STRUCTURAL MODEL UPDATING OF STEEL BOX GIRDER BRIDGE USING MODAL FLEXIBILITY BASED DEFLECTIONS

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Abstract. The finite element models need to be updated to simulate the actual behavior of the structures. A simplified approach has been presented in this paper for model updating of steel box girder bridge using the virtual static deflections. The approach is based on the conversion of dynamic model to static model. The deflections used for the model updating are estimated from the modal flexibility matrix which requires only a small number of lower modes for accurate estimation. An optimization process is adopted for the model updating. Dynamic tests have been performed on a two span continuous bridge for the verification of the proposed approach. It has been found that the updated numerical deflections obtained by the proposed method reasonably agree with the estimated deflections.

Keywords: model updating, modal flexibility, dynamic experiment, deflection, bridge.

1. Introduction

The actual behavior of the structure may not be reflected by finite element model (FEM) and it may require updating. The difference between the results obtained by the FEM and those obtained by carrying out the experiments is reduced by model updating. It involves the modification in the properties like mass, stiffness and damping parameters.

Many studies are available for the model updating of the bridges. Experiments were conducted on a reinforced concrete bridge and the FEM was updated by El-Borgi *et al.* (2005) to reduce the difference between the results obtained by FEM and the experiments.

A FEM of a real arch bridge under use was updated by Jaishi *et al.* (2007). The parameters were updated in such a manner that the natural frequencies obtained from the updated model became quite close to the experimental values. A methodology was suggested by Bayraktar *et al.* (2009) to update the FEM for an arch bride using ambient vibration tests. The analytical modeling was carried out using three dimensional finite element models. The peak picking method in the frequency domain was used for the modal identification. Updating of the FEM led to a good agreement between the natural frequencies obtained by numerical method and experiments. Similar studies were reported recently by Bayraktar *et al.* (2010) for a reinforced concrete box girder bridge constructed by balanced cantilever method. A good agreement was found between natural frequencies and mode shapes after model updating.

Feng *et al.* (2004) proposed a method to build baseline models for bridge performance monitoring using neural networks. The method was applied to a pre-stressed post tension box girder bridge. The natural frequencies of the baseline model were found to be in good agreement with the reality than those of the model before the updating. Lee *et al.* (2005) have also proposed a technique of neural network based damage detection for bridges. The modes shape ratios before and after damages are used as input

for the networks. The training is carried out for reducing the effect of errors in baseline finite element model. Kim *et al.* (2009) proposed an iterative training neural network strategy for static model identification of a fiber-reinforced polymer bridge deck. The methodology was applied to the example bridges and was found to be very effective in system identification of the fiber-reinforced polymer bridge deck.

The modal flexibility has been found to be more sensitive to damage by Zhao and Wolf (1999) than natural frequencies and mode shapes. It has been shown by Lenett *et al.* (1999) that condition of structures is established quite accurately by carrying out modal analysis and evaluating their flexibility. An inverse relationship exists between the flexibility and square of modal frequencies. The higher frequency modes therefore have a very small contribution to the flexibility matrix and may be neglected. The flexibility matrix may be represented accurately by only small number of modes. This feature of flexibility matrix has therefore been used extensively for detecting damages in the structures (Aimin, Golinval 2005; Bernal, Gunes 2002; Gao, Spencer 2002; Jaishi, Ren 2006; Jaishi *et al.* 2007; Koo *et al.* 2008; Pandey, Biswas 1994, 1995; Patjawit, Kanok-Nukulchai 2005; Stutz *et al.* 2005; Toksyo, Aktan 1994). Wang *et al.* (2008) discussed about the vibration-based damage detection with structural modal characteristics. A novel technique, requiring the use of vibration data, was suggested for locating the damage and also for evaluating the condition of the already known damage.

A simplified approach has been adopted in this paper for model updating of steel box-girder bridge using the virtual static deflections based on modal flexibility matrix. The approach is based on the conversion of dynamic model to static model. The deflections used for the model updating are estimated from the modal flexibility matrix. As stated above, a small number of lower modes are sufficient for accurate evaluation of flexibility matrix. An optimization process is adopted for the model updating. Dynamic tests have been performed on a two span continuous bridge for the verification of the proposed approach.

2. Modal flexibility estimation from dynamic test

The flexibility matrix ideally consists of flexibility coefficients which are deformations corresponding to static sources of unit magnitude acting at the coordinates as in this case the error is reduced to imprecision of measurements. However, in some cases, the application of static forces to produce a displacement field that is measurable with sufficient accuracy is quite a difficult task and flexibility may be synthesized from the vibration measurements in those cases. When the flexibility is extracted from vibrations, the error due to truncation is to be accepted because all the modes are not obtained from experimental data.

Consider the response of a structure described by the following linear equations of motion:

$$M\ddot{x} + C\dot{x} + Kx = f, \tag{1}$$

where M – the mass matrix; C – the damping matrix; K – the stiffness matrix; x – the displacement vector; f – the force vector.

Assuming proportional damping and taking into account the fact that the mode shapes are orthogonal with respect to mass and stiffness matrix the following equation is obtained.

$$\bar{M} = \Phi^T M \Phi, \bar{K} = \Phi^T K \Phi, \tag{2}$$

where $\bar{M}$ – the generalized mass matrix which has only diagonal components; $\bar{K}$ – the generalized stiffness matrix which has only diagonal components; $\Phi = [\phi_1, \phi_2, ..., \phi_n]$ – the mode shape matrix. The modal frequencies are denoted as ω_j^2. The matrix of the square of the modal frequencies, Λ, comprising of ω_j^2 on main diagonal is obtained as:

$$\Lambda = \bar{M}^{-1}\bar{K}. \tag{3}$$

Combining Eqs (2) and (3) gives

$$\Phi^T K \Phi - \Phi^T M \Phi \Lambda = 0. \tag{4}$$

Then

$$\nu = (\Phi^T M \Phi)^{\frac{1}{2}} = \bar{M}^{\frac{1}{2}}, \tag{5}$$

where ν – the diagonal matrix which is comprised of mass-normalized indices. Substituting Eq (5) into Eq (4) and keeping in mind that ν is a diagonal matrix, yields

$$\Phi^T K \Phi - \nu^2 \Lambda = 0 \text{ or } \Phi^T K \Phi = \nu^T \Lambda \nu. \tag{6}$$

The stiffness matrix is now given from Eq (6) as:

$$K = (\Phi^T)^{-1} \nu^T \Lambda \nu \Phi^{-1}. \tag{7}$$

Taking into account that the flexibility matrix is inverse of stiffness matrix, i.e. $G = K^{-1}$, the flexibility matrix obtained from Eq (7) is:

$$G = \Phi \nu^{-1} \Lambda^{-1} \nu^{-1} \Phi^T. \tag{8}$$

If the origin of the physical coordinate system is established as a reference, the orthogonal property for mass-normalized mode shapes is expressed as:

$$\bar{M} = \Phi^T M \Phi = I. \tag{9}$$

It has been demonstrated by Doebling (1996) that the stiffness and flexibility matrices based on the modal data are derived from Eqs (7) and (8):

$$K = M\Phi\Lambda\Phi^T M = M(\sum_{i=1}^{n} \omega_i^2 \phi_i \phi_i^T)M, \tag{10}$$

$$G = \Phi \Lambda^{-1} \Phi^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \phi_i \phi_i^T, \qquad (11)$$

where ϕ_i means the ith mode shap; ω_i^2 means the ith modal frequency; $\Lambda = diag(\omega_i^2)$ means the modal stiffness matrix, and n means the number of degrees of freedom. Analytical model may be used or the experiments may be carried out to obtain the mode shapes and the natural frequencies.

It is observed from Eq (10), that the modal contribution to the stiffness matrix varies with the frequency, and an increase in frequency causes increase in the modal contribution to stiffness matrix. The accurate estimation of stiffness from the experiments requires the measurement of all modes or at least the high frequency modes. In practice, in any experimental survey of complex structures, it is difficult to measure all the modes, particularly the higher modes. However, Eq (11) shows that the modal contribution decreases with increase in frequency and for higher values of frequencies, the modal contribution is insignificant. Therefore, the flexibility matrix is obtained reasonably accurately by considering only the small number of lower frequency modes (Gao, Spencer 2002).

3. Identification method by deflections

Suppose there is a deflection u arising from an arbitrary load f. The deflection may be obtained using the flexibility matrix, G as

$$u = Gf. \qquad (12)$$

In this study, deflections are obtained from Eq (12) by applying Positive Bending Inspection Loads (PBILs). A PBIL which is defined as a load or a system of loads which guarantees positive bending moments in the inspection region makes the deflection change sensitive to structural changes so that the optimization is carried out rapidly. Examples of PBILs are shown in Table 1 for a two span continuous beam used in the experimental study. To update a FEM, the following cost function is used based on deflections obtained by modal flexibility.

$$C(EI_i) = \sum_{j=1}^{2} \frac{\left\| d_j^{FEM}(EI_i) - d_j^{EXP} \right\|}{d_j^{EXP}} 100\%, \qquad (13)$$

where $C(EI_i)$ – the cost function; E – Young's modulus; I_i – bending rigidity at the ith beam element; d_1^{EXP}, d_2^{EXP} – experimental deflections under two PBILs (f^1 and f^2) obtained by modal flexibility; $d_1^{FEM}(EI_i)$, $d_2^{FEM}(EI_i)$ – corresponding deflections from the FEM.

To update the model is to estimate the representative stiffness of the steel bridge. The optimization toolbox in MATLAB is employed which use the cost function as Eq (13) as the objective function to perform the model updating (an optimization process actually). The process of model updating method is a process of optimization (Fig. 1).

4. Experimental study and model updating

A two-span continuous box-girder bridge was casted in the laboratory for performing the experiments. The spans

Table 1. Schematic representation of PBILs (Koo *et al.* 2008)

Beam types	Inspection regions	PBILs	Definitions of inspection loads
2-span continuous beam	1st span-region 2nd span-region	f^1 $-f^1$	(f^1) w_1, L_1, I_1 w_2, L_2, I_2 where $\dfrac{w_1}{w_2} = \dfrac{I_1}{I_2}\left(\dfrac{L_2}{L_1}\right)^3$
	Intermediate support-region[**]	f^{2*}	(f^2) w_1, L_1, I_1 w_2, L_2, I_2 where $\dfrac{w_1}{w_2} = \left(\dfrac{L_2}{L_1}\right)^2$
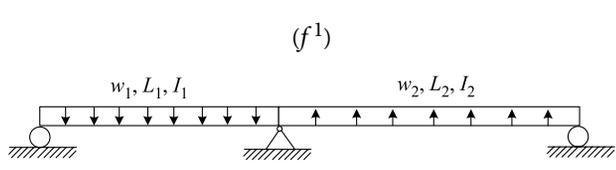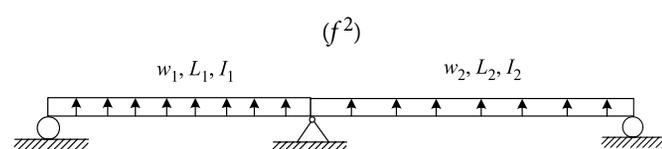
[*] f^1 and f^2: Span-region inspection load and intermediate support-region inspection load.
[**] Intermediate support-region may cover 1/4 of the span regions on both sides of the support.
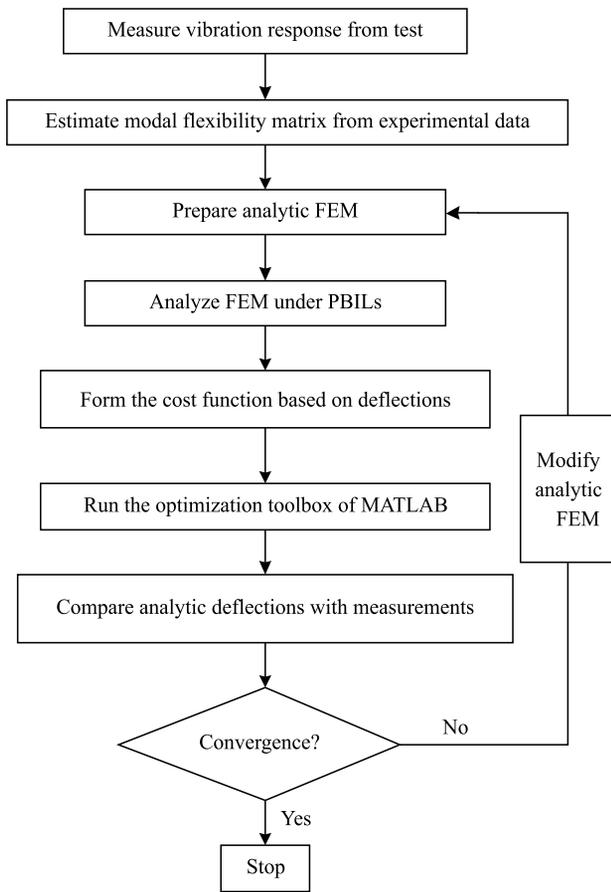
Fig. 1. Flowchart of the identification method based on deflections
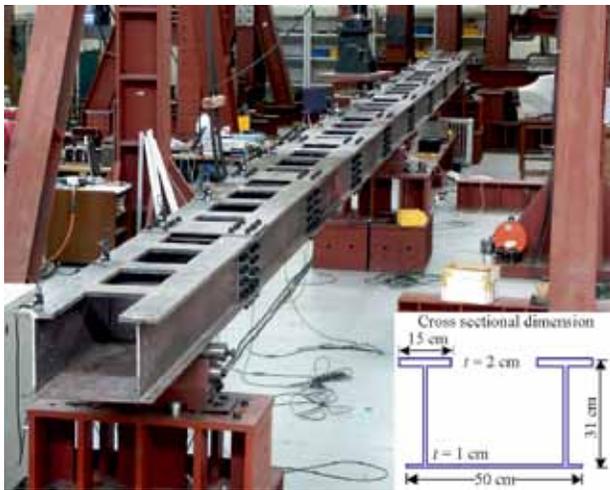
Fig. 2. Experimental setup and cross-section dimensions (Koo *et al.* 2008)
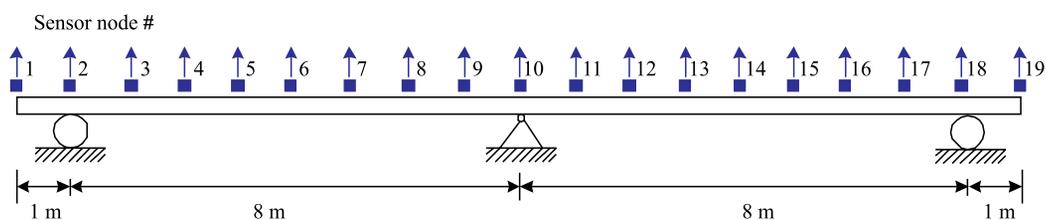
were of equal length with the length of each span being 8.0 m and 1.0 m overhang for each span. The bridge was comprised of total nine segmental boxes. The boxes were connected by the bolts. Fig. 2 shows the cross-section of the bridge along with the experimental set up. Nineteen accelerometers were evenly placed along one side on the upper surface of the box-girder as shown in Fig. 3. The impact loads at the regular time interval of four minutes were applied to vibrate the bridge. The measurement of acceleration responses was obtained at the sampling rate of 200 Hz. The measurements were carried out a number of times (eighteen times) to ascertain the intact. Efforts were made to keep the temperature constant during the experiments of about total six hour duration so as to avoid the thermal effects.

The modal flexibility matrices were constructed using Eq (11) based on the mass-normalized mode shapes and natural frequencies obtained from the acceleration measurements. The deflections under the PBILs f^1 and f^2 were obtained for 19 sets of the acceleration measurements. The results were scaled so as to keep the absolute max value as 1.0. As seen in Fig. 4, the eighteen results almost coincide with each other and the deviation levels were less than ±0.0008, which indicates the excellent repeatability of the estimated deflections.

A FEM was developed for the bridge. The model consisted of beam elements of one meter length. The rubber pads below the supports were modeled as the point springs. The cross-section properties were converted in the equivalent properties for use in the FEM. The area of the equivalent section was 0.0166 m^2 and the moment of inertia was 0.0002751 m^4. The material was modeled as steel.

The MATLAB software was used for coding the FEM and its optimization tool box was used for the updating of the model through optimization. The optimized values at the final step during the optimization process are shown in Fig. 6. The value of the cost function decreased from 11.215 to 5.206 after the optimization process shown in Fig. 7, concluding that the optimization process worked effectively.

As there were totally 18 records for the acceleration measurements, optimization processes were performed 18 times to compare the accuracy of the estimated stiffness of the steel bridge. Fig. 8 shows the mean values and standard deviations of the stiffness of 18 beam elements, which is the FEM for the test steel bridge. From the graphs it is noted that the standard deviations are small compared with the mean values. Thus, it could be concluded that the

Fig. 3. Accelerometer layout of experimental test

Mean of scaled deflections under f^1

Mean of scaled deflections under f^2

Deviations of scaled deflections under f^1

Deviations of scaled deflections under f^2
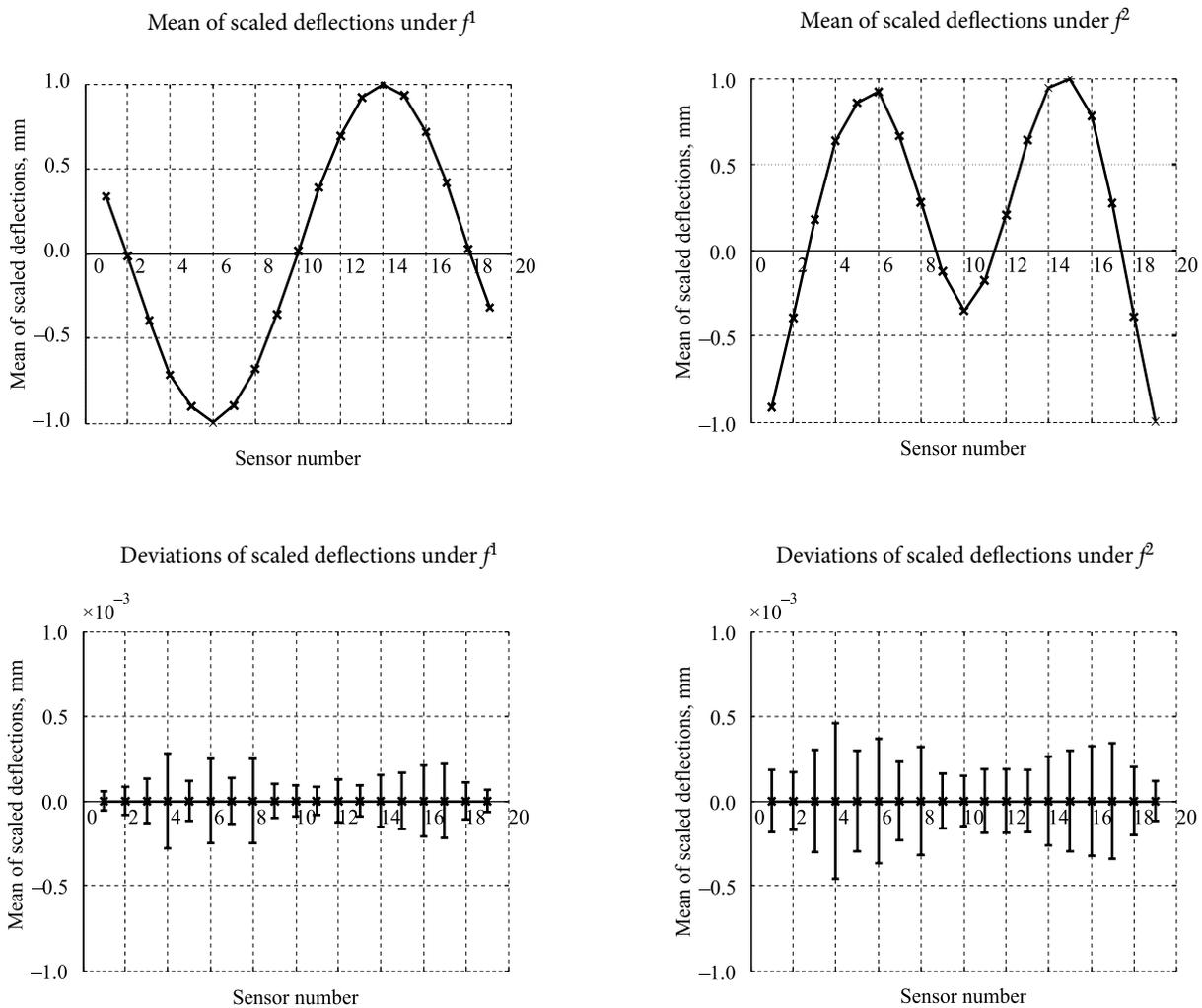
Fig. 4. Mean deflections and their deviation level under PBILs before identification
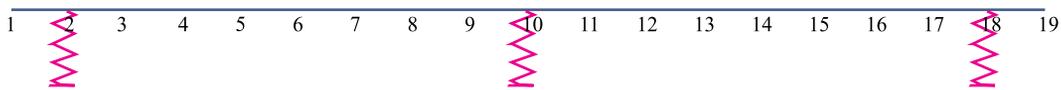
Fig. 5. FEM of the steel bridge

experimental results are acceptable and the estimated stiffness is reasonable.

The deflection obtained by the experiment at the sensor locations and the deflection obtained by the FEM before updating corresponding to these locations are shown in Fig. 9. Similarly, the deflection obtained by the experiment at the sensor locations and the deflection obtained by the FEM after updating corresponding to these locations are shown in Fig. 10. The values shown in the Figs 9 and 10 are the average values both for the FEM and experiments. Fig. 11 shows the mean deflections and their deviations which are obtained from FEM analysis. The results are scaled in a way that the absolute max value is 1.0. The identified deflections almost coincide with each other and
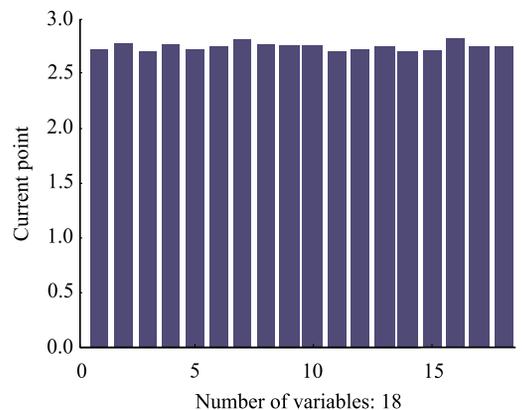
Fig. 6. Optimized values at the final step
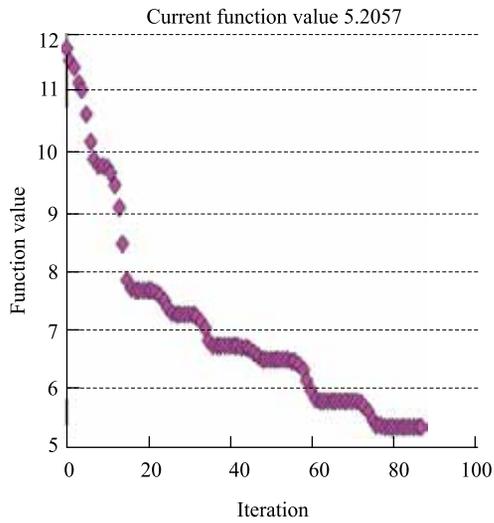
Fig. 7. The cost function values during optimization process
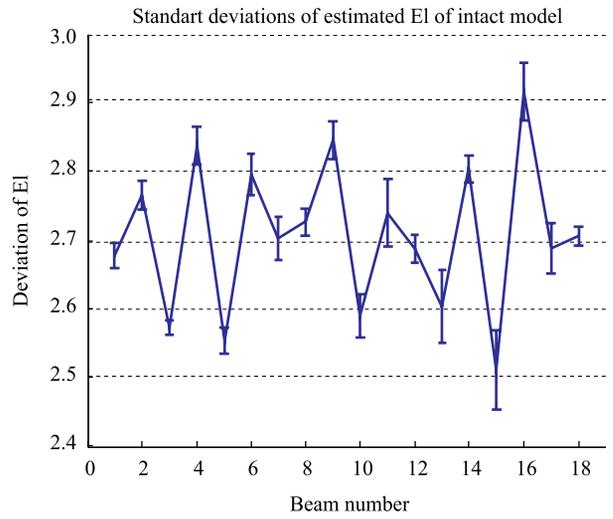
Fig. 8. Mean values and standard deviations of the estimated stiffness
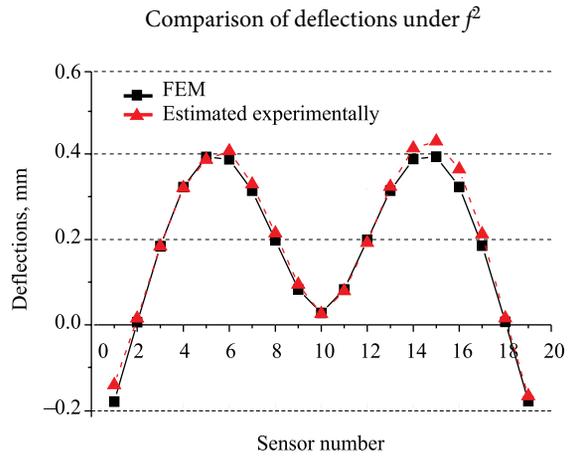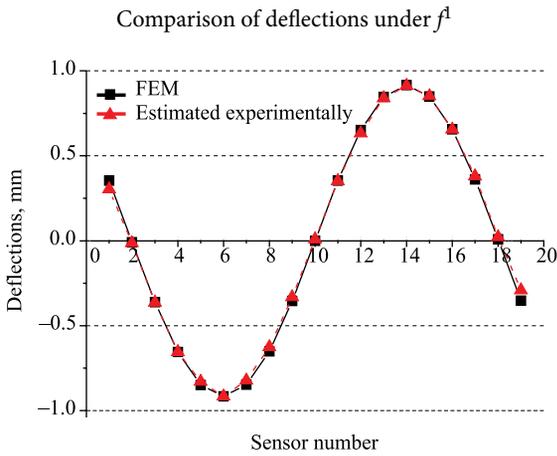
Fig. 9. Comparison of deflections under PBILs before identification
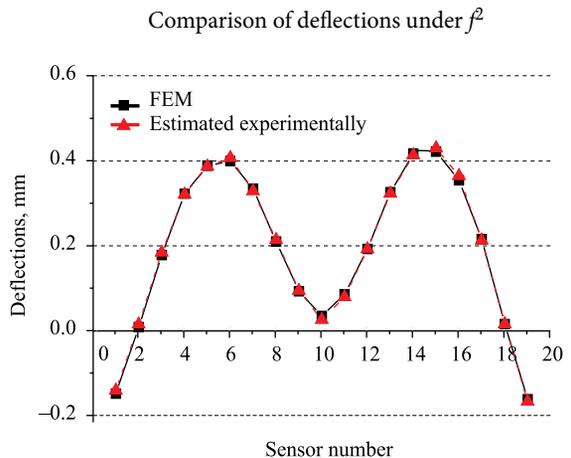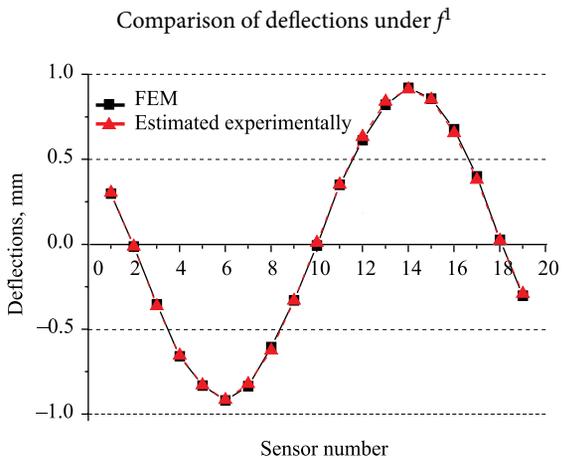
Fig. 10. Comparison of deflections under PBILs after identification

Mean of scaled deflections under f^1

Mean of scaled deflections under f^2

Deviations of scaled deflections under f^1

Deviations of scaled deflections under f^2
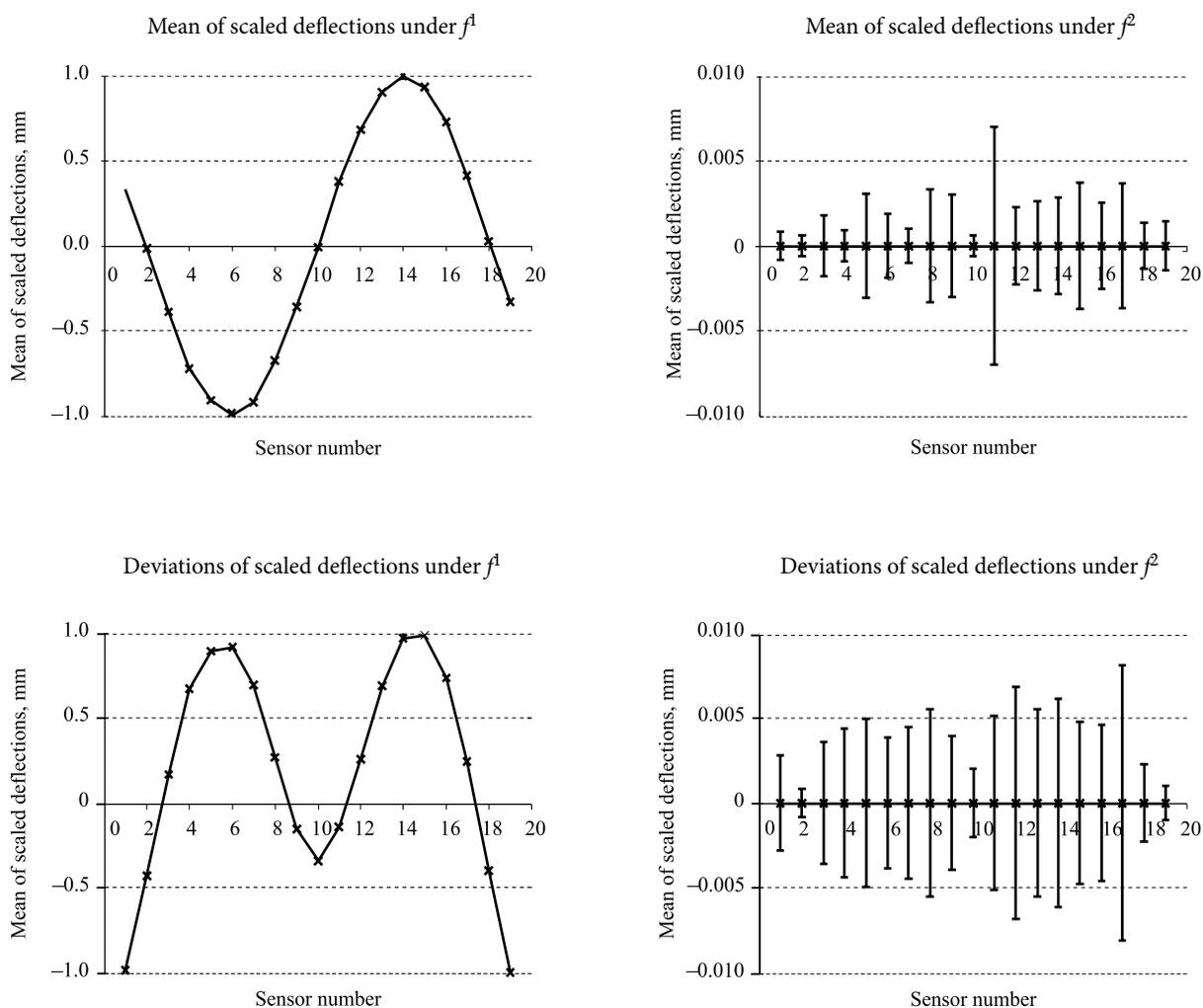
Fig. 11. Mean deflections and their deviation level under PBILs after identification

the deviation levels are less than ±0.008, showing the excellent analytic deflections after identification.

It is observed from Fig. 10, that the deflections obtained by the FEM after updating are in good agreements with the deflections obtained by the experiments. This shows that the proposed method which updates the FEM by the deflections is effective and may be used for obtaining a baseline model of the bridge based on dynamic data obtained by experiments. This method is better since it requires the use of deflections as compared to other methods which require natural frequencies and mode shapes.

5. Conclusions and discussions

The important conclusions based of this paper are as follows:

A FEM updating method has been presented for steel box girder bridges. The method utilizes virtual static deflection estimated by the modal flexibility matrix which, in turn, is formed by using the modal parameters, such as natural frequencies and mode shapes.

The proposed method has been validated by performing the dynamic tests on a two-span continuous box girder bridge. The numerical deflections obtained after using the proposed method for model updating are in close agreements with the experimental results. This shows that the presented method enables to carry out efficiently and accurately the model updating of the structures.

The presented model updating method may be extended in the future for condition assessment and damage detection in bridges.

Acknowledgements

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