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STRESS-STRAIN ANALYSIS OF SAND SUBJECTED TO TRIAXIAL LOADING

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Abstract. An influence of the end boundary conditions to distribution of stress and strains in a soil specimen during conventional compression triaxial tests is analyzed by the experimental and numerical methods. An evaluation of actual stress state is important when determining the shear strength parameters of soil. These methods are used in this paper to investigate and simulate the contact between the testing equipment and ends of sand specimen during the test. Two different conditions of sample boundaries are analyzed: the first case, when the friction between the sample ends and testing machine is not eliminated (fixed ends); the second case, when the friction between the sample ends and testing machine is eliminated (free ends). The friction is eliminated by allowing the sample base to move freely in any horizontal direction. Simulation results of stress-strain distribution in the sample by using the finite element method show that the shear stress at the contact plane increases for the sample with fixed ends. The stress restricts the displacement of sample ends in the horizontal direction. In the case of free ends the horizontal displacement of sample base occurs. Similar to simulation results have been obtained from the laboratory tests performed with triaxial compression apparatus.

Keywords: triaxial test, effect of end restraint, free ends, soil shear strength parameters, angle of internal friction, numerical simulation.

1. Introduction

Necessary information on soil properties, needful for designers and constructors, is obtained in each construction site by examining the physical and mechanical properties of soils (Chang, Meidani 2012; Sulewska 2012). A triaxial test is recognized to be the most widely used method for the determination of the strength properties and the stress-strain state of soil. However, applying this test one should evaluate the peculiarities of the testing equipment in terms of the actual boundary conditions. One of these peculiarities to be taken into account in triaxial test is the effect of end restraint prescribing impact to the stress and strain distribution in sample. This subsequently results in an accuracy of the determined mechanical properties of soil. Thus, the contact between the testing equipment and the ends of soil specimen should be properly evaluated. The developed friction at the ends of specimen limits the deformation of the specimen ends. It obviously has the influence to a non uniform distribution of the stress and strain inside the specimen and finally affects the test results. The finite-element method simulation as well as the analysis of experimental tests yields that stress and strain distribution within the sample is non-uniform during triaxial testing (Airey 1991; Jeremić *et al.* 2004; Liyanapathirana *et al.* 2005; Peric, Su 2005; Sheng *et al.* 1997; Vervečkaitė *et al.* 2007; Widulinski *et al.* 2009). Thus, one must identify the actual stress and strain distribution in the soil sample when a load is transmitted in a provided way. Summarizing, one should determine the influence of the non-uniformity of stress and strains for the shear strength parameters.

One can list many investigations in this field with the different results, proposals, recommendations. Yang and Ge (2012) concluded that the influence of the end effect decreases with an increasing distance away from the specimen ends. It is known from simulation results that the stress and strain distributions are uniform within the 1/3 zone in the middle of the specimen, thus, one can employ this finding to reduce the influence of the end effect. One cannot directly use the force applied to the specimen

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before it is deducted when measuring and calculating the stress of the zone. By calculating and comparing, it is suggested that the vertical stress applied to the specimen should be deducted 10% for calculating the stress within the 1/3 zone in the middle of the specimen, the horizontal stress remaining the same (Yang, Ge 2012).

Liyanapathirana and his co-workers studied the effect of end restraint on the non-homogeneous behavior of the triaxial specimen. They obtained results for the ideal case, i.e. when there is no friction between the triaxial specimen ends and the platens. The obtained results have been compared with the one of the case where no displacement is allowed between the specimen and the ends platens. It could be seen that a destruction phase of the structured soil has not been influenced significantly by the ends restraints but after the destruction, during the hardening, the stress–strain behavior of the soil was significantly influenced by the end restraint effect (Liyanapathirana *et al*. 2005).

Su and his colleagues concluded that the stress in the specimen with free ends was uniform while the stress in the specimen with the fixed ends was not uniform for non-dilative soil. The investigations showed that the end restraints influence only the stress distribution in the specimen and do not influence the overall behaviour in case of the drained triaxial tests (Su *et al.* 2011).

Bishop and Green proved that the same strength properties are obtained even for dense sand sample when the sample dimensions ratio is 1 and 2 in the case when the friction is eliminated at the top and the bottom of the sample (Bishop, Green 1965). Other researches state that the eliminating of the friction by applying the silicone between two rubber membranes is a sufficiently reliable method to eliminate the friction, that develop between the sample and the platens during the triaxial test (Tatsuoka *et al.* 1984). Rowe and Barden (1964) found that the usage of lubricated end platens led to a much greater uniformity of stress and deformation during the test.

Current investigation analyses the influence of constraining horizontal displacements at the sample top and bottom on the soil shear strength parameters. The method to reduce this effect is proposed. The influence of movable support on the soil shear strength parameters is analyzed experimentally. This influence and the distribution of stress and strain within the soil sample were also simulated using the commercial program COSMOS/M (Finite Element Analysis System, Version 1.75).

2. Experimental analysis

2.1. Identification of tested soil

The type of soil analyzed experimentally and via numerical simulation is sand. According to Unified Soil Classification System it is recognized as poorly-graded sand with fine SP–SM. Sand is described by the following properties: the uniformity coefficient is 3.03, the curvature coefficient is 1.47, the specific gravity of soil particles is 2.671 g/cm^3, the maximum void ratio is 0.745, the minimum void ratio is 0.502.

2.2. Triaxial testing

The testing process of the dense sands by triaxial apparatus leads to the formation of a shear plane. The specimen parts, located below and above this plane, displace in respect of each other not only in the vertical but also in the horizontal directions. The horizontal displacements are resisted by the friction forces, developed between the specimen ends and the platens on the top and the bottom of the specimen. Thus, the normal and the shear stresses are induced at the specimen ends. This should be taken into account when determining shear strength parameters of the soil. When the horizontal displacements at the ends of the specimen are constrained, the larger normal stress magnitudes are necessary for the sheared specimen parts to displace in respect of each other. The ability of free displacements at the specimen base cancels here the shear stress.

The triaxial tests were performed at Geotechnical Research Laboratory of Vilnius Gediminas Technical University on the specimens of height/diameter ratio of $H/D = 2$. The tested samples of low water contents ($W = 6\%$) have been performed by compacting. The triaxial tests have been performed for the samples of two densities, namely: for the dense sands of density $\rho = 1.871$ g/cm^3 and the void ratio $e = 0.51$; and that of the loose sands of the $\rho = 1.610$ g/cm^3 and the $e = 0.74$. Each type of the prepared samples has been cut leastwise three times. The boundary conditions being employed for the tests are: in the first case, when the sample top cap can turn and the friction between specimen ends and the platens is not eliminated (Fig. 1a); and in the second case, when the above described friction is eliminated. The friction at the ends of specimen is eliminated by introducing the movable support which allows the lateral displacement of the specimen base in any direction at the horizontal plane (Fig. 1b).

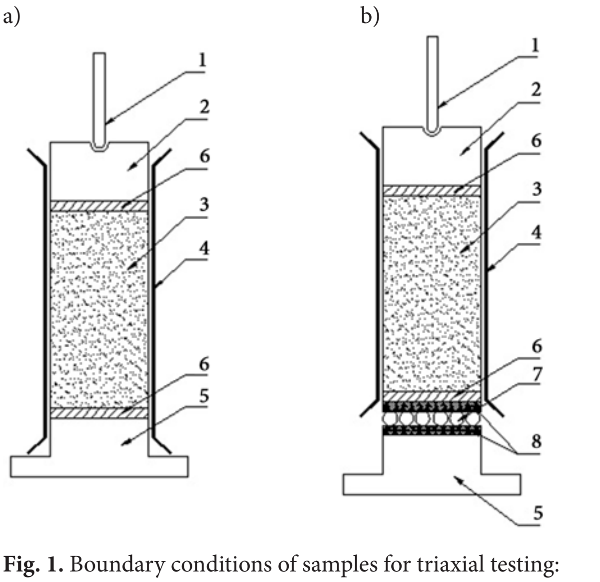

Fig. 1. Boundary conditions of samples for triaxial testing: a – with fixed ends; b – with free ends: 1 – rod, 2 – cap (platen), 3 – specimen, 4 – latex membrane, 5 – pedestal (platen), 6 – porous stone, 7 – thrust bearing, 8 – stainless steel plates

In the first case of the sample with the fixed ends, when the failure plane starts to develop the vertical component of the stress decreases evenly as the axial strain increases. In the second case of the free ends when the axial strain of 4–5% is achieved a significant vertical component of stress decrease is observed, after that the curve declines insignificantly and remains stable (Fig. 2). The deviatoric stress achieves the minimal value for dense samples with free ends faster than for the sample with the fixed ends. In this case the vertical component of the stress is by 16% smaller when compared with the vertical stress for the sample with the fixed ends (here the relative axial deformation ε_1 is equal to 15%) (Dirgėlienė *et al.* 2007a; 2007b).

When testing the loose sand samples of ratio H/D = 2 for the fixed and the free ends cases, the shape of the graphs of strain versus stress $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ is similar in both cases under consideration (Fig. 3). The vertical component of the stress varies similarly for the fixed and the free ends. The deviatoric stress increases until the axial relative strain magnitude reaches 12%.

2.3. Determining the shear strength parameters

The mean value of the stress vertical component for the dense samples in the case of free ends is approximately 16% less compared to the one obtained by the standard triaxial apparatus (fixed ends) for relative 15% axial deformation to be reached (Table 1). The obtained residual mean values of σ_1 for the loose samples are similar in the case of the standard and the improved triaxial apparatuses to be employed.

The variation coefficients for the stress vertical components for two densities of sample obtained by both types of apparatuses are presented in Table 2. The analysis of these data confirms that the variation coefficients are similar in all cases under investigation. This is valid for the loose and the dense samples, also and for the free and the fixed ends. Thus, the movable support has no influence on the variation of σ_1.

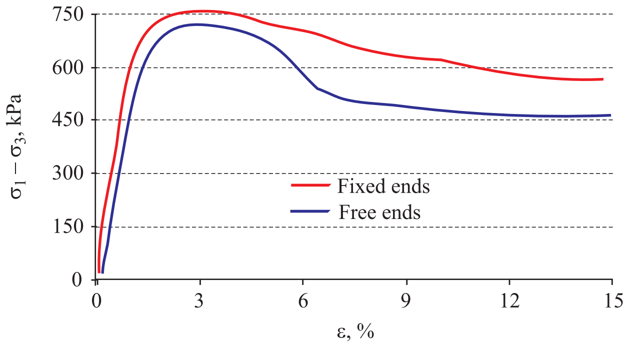

Fig. 2. Stress-strain graph of dense sand, when σ_3 = 200 kPa

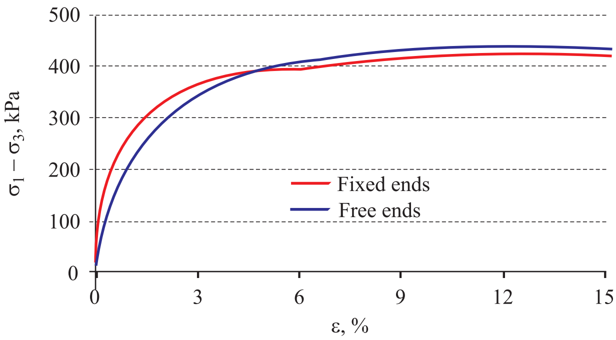

Fig. 3. Stress-strain graph of loose sand, when σ_3 = 200 kPa

The mean values of the angle of internal friction for the dense sand, obtained from max values of the vertical stress σ_1 in both considered cases of testing, differ insignificantly. The max difference is 5%. The mean values of cohesion are approx 31% less for the samples with the free ends. The values of the angle of internal friction obtained from the values of the vertical component of stress when ε_1 = 15% for the sample with the free ends are up to 15% smaller compared to those obtained from the sample with

Table 1. Mean values of stress vertical component σ_1

Void ratio, e	Stress vertical component σ_1	Values of σ_3, kPa	Values of σ_1, kPa	
			Standard apparatus	Improved apparatus
0.51	Peak	50	304.71	308.79
		100	541.92	489.94
		200	907.61	930.93
	Residual	50	234.30	197.71
		100	423.32	339.75
		200	760.05	661.73
0.74	Residual	50	183.15	184.10
		100	331.93	309.51
		200	583.35	601.15

Table 2. Variation coefficients of stress vertical component σ_1

Void ratio e	Stress vertical component σ_1	Values of σ_3, kPa	Values of variation coefficients V of stress vertical component σ_1, kPa	
			Standard apparatus	Improved apparatus
0.51	Peak	50	0.090	0.103
		100	0.088	0.114
		200	0.053	0.037
	Residual	50	0.072	0.061
		100	0.047	0.057
		200	0.020	0.036
0.74	Residual	50	0.064	0.065
		100	0.058	0.056
		200	0.024	0.048

the fixed ends (Table 3). The values of cohesion are less by approx 42% (Dirgėlienė *et al.* 2007a; 2007b).

The calculated mean residual values of tan φ for the loose sands are similar both of the standard and the improved apparatuses test results. The values are approx 6% larger for the samples tested with the movable support, and the mean residual values of c are approx 35% larger for the samples tested with the fixed support. Thus, the movable support has no essential influence on identifying the shear strength parameters of the loose sands by processing the data tests.

Table 3. Mean values of the soil shear strength parameters φ and *c*

Void ratio *e*	Shear strength parameters		Mean values of soil shear strength parameters	
			Standard apparatus	Improved apparatus
0.51	Peak	φ, °	36.6	38.0
		c, kPa	30.9	21.4
	Residual	φ, °	33.6	31.0
		c, kPa	17.8	10.3
0.74	Residual	φ, °	26.8	28.3
		c, kPa	17.7	11.4

Table 4. Soil strength parameters

Soil properties		For whole specimen	For failure plane
Elasticity modulus	*E*, MPa	50.0	30.0
Poisson's ratio	υ	0.30	0.45
Angle of internal friction	φ, °	37.9	30.0
Cohesion	*c*, kPa	26.0	17.0

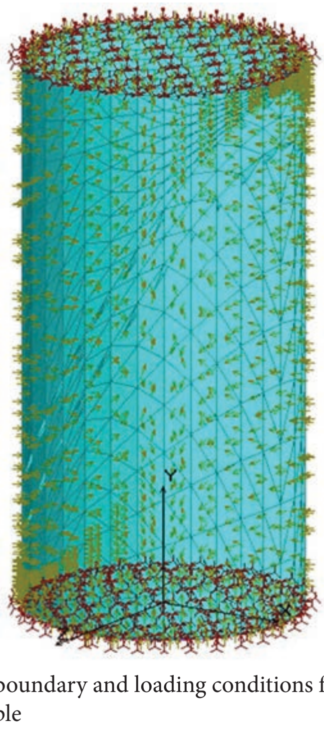

Fig. 4. Mesh, boundary and loading conditions for the discrete model of sample

3. Theoretical analysis of stress distribution in soil sample via numerical simulation

The stress and strain state of soil sample has been simulated applying the computer software COSMOS/M (Finite Element Analysis System, Version 1.75). The nonlinear behavior of the sample is described by the Drucker-Prager physical model of an elastic-perfectly plastic material. The material of the discrete model of the sample responses in an elastic way prior the yielding limit is reached, after that it responses in the perfectly plastic way. The yielding criterion is described by:

$$F = 3\alpha\sigma_m + \overline{\sigma} - k = 0, \quad (1)$$

where α and *k* – material constants; σ_m– the stress mean value; $\overline{\sigma}$ – an effective stress value. The constants α and *k* obtained experimentally.

During the triaxial test the specimen of the dense soil is sheared, the sand particles at the shear plane slide in respect of each other, i.e. the dilatancy effect is recognized. The dilatancy causes the change of the sample volume during shear. The dense soil is loosened at the shear plane (volume), i.e. its properties change and therefore the material properties here are different when compared to that of the remaining volume of the tested specimen. Thus, the different material properties have been chosen for the specimen and the shear plane (volume), respectively (Table 4) to represent the actual situation.

The description of the discrete model of the soil specimen and the loading for simulation of the triaxial test is given below. The geometry of the specimen: the diameter *D* = 5 cm, the height *H* = 10 cm (Fig. 4). The specimen is divided to solid tetrahedral finite elements with four nodal points of three degrees of freedom. Two different design schemes have been simulated. At the first case the nodes at the top plane of the specimen are subjected by the equal vertical displacement u_y = const, while that of the nodes at the bottom plane are fixed. The horizontal (lateral) displacements of the top and the bottom plane of the specimen are also fixed. The pressure is applied isotropically to the soil sample. In the second case the design scheme corresponds to the first case described above with an exception that the horizontal displacements at the bottom plane are free (the movable support links are introduced).

The comments on the obtained numerical simulation results are given below. The shear stress components develop in the contact planes of the platens and the specimen ends. The latter does not allow (or constrain) the horizontal displacements in the case when the changed properties at the shear plane (volume) are introduced to the design scheme (Figs 5, 7, 9, 11). The horizontal displacements

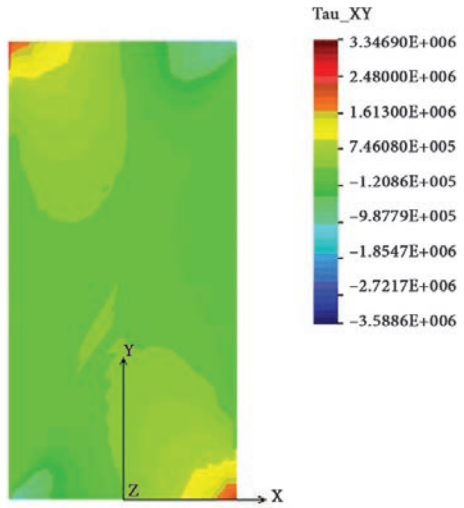

Fig. 5. Distribution of shear stress component τ_{xy} in the sample with fixed ends

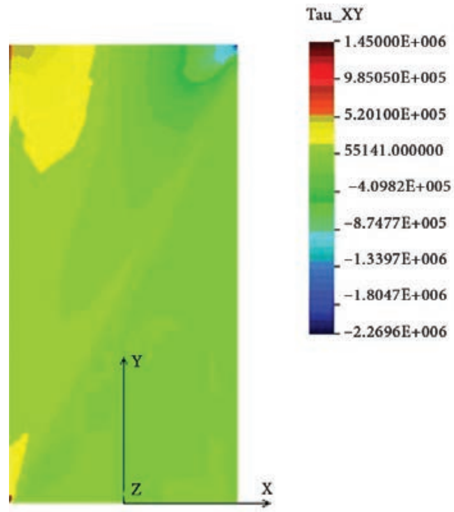

Fig. 6. Distribution of shear stress component τ_{xy} in the sample with free ends

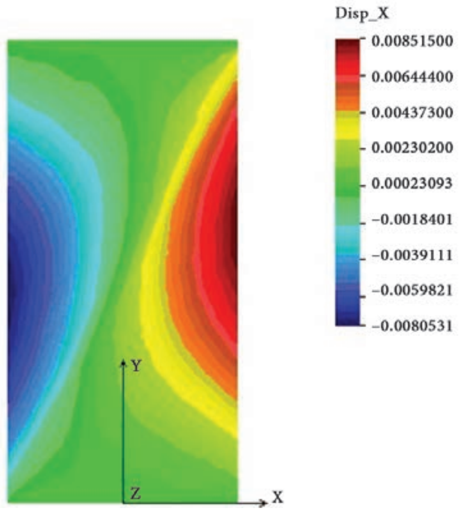

Fig. 7. Distribution of horizontal soil displacements u_x in the sample with fixed ends

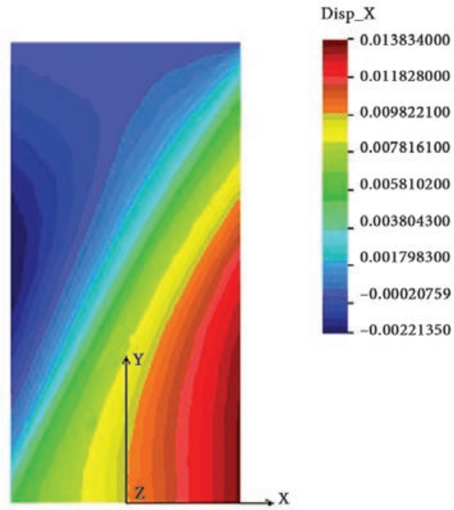

Fig. 8. Distribution of horizontal soil displacements u_x in the sample with free ends

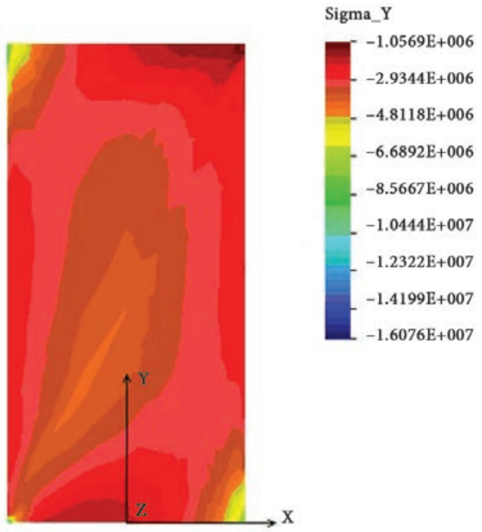

Fig. 9. Distribution of vertical component of stress σ_y in the sample with fixed end

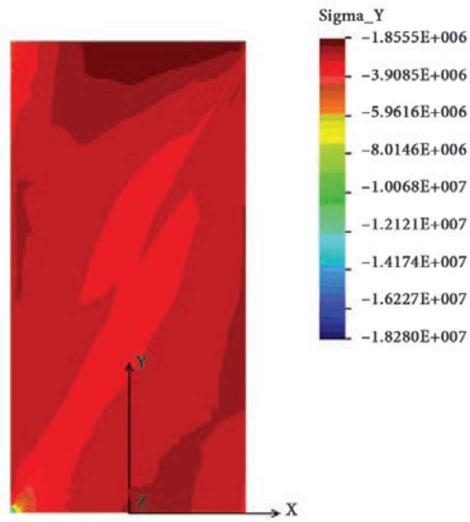

Fig. 10. Distribution of vertical component of stress σ_y in the sample with free ends

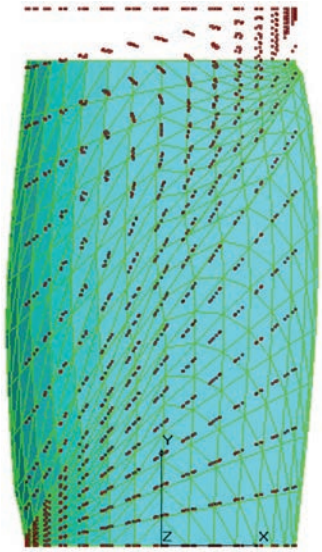

Fig. 11. Deformed shape of sample of triaxial test with fixed ends

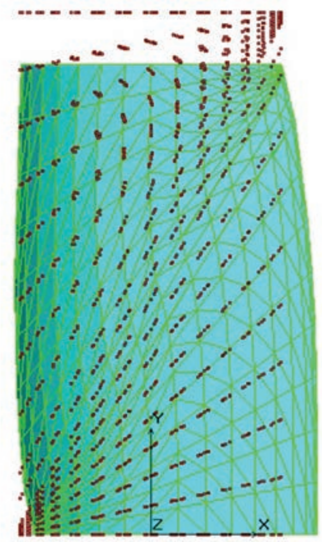

Fig. 12. Deformed shape of sample of triaxial test with fixed ends

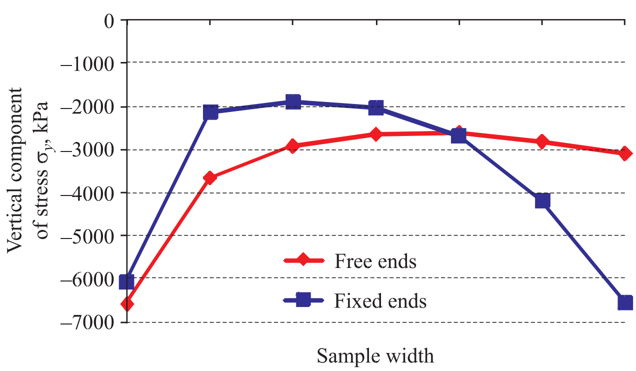

Fig. 13. Distribution of vertical stress component σ_y of specimen base along the section of sample (along X axis)

develop in the case when the design scheme of the free bottom plane is employed (Figs 6, 8, 10, 12).

The case of the free ends yields the stress vertical component reduction of approx 10% when compared to the case of the fixed ends (Fig. 13). The similar data have been obtained experimentally.

4. Conclusions

1. The movable support between the sample base and the platen aiming to reduce the friction between the specimen and the platen is recommended to be introduced for obtaining the more accurate soil shear strength parameters from triaxial test results.

2. The experimental investigations showed that the mean values of the angle of internal friction obtained by processing the values of σ_1 when ε_1 = 15% are approx 15% smaller in the case of the free specimen ends compared to the one of the fixed specimen ends. The mean values of the residual cohesion are smaller by 42%.

3. The FEM simulation showed that the shear stress components develop in the contact planes of the platens and the specimen. They do not allow the horizontal displacements. This is not evaluated when processing the test data to determine the shear strength properties of the tested soil.

4. The FEM simulation proved that the case of the free specimen ends yields the stress vertical component reduction of approx 10% compared to the case of the fixed ends. The similar results have been obtained experimentally.

The equipment and infrastructure of Civil Engineering Scientific Research Center of Vilnius Gediminas Technical University were employed for investigation.

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