying the least squares method. For example, for the IPE-type profile,  $a_1 = 0.8036, b_1 = 2.3165, a_2 = 1.36568, b_2 = -0.6637$ . However, for the IPEA-type or other profiles these ratios are different.

Since it is hardly possible to describe the buckling ratios by the analytical (18) or other functions (herewith relating these ratios with the unknowns) the problem is solved by the iterative method. In the first iteration, any buckling ratio values  $\varphi_j \leq 1$  are chosen. In other iterations, the values of the buckling ratios are determined depending on the slenderness values of the elements under compression, which are calculated for the cross-sections selected from the profiles' assortments according to the areas' values  $A_j$ , which were used in the previous iteration. The buckling ratios of non-centrally compressed elements are calculated, taking into account the eccentricity of load (Kala et al. 2010; Merkevičiūtė et al. 2006). The requirements of the profiles' assortment are satisfied by applying the method of branch and bound (Kalanta 2007). The iterative process is completed when the values of the vectors  $A_0, \mathbf{h}$  obtained in iteration, match the specified values.

**4. Optimization results of the simple truss**

The elastic truss subjected by two load cases is given (Fig. 3):

load case 1:  $F_{1,1} = 100 \text{ kN}, F_{1,2} = 100 \text{ kN}, F_{1,3} = 100 \text{ kN}$ ;

load case 2:  $F_{2,1} = 100 \text{ kN}, F_{2,2} = 70 \text{ kN}, F_{2,3} = 40 \text{ kN}$ .

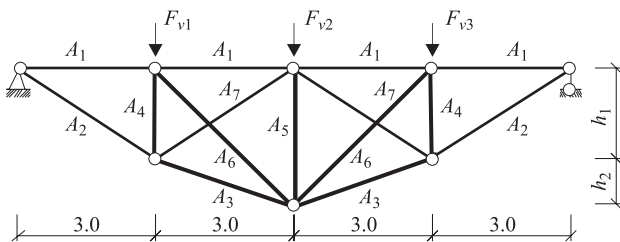


Fig. 3. Truss design diagram

All truss elements are designed using the hot rolled steel pipes, steel S275 and the elasticity modulus  $E = 2.10 \times 10^5 \text{ MPa}$ . The admissible vertical displacement of the middle node is  $u_y \leq 0.05 \text{ m}$ .

The design strength of the steel  $R = \frac{f_y}{\gamma_m} = \frac{275}{1.1} =$

250 MPa. The bars of the truss are modelled by using the first-order finite elements  $k = 1, 2, \dots, r$  (Fig. 1a). The symmetric truss is designed with the optimization parameters  $h_1, h_2$  and  $A_1, A_2, \dots, A_7$  (Fig. 3). The aim is to find the optimal truss height, the optimal layout of the bars and areas of bar cross-sections, while minimizing the expenditure of steel, when the truss is subjected only to the first or only to the second load case or to both load cases. Moreover, all calculations are performed with the evaluation of the height constraints,  $h_1 + h_2 = 2 \text{ m}$  and  $h_1 + h_2 = 1.5 \text{ m}$ . The optimization results are compared to the data, obtained by using the finite element software *Autodesk Robot Structural Analysis*.

The results obtained for the truss subjected to the first load case are as follows: the truss height values are  $h_1 = 2.911 \text{ m}$  and  $h_2 = 0$ ; the cross-section areas are  $A_1 = 11.08 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}101$ ),  $A_2 = A_3 = A_4 = 8.616 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}88.9$ ),  $A_5 = 0, A_6 = 0, A_7 = 8.616 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}88.9$ ). Steel expenditure  $V = 37888.34 \times 10^{-6} \text{ m}^3$ . Optimal scheme is shown in Fig. 4.

The same truss height and cross-sections areas values were obtained for the truss subjected by both load cases.

Results for the second load case: truss height –  $h_1 = 1.649 \text{ m}, h_2 = 0.312 \text{ m}, h = h_1 + h_2 = 1.961 \text{ m}$  cross-sections  $A_1 = 12.52 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}114.3$ ),  $A_2 = 10.67 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}88.9$ ),  $A_3 = 8.616 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}88.9$ ),  $A_4 = 5.74 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}60.3$ ),  $A_5 = 3.733 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}48.3$ ),  $A_6 = 0, A_7 = 6.004 \times 10^{-4} \text{ m}^2$  ( $\text{Ø}76.1$ ). Steel expenditure is  $V = 34261.43 \times 10^{-6} \text{ m}^3$ . Optimal scheme is presented in Fig. 5.

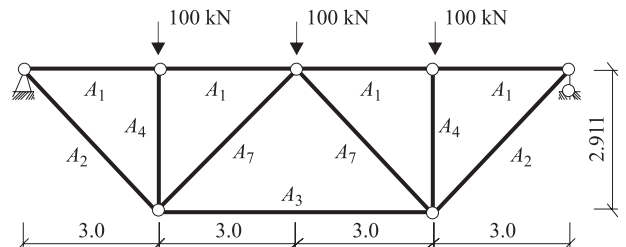


Fig. 4. The optimal truss structure for the first load case and for both the first and the second load cases

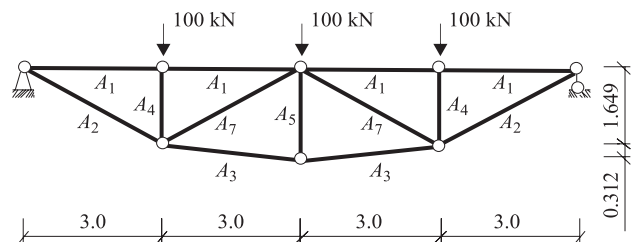


Fig. 5. The optimal truss structure for the second load case

Similar optimal diagrams were obtained in designing 2 m and 1.5 m high trusses. The values of steel expenditure for both load cases are  $41\,220.0 \times 10^{-6} \text{ m}^2$  and  $45\,956.0 \times 10^{-6} \text{ m}^2$ , respectively. The analysis of the results shows that steel expenditure is increasing when the truss height is decreasing. Thus, the conclusion of prof. Rabinovich (1933) that for a single load case a statically determinate truss is optimal, is confirmed. With reference to two load cases, the statically determinate truss is obtained (Fig. 4), though, in general, it could be statically indeterminate as well.

The comparison of the optimization results with the results obtained by using the software *Autodesk Robot Structural Analysis* is presented in Figs 6–8.

### 5. Optimization results of bridge truss

The bridge truss with the span of 24 m and the continuous top chord (Fig. 9) is considered. The truss is subjected to the equally distributed dead load  $p_1 = 12 \text{ kN/m}$  and live load  $p_2 = 13 \text{ kN/m}$ . The design strength of steel is  $R = 250 \text{ MPa}$ , while the elasticity modulus is  $E = 2.10 \times 10^5 \text{ MPa}$ . The aim is to find the optimal designs for three load cases:

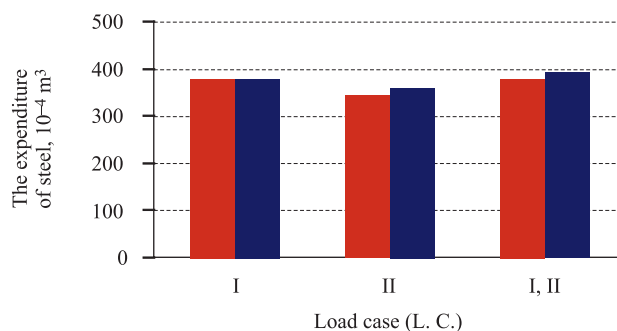
- the whole bridge is subjected to the dead and live loads; thus,  $p = p_1 + p_2 = 25 \text{ kN/m}$ ;
- the whole bridge is subjected to the dead load, while the live load acts only on the half of the bridge;
- with reference to the first and second load cases.

The cross-section of the top chord is designed using the IPE profiles, while other bars are designed using hot rolled pipes. The admissible vertical displacement of the middle node is  $u \leq 0.010 \text{ m}$ .

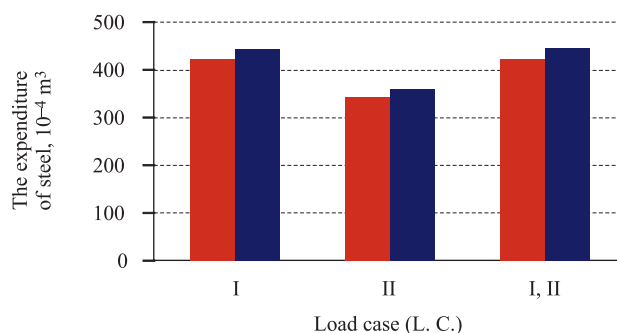
The top chord is modeled by four second order elements (Fig. 1b), while other bars are modeled by elements under tension or compression (Fig. 1a). The symmetric truss is designed, optimal parameters are  $h_1, h_2$  and  $A_1, A_2, \dots, A_5$  (Fig. 10). Optimal truss height values  $h_1$  and  $h_2$  are defined for the cases without height constraint and with height constraint  $h = h_1 + h_2$ . Optimization results are presented in Table 1. Steel expenditures for different truss heights and load cases are shown in Figs 10–14.

**Table 1.** Optimization results of the bridge truss

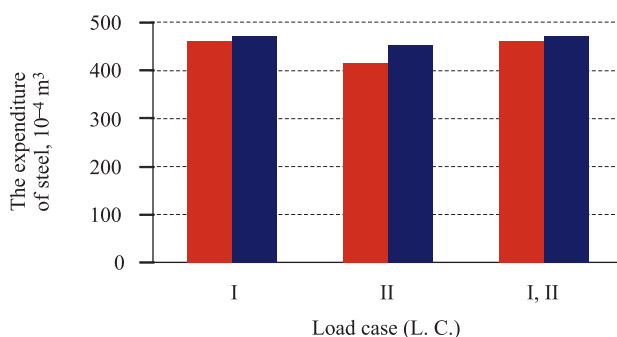
Truss height, $10^{-2} \text{ m}$	Load case L.C.	$h_1, 10^{-2} \text{ m}$	$h_2, 10^{-2} \text{ m}$	$h, 10^{-2} \text{ m}$	$A_1, 10^{-4} \text{ m}^2$	$A_2, 10^{-4} \text{ m}^2$	$A_3, 10^{-4} \text{ m}^2$	$A_4, 10^{-4} \text{ m}^2$	$A_5, 10^{-4} \text{ m}^2$	$V, 10^{-6} \text{ m}^3$
unlimited	I	469.80	146.64	616.44	45.94	17.17	12.52	17.17	21.38	181 202.8
	II	509.53	179.72	689.25	62.61	12.52	15.39	12.52	15.39	212 619.4
	I, II	358.83	179.35	538.18	72.73	17.17	15.39	10.67	17.05	234 668.4
$h \leq 500$	I	387.84	101.74	489.58	53.81	18.86	15.17	15.17	15.17	193 750.4
	II	386.64	109.53	496.17	72.73	13.18	11.08	11.08	11.08	220 948.9
	I, II	372.25	125.80	498.05	72.73	18.86	15.39	11.08	17.05	236 796.3
$h \leq 400$	I	330.35	69.65	400	62.61	20.33	18.86	15.17	10.67	215 188.1
	II	318.44	77.03	395.47	72.73	15.17	15.17	10.67	8.616	223 717.2
	I, II	303.62	92.17	395.79	72.73	21.38	18.86	11.08	11.08	237 316.7



**Fig. 6.** The expenditure of steel, when  $h = 2.911 \text{ m}$



**Fig. 7.** The expenditure of steel, when  $h = 2.0 \text{ m}$



**Fig. 8.** The expenditure of steel, when  $h = 1.50 \text{ m}$ : ■ – Matlab; ■ – autodesk robot structural analysis

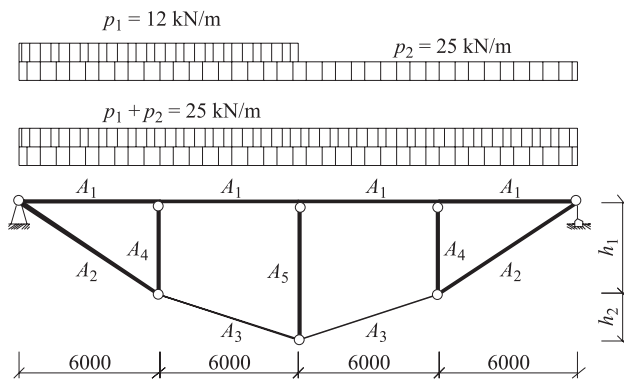


Fig. 9. Design diagram of the bridge truss

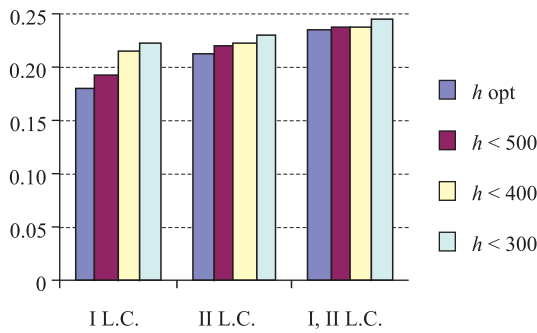


Fig. 10. Comparison of steel expenditures in  $m^3$  for different load cases

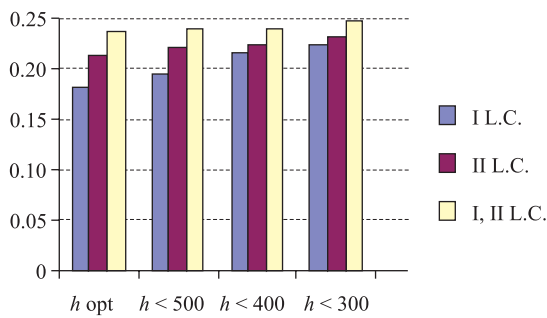


Fig. 11. Comparison of steel expenditures in  $m^3$  for different truss height cases

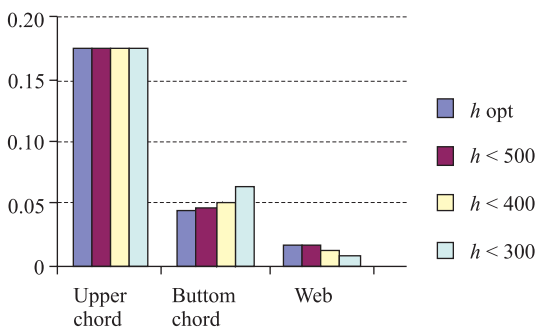


Fig. 12. Steel expenditures in  $m^3$  for chords and webs with reference to both load cases

The analysis of the results shows that, independent of the truss height, steel expenditure for the first load case (symmetric loading) is lower for the top chord and higher for the bottom chord and webs, compared to the load case of non-symmetric loading (the second load case). Furthermore, it can be seen from the pictures of steel expenditure that the demand for steel is increasing for chords and decreasing for webs, when the truss height is decreasing. In the case of non-symmetric loading, the mean intensity of loading is lower, compared to the symmetric load case; however, the non-symmetric load case is more dangerous because steel expenditure is higher.

### 6. Conclusions

1. The optimization problems of elastic trusses modelled by using the equilibrium finite elements are formulated and solved as nonlinear problems of discrete mathematical programming. Since the height of a truss is unknown, the objective function, equilibrium and geometric equations, as well as strength and stability conditions, are nonlinear.

2. Computational research has shown that the mathematical model of the optimization problem allows for determining the optimal cross-sections of the truss bars, as well as the optimal height and layout of the truss bars subjected to several load cases and taking into account steel design code requirements.

3. It is shown that the optimal truss, composed only of the bars under bending or compression, should have the

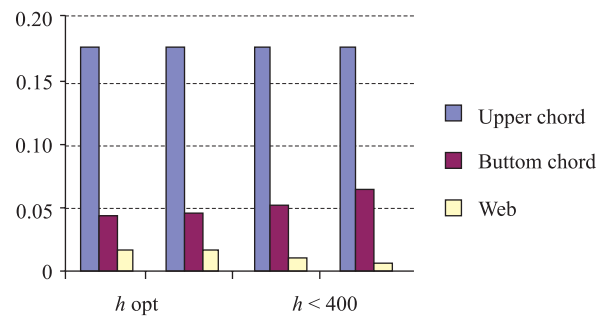


Fig. 13. Steel expenditures in  $m^3$  for chords and webs with reference to the first load case

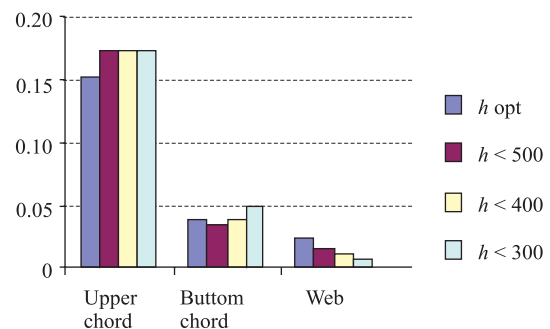


Fig. 14. Steel expenditures in  $m^3$  for chords and webs with reference to the second load case

statically determinate structure. The optimal height values are  $h_1 = 2.911$  m and  $h_2 = 0$  m for the truss subjected to both load cases.

4. The computational research has shown that steel expenditure for the truss of symmetric geometry and loading is smaller for the top chord and larger for the bottom chord and webs, compared to steel expenditure in the case of non-symmetric loading. Moreover, the demand for steel is increasing for chords and decreasing for webs, when the truss height is decreasing.

5. It has been shown that the design of trusses, based on the use of the suggested method, is more economical compared to the design based on the use of the finite element software *Autodesk Robot Structural Analysis*.

## References

- Atkočiūnas, J.; Venskus, A. 2011. Optimal Shakedown Design of Frame under Stability Condition According to Standards, *Computer & Structures* 3–4(89): 435–443. <http://dx.doi.org/10.1016/j.compstruc.2010.11.014>
- Feng, F. Z.; Kim, Y. H.; Yang, B.-S. 2006. Application of Hybrid Optimization Techniques for Model Updating of Rotor Shafts, *Structural and Multidisciplinary Optimization* 32(1): 67–75. <http://dx.doi.org/10.1007/s00158-006-0003-4>
- Goremikins, V.; Serdjusks, D. 2010. Rational Structure of Trussed Beam, in *The 10<sup>th</sup> International Conference "Modern Building Materials, Structures and Techniques": selected papers*. Ed. by Vainiūnas, P.; Zavadskas, E. K. May 19–21, 2010, Vilnius, Lithuania. Vilnius: Technika, 513–518.
- Gutkowski, W. 1997. *Discrete Structural Optimization*. Springer-Verlag, 250 p.
- Hayalioglu, M. S.; Degertekin, S. O. 2004. Design of Non-Linear Steel Frames for Stress and Displacement Constraints with Semi-Rigid Connections via Genetic Optimization, *Structural and Multidisciplinary Optimization* 27(4): 259–271. <http://dx.doi.org/10.1007/s00158-003-0357-9>
- Hayalioglu, M. S. 2000. Optimum Design of Geometrically Non-Linear Elastic-Plastic Steel Frames Via Genetic Algorithm, *Computers & Structures* 77(5): 527–538. [http://dx.doi.org/10.1016/S0045-7949\(99\)00221-7](http://dx.doi.org/10.1016/S0045-7949(99)00221-7)
- Haug, E. J.; Arora, J. S. 1980. *Applied Optimal Design: Mechanical and Structural Systems*. New York: John Wiley & Sons. ISBN 047104170X.
- Hyberyan, K. M. 1960. Usilija v staticheski neopredelimoj ferme, otvechajushchej naimenshemu ee vesu v raschiote na mnogie zagruzheniia, *Izvestiia AN SSSR, OTN. Mekhanika i mashinostroenie*, No. 3.
- Kala, Z.; Puklický, L.; Omishore, A.; Karmazínová, M.; Melcher, J. 2010. Stability Problems of Steel-Concrete Members Composed of High-Strength Materials, *Journal of Civil Engineering and Management* 16(3): 352–362. <http://dx.doi.org/10.3846/jcem.2010.40>
- Kalanta, S.; Atkočiūnas, J.; Venskus, A. 2009. Discrete Optimization Problems of the Steel Bar Structures, *Engineering Structures* 31(6): 1298–1304. <http://dx.doi.org/10.1016/j.engstruct.2009.01.004>
- Kalanta, S. 2007. *Taikomosios optimizacijos pagrindai*. Vilnius: Technika. 480 p. ISBN 9789955281603. <http://dx.doi.org/10.3846/924-S>
- Kalanta, S. 1995. Element konechnogo ravnesiia v vychislenii uprugikh konstrukcij, *Stroitelstvo* 1(1): 25–47.
- Karkauskas, R. 2004. Optimization of Elastic-Plastic Geometrically Non-Linear Lightweight Structures under Stiffness and Stability Constraints, *Journal of Civil Engineering and Management* 10(2): 97–106. <http://dx.doi.org/10.1080/13923730.2004.9636293>
- Maciulevicius, D. A. 1966. Algoritm vypuklogo programmirovaniia dlia sinteza uprugoj sharnirno-sterzhnevoj konstruktsii minimalnogo vesa v sluchae mnogikh zagruzhenij, in *Stroitelnaia mekhanika i konstruktsii*. Vilnius: Mintis, 79–85.
- Maciulevicius, D. A. 1964. Algoritm lineinogo programmirovaniia dlia sinteza sterzhnevykh staticheski opredelimykh konstruktsij minimalnogo vesa, in *Stroitelnaia mekhanika i konstruktsii*. Vilnius: Mintis, 33–49.
- Majid, K. I. 1974. *Optimum of Design Structures*. New York: Wiley. 264 p. ISBN 0470565330.
- Manickarajah, D.; Xie, Y. M.; Steven, G. P. 2000. Optimum Design of Frames with Multiple Constraints Using an Evolutionary Method, *Computers & Structures* 74(6): 731–741. [http://dx.doi.org/10.1016/S0045-7949\(99\)00083-8](http://dx.doi.org/10.1016/S0045-7949(99)00083-8)
- Merkevičiūtė, D.; Atkočiūnas, J. 2006. Optimal Shakedown Design of Metal Structures under Stiffness and Stability Constraints, *Journal of Constructional Steel Research* 62(12): 1270–1275. <http://dx.doi.org/10.1016/j.jcsr.2006.04.020>
- Rabinovich, I. M. 1933. *K teorij staticheski neopredelimykh ferm*. Moskva.
- Rao, S. S. 2009. *Engineering Optimization: Theory and Practice*. 4<sup>th</sup> edition. New York: John Wiley & Sons. 840 p. ISBN 9780470183526
- Yuge, K.; Iwai, N.; Kikuchi, N. 1999. Optimization of 2D Structures Subjected to Non-Linear Deformations Using the Homogenization Method, *Structural Optimization* 17(4): 286–299. <http://dx.doi.org/10.1007/BF01894077>
- Zheng, Q. Z.; Querin, O. M.; Barton, D. C. 2006. Geometry and Sizing Optimisation of Discrete Structure Using the Genetic Programming Method, *Structural and Multidisciplinary Optimization* 31(6): 452–461. <http://dx.doi.org/10.1007/s00158-005-0561-x>

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