



ANALYSIS OF MODERN THREE-SPAN SUSPENSION BRIDGES WITH STIFF IN BENDING CABLES

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Abstract. Suspension bridges are one of the most effective bridge structures used for large spans to overlap. Recently, not only the single-span bridges but also multi-span suspension bridges have been used. The main issue of design and analysis shall consider the excessive deformability induced by the interaction of flexible and kinematic displacement. Stabilization of suspension bridges could be ensured by various means. The most effective and up-to-date measure applied to reduce displacements of suspension bridges is the application of the so-called “rigid” cables with appropriate bending stiffness instead of conventional flexible cables. These retaining elements demonstrate high corrosion stability; their cross-sections are designed using conventional structural steel sections, factory and fabricated connections are simple and firm. The main advanced feature of these cables is that these elements along with the suspended stiffening girder stabilize the initial shape of the bridge effectively. It shall be noted that analysis methods applied for these innovative three-span suspension bridges with “rigid” cables are still under development. There are only few individual publications describing the behaviour of a single-span suspension bridge. The article describes the modern suspension three-span bridge and provides analytic expressions of internal forces and displacements calculated considering the erection sequence.

Keywords: suspension bridge, steel bridge, rigid cable, symmetric loadings, non-linear analysis.

1. Introduction

Suspension bridges, due to their excellent technical performance, are widely used for centuries. Efficiency of these composed-structure bridges is highlighted (distinguished) by the availability to overlap record spans (Ryall *et al.* 2000; Troyano 2003; Song, Wang 2010). Comparison of these bridges with conventional standard bridges shows the substitution of bending loads with tension loads transferred to the main carrying element – suspension cable (Gimsing, Georgakis 2012; Göpper *et al.* 2005; Katchurin *et al.* 1971). Cables of modern suspension bridges are designed using exceptionally strong steel wires. Bending stiffness of these cables is virtually close to zero. At the same time, the cables shall be coated with special anticorrosion protection and are rather complex in terms of construction due to their anchorage and intermediate connections (Daniūnas, Urbonas 2013; Gimsing, Georgakis 2012; Nakamura, Suzumura 2009; Xu, Chen 2013; Ryall *et al.*

2000; Yanaka, Kitagawa 2002). Such factors induce appreciation of bridge construction and operation.

The main disadvantage of suspension bridges, i.e., the excessive deformability, is familiar to the designers of suspension bridges or structures (Horokhov *et al.* 2006; Jennings 1987; Kiisa *et al.* 2012). It is preconditioned not by flexible deformations of carrying suspension elements (cables) but by kinematic displacements under the impact of asymmetric or local loads (Kulbach 2007; Sandovič *et al.* 2011). Generally, the initial shape of the suspension bridge could be stabilized using suspended stiffening girder. This stabilization method could not be deemed as extremely effective due to the high altitude of the girder cross-section and, accordingly, the mass required under particular bridge operation conditions (Grigorjeva *et al.* 2010; Lewis 2012; Wollman 2001). Various additional structural means enabling minimization of displacements of kinematic origin could be used as well (Jennings 1987;

Katchurin *et al.* 1971; Strasky 2005). Some of them are complicated or ineffective from technical – economical point of view.

Recently, the required rigidity of suspension bridge (minimization of kinematic displacements) is ensured by the application of the so-called “rigid” cables instead of conventional flexible cables (Grigorjeva *et al.* 2010; Juozapaitis *et al.* 2006, 2010). These cables are corrosion-resistant and made of standard hot-rolled or welded steel cross sections and their factory and fabricated connections are simple and firm. It simplifies both fabrication and installation of such elements (Gorokhov *et al.* 2013).

There are numerous publications presenting the analysis of the behavior of standard suspension bridges with absolutely flexible suspension cable (Cobo del Arco, Aparicio 2001; Clemente *et al.* 2000; Gimsing, Georgakis 2012; Katchurin *et al.* 1971; Kim, Thai 2010; Wollman 2001). In particular, the articles describing the analysis of suspension bridge flexible cables in terms of local bending stresses having occurred at certain sections shall be mentioned (Caballero, Pose 2010; Chen *et al.* 2011; Fürst *et al.* 2001; Gimsing, Georgakis 2012; Prato, Ceballos 2003; Strasky 2005).

It shall be noted that analysis methods applied for these innovative three-span suspension bridges with “rigid” cables are still under development. There are only few individual publications describing the behaviour of single-span suspension bridge (Grigorjeva *et al.* 2010; Juozapaitis *et al.* 2010). Between them a simplified engineering method could be mentioned for the analysis of internal forces and displacements of suspension bridge with “rigid” cables (Grigorjeva *et al.* 2010). However, the methodology could not be deemed as completely accurate. Recently, the internal forces and displacements of suspension bridge with “rigid” cables are calculated applying digital techniques (Neviril, Kytir 2001; Kala 2012). Still, the above-mentioned techniques could not provide unexceptional results of the analysis of modern suspension bridges with “rigid” cables. Undoubtedly, the analysis of modern suspension three-span bridges with stiff in bending cables shall be developed.

The article describes the behaviour of suspension multi-span bridge with “rigid” cables under the impact of symmetric loads, provides formulas to be applied for calculation of internal forces and displacements of the aforementioned bridge considering the non-linear behavior, and deals with analysis method considering the erection sequence of “rigid” cables.

2. Analysis of suspension three-span bridge with “rigid” cable

According to the provided, the initial shape of suspension bridges could be stabilized and displacements minimized by applying the so-called “rigid” cables with calculated bending stiffness ($EJ_c \neq 0$) instead of flexible ones. These “rigid” cables are made of conventional structural steel sections or complex welded steel cross sections (Grigorjeva *et al.* 2010; Juozapaitis *et al.* 2010). Value of the cable bending strength EJ_c shall be selected in accordance with

the imposed loads and operational requirements. It shall be noted that cross-sectional area of this “rigid” cable is approximately equal to the cross-sectional area of a flexible cable, and its application does not increase the mass of carrying structures. On the contrary, while comparing it with classical bridge (with flexible cable), total demand for steel could be reduced due to the “rigid” cable resistance to the impact of asymmetric loads and “unloading” of the suspended stiffening girder. Thus the cross section of the girder could be reduced.

One more characteristic property of rigid cable shall be noted. There are two variants of cable installation. The first one when the cable is provided with bending stiffness ($EJ_c \neq 0$) in case of both dead load g and temporary load p . The second method foresees minimization of initial loads applying the bending stiffness after the bridge erection, i.e. only under the impact of live loads. This article describes a second method of bridge erection.

The structure of new suspension bridge (with „rigid” cable) is analogous to that of conventional bridge. Stiffening girders, pylons and hangers connecting bearing cables and stiffening girders shall be erected for this type of bridge. Analysis of these new suspension bridges shall be made applying the same assumptions (Gimsing, Georgakis 2012; Katchurin *et al.* 1971; Wollman 2001). First of all, it is assumed that the height of a stiffening girder remains constant along the entire bridge ($h_b = const$). Secondly, it is assumed that the distance between hangers could ensure random distribution of loads imposed on the cable. Thirdly, it is assumed that the hangers are absolutely rigid, i.e. the elongation of hangers is disregarded. Fourthly, it is assumed that rigid cable under the dead load takes the shape of parabola. This assumption could be used for the second method of “rigid” cable formation without any exceptions. Still, it shall be noted that in case of first erection method the “rigid” cable could be provided both with square parabola shape and other shape foreseen and described by the designer.

2.1. Initial shape of suspension bridge

Structural and estimated diagrams of considered suspended three-span bridge with “rigid” cables, stiffening girders, connection hangers, and pylons are provided in Fig. 1. Stiffening girders are split, i.e. flexibly connected at the level of intermediate supports (Fig. 1c). Certainly, high bending loads could be prevented by applying flexible connection of “rigid” cables and the upper part of pylons.

According to the aforesaid, the second method of “rigid” cable formation shall be analyzed, i.e., bearing cable during its installation will adopt the entire dead load as an absolutely flexible cable, and after the completion of bridge erection it will behave as “rigid” cable ($EJ_c \neq 0$). In this case, the bridge loading history will be obvious: the overall dead load g will be imposed on flexible cable, and live loads p will be distributed among the stiffening girder (p_b) and “rigid” cable (p_b).

Deformation diagram of suspension multi-span bridge under the loads g and p is shown in Figs 1b, 1c. The bridge shall be conditionally divided into individual structural elements – cables (1, b) and stiffening girders (1, c). Mid span

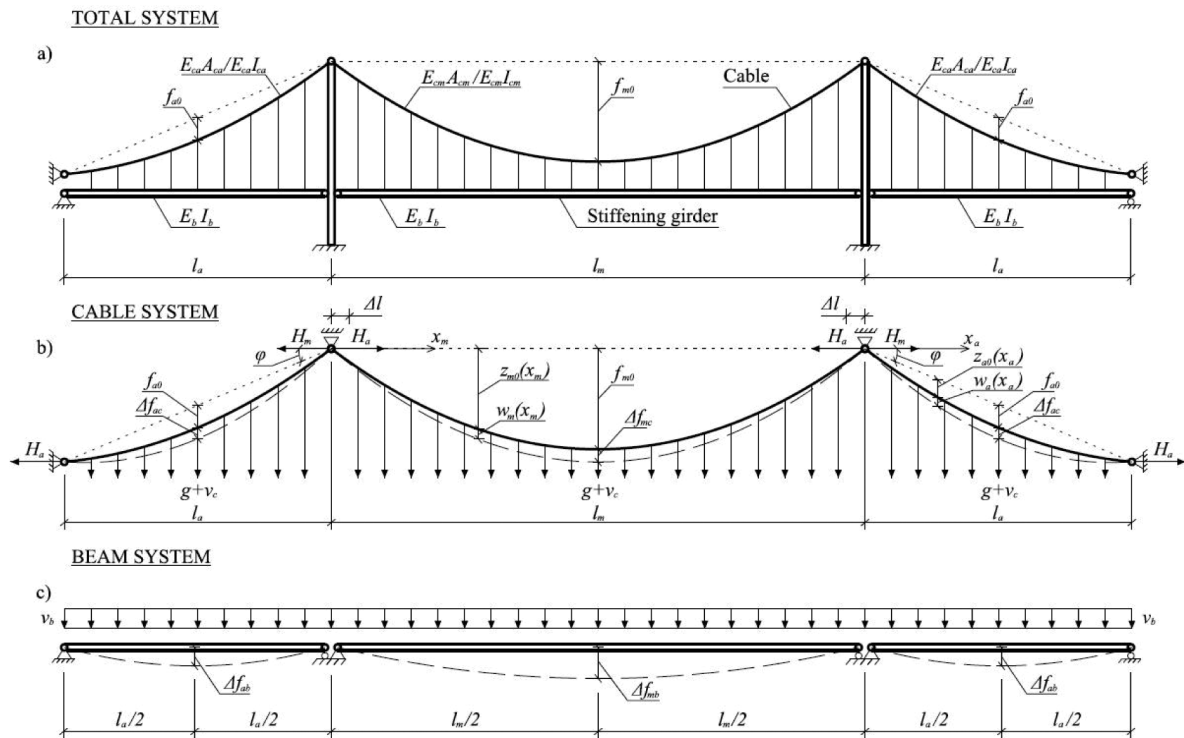


Fig. 1. Structural and estimated diagrams of considered suspended bridge

(main span) (l_m) and side spans (l_a) of the bridge shall be analyzed individually.

2.1.1. Mid span

Supports of the mid span cable shall be installed at the same level. The initial shape of flexible cable under the dead load g , considering the erection sequence, shall be expressed by the following equilibrium equation:

$$H_{m0} z_{m0}(x_m) = M_{mg}(x_m), \tag{1}$$

where H_{m0} – horizontal component of cable tension under the load g ; $M_{mg}(x_m) = 0.125 g l_m^2 \left(\frac{4x_m}{l_m} - \frac{4x_m^2}{l_m^2} \right)$ – moment caused by the dead load g in the analogous girder, i.e., girder with the equal length of the span; $z_{m0}(x_m)$ – initial curve of the cable (square parabola).

Cable straining force at the center of the mid span ($x_m = 0.5l_m$) shall be calculated as follows:

$$H_{m0} = \frac{g l_m^2}{8 f_{m0}}, \tag{2}$$

where f_{m0} – initial sag of flexible cable.

It shall be repeated that in case of the second suspension bridge erection sequence, stiffening girders (both of mid span and side spans) will not adopt the dead load, and the bending moments will be equal to zero ($M_{bg} = 0$).

2.1.2. Side spans

Supports of the side span cable shall be installed at different levels (Fig. 1). These inclined cables under the impact

of the dead load g , considering the erection sequence, will be flexible as well. If the inclined cables are analyzed applying the global reference system, their initial shape could be approximately expressed by the analogous equation:

$$H_{a0} z_{a0}(x_a) = M_{ag}(x_a), \tag{3}$$

where H_{a0} – horizontal component of cable tension under the load g ; $M_{ag}(x) = 0.125 g l_a^2 \left(\frac{4x_a}{l_a} - \frac{4x_a^2}{l_a^2} \right)$ – moment caused by the dead load g in the analogous girder of a side span; $z_{a0}(x_a)$ – initial curve of the side span cable.

It shall be highlighted that $z_{a0}(x_a)$ shall be calculated from the line connecting the top and the bottom supports (Fig. 1b). This means that the initial straining force of side cable at the center of the span ($x_a = 0.5l_a$) shall be calculated as follows:

$$H_{a0} = \frac{g l_a^2}{8 f_{a0}}, \tag{4}$$

where: f_{a0} – initial sag of flexible cable, calculated from the line connecting the top and the bottom supports.

Initial tension of the side cable H_{a0} is horizontally directed (Fig. 1b).

2.2. Selection of geometrical parameters of suspension bridge

The structure of suspension three-span bridge is more complex in comparison to the single-span suspension bridge. This type of bridge structure requires the selection of appropriate geometrical parameters of individual structural

span elements in order to eliminate occurrence of additional vertical and horizontal displacements in case of a dead load. In other words, it is essential to ensure the appropriate balance of the entire structural system. For this reason the upper part of pylons shall meet the following condition:

$$H_{a0} = \frac{gl_a^2}{8f_{a0}} = H_{m0} = \frac{gl_m^2}{8f_{m0}}. \quad (5)$$

Under the given lengths of main and side spans l_m and l_a , the following dependency (relation) of the initial sag of mid and side cables could be discovered:

$$f_{m0} = f_{a0} \frac{l_m^2}{l_a^2}. \quad (6)$$

Bridge engineering practice describes the most frequent length of side spans as the half of the mid one ($l_a = 0.5l_m$). In this case $f_{a0} = 0.25f_{m0}$.

Lately, the so-called asymmetric multi-span bridges with side spans of different length are being used, i.e. $l_{a1} \neq l_{a2}$. In this case, instead of Eq (6), the following generalized expression should be used:

$$f_{m0} = f_{a0.1} \frac{l_m^2}{l_{a1}^2} = f_{a0.2} \frac{l_m^2}{l_{a2}^2}. \quad (7)$$

It shall be noted that the described conditions will be correct if in case of initial shape (when the overall length of a bridge is subjected to the dead load ($g = const$), the cables are flexible ($E_{ca}J_{ca} = 0$ and $E_{cm}J_{cm} = 0$) and the pylons rest flexibly on the foundation.

2.3. Analysis of suspension bridge under temporary load

The following analysis of suspension three-span bridge will be discussed. Symmetric load in case of equal temporary load value of all spans ($v_a = v_m = v$) will be investigated. Deformed diagram of this bridge under both dead g and temporary v loads is provided in Figs 1b, 1c. Let's refresh the fact that in course of bridge behavior analysis the second erection variant shall be applied. The initial equilibrium state of these "flexible" cables under the g load shall be described by (1), (2) and (3), (4) equations. Prior the impact of permanent load particular bending stiffness shall be set for all cables applying appropriate structural means, i.e. cable will become the so-called "rigid". The case will be analyzed when the bending stiffness of mid and side cables is equal ($E_{ca}J_{ca} = E_{cm}J_{cm} = E_cJ_c \neq 0$). It means that the temporary load imposed on the above cables (v_c) will induce additional axial forces H and bending moments $m_c(x)$. Stiffening girders will take remaining part of the above loads v_b .

2.3.1. Analysis of the main span of suspension bridge

"Rigid" cables and stiffening girders of the entire bridge under the temporary load v will be deformed (Fig. 1). Suspension cable will receive the dead load and a part of a temporary load ($g+v_c$), and the stiffening girder – sharing part of a temporary load (v_b). The above mentioned third assumption enables the following even displacement condition of stiffening girder and "rigid" cable

$w_{bm}(x_m) = w_{cm}(x_m) = w_m(x_m)$. According to the above-mentioned condition, the equilibrium equation of stiffening girder will be as follows:

$$-E_bJ_b w_m''(x_m) + M_{b,m}(x_m) = 0, \quad (8)$$

where E_bJ_b – bending stiffness of stiffening girder; $M_{b,m}(x_m) = 0.125v_b l_m^2 \left(\frac{4x_m}{l_m} - \frac{4x_m^2}{l_m^2} \right)$ – imposed live loads (v_b) imposed bending moment.

Deformation state of "rigid" cable tension will be more complicated. It will be subjected to bending moment as well. Consequently, the equilibrium equation of the abovementioned cable will be as follows:

$$H_m [z_{m0}(x_m) + w_m(x_m)] + m_{cm}(x_m) + M_{c,m}(x_m) = 0, \quad (9)$$

where $m_{cm}(x_m) = -E_cJ_c w_m''(x_m)$ – bending moment of "rigid" cable; $M_{c,m}(x_m)$ – moment caused by dead load g and temporary load v_c inside the analogous girder.

Taking into consideration that the estimated general load imposed on the bridge structures is equal to $g + v = g + v_c + v_b$, and based on the third assumption the following equilibrium equation could be made:

$$(E_{bm}J_{bm} + E_{cm}J_{cm})w_m''(x_m) - H_m [z_{m0}(x_m) + w_m(x_m)] + M_m(x_m) = 0. \quad (10)$$

Eq (10) is similar to the equilibrium equation of standard single span bridge with flexible cable (Juozapaitis et al. 2010; Wollman 2001). In this case the overall strength of calculated bridge will increase due to the member $E_{cm}J_{cm}$, i.e. due to the bending stiffness of a cable. It could be stated that displacements of new suspension bridge with "rigid" cables under the equal initial conditions will be lower than displacements of a suspension bridge with flexible cable (Juozapaitis et al. 2010). Eq (10) could be rewritten as follows:

$$w_m''(x_m) - k_m^2 w_m(x_m) = k_m^2 \left[z_{m0}(x_m) - \frac{M_m(x_m)}{H_m} \right], \quad (11)$$

where $k_m^2 = \frac{H_m}{(E_{bm}J_{bm} + E_{cm}J_{cm})}$ – total flexibility factor of the entire system.

In order to find simpler solution and minimize the volume of iterative calculation the following concept of fictive displacement of "rigid" cable shall be used (Juozapaitis et al. 2010; Moskalev 1981):

$$w_{fic,m}(x_m) = z_{fic,m}(x_m) - z_{0m}(x_m), \quad (12)$$

where $z_{fic}(x) = \frac{M_m(x_m)}{H_m}$ – fictive curve complying with the curve of deformed axle of absolute flexible cable with the equal tension force (Moskalev 1981).

In this case (11) the equation will be as follows:

$$w_m''(x_m) - k_m^2 w_m(x_m) = -k_m^2 w_{fic,m}(x_m). \quad (13)$$

Solution of this equation, considering the ultimate condition will be as follows:

$$w_m(x_m) = \Delta f_{fic,m} \left[\frac{4x_m}{l_m} - \frac{4x_m^2}{l_m^2} + \frac{8}{k_m^2 l_m^2} \beta_m \right], \quad (14)$$

where $\Delta f_{fic,m} = z_{fic,m}(x_m^*) - z_{0m}(x_m^*)$ – fictive displacement of “rigid” cable at the middle of the span ($x_m^* = 0.5l_m$);

$$\beta_m = ch k_m x_m - \frac{1 - ch k_m l_m}{sh k_m l_m} sh k_m x_m - 1.$$

Analysis of this solution (14) shows its analogy to the formula applied for the displacement of “rigid” cable ($EJ_c \neq 0$) (Juozapaitis *et al.* 2006). It shall be stated that the new suspension bridge could be calculated as a single “rigid” cable replacing its flexibility factor with the total flexibility factor of the entire system (Eq (11)). It could be stated that behaviour of new suspension bridge is described by the interaction of cable with stiffening girder. Level of initial bridge shape stabilization applying bending stiffness is described by the flexibility factor of the entire structural system. It is essential to highlight the fact that in case of the same general bridge strength and varying values of girder bending stiffness $E_{bm}J_{bm}$ and cable bending stiffness $E_{cm}J_{cm}$, the ratio of values shall ensure minimum tension forces inside bridge structures. It means that deformation behaviour of the entire bridge could be regulated in accordance with the ultimate shape conditions.

In order to determine displacements of “rigid” cable $w_m(x_m)$, and bending moment $m_{cm}(x_m)$ it is required to calculate appropriate fictive displacements $\Delta f_{fic,m}$ and cable tension force H_m . The abovementioned values are correlated by the following dependency:

$$H_m = \frac{(g + v_c)l_m^2}{8(f_{m0} + \Delta f_{fic,m})}. \quad (15)$$

In order to determine $\Delta f_{fic,m}$ and H_m , it is required to apply additional equation describing the relation (dependency) of the initial length of the cable (s_0), its flexible elongation (Δs_{el}), and length after the deformation (s):

$$s = s_0 + \Delta s_{el}. \quad (16)$$

Analysis of the mid span requires the evaluation of possible horizontal displacement of the pylon upper part Δl (Fig. 1b).

Initial length of the cable, considering its shape, shall be calculated applying the following expression $s_{0m} = l_m + \frac{8f_{m0}^2}{3l_m}$. In case of temporary load of symmetrically stressed (v_c) length of a “rigid” cable after the deformation (considering pylon displacements Δl), will be equal to:

$$s_m = l_m - \Delta l + \frac{8(f_{m0} + \Delta f_{fic,m})^2}{3(l_m - \Delta l)} \phi(k_m l_m), \quad (17)$$

where $\phi(k_m l_m)$ – function evaluating the impact of cable bending stiffness on its deformation.

It shall be noted that application of a “rigid” cable length after the deformation fictive bending concept expression (17) is identical to the formula applied for calculation of a flexible cable length (Moskalev 1981). It simplifies iterative analysis. Provided formulas show that decreased bending stiffness of a “rigid” cable EJ_c , values of fictive bending $\Delta f_{fic,m}$ approximate the values of a flexible cable bending, if $EJ_c \rightarrow 0$, the value of fictive bending of rigid cable will be equal to the value of flexible cable fictive bending.

It shall be noted that horizontal displacement of cable supports (the upper part of pylons) Δl will increase both vertical displacements and additional bending moments inside the “rigid” cable and stiffening girder.

It is essential to apply structural means intended for minimization of cable support (the upper part of pylons) displacements.

Iterative calculation shall be performed applying the following sequence. First of all, values of the initial main indeterminates $\Delta f_{fic,m}$ and H_m shall be accepted for flexible cable. Then, flexibility factor of the entire system k_m and function $\phi(k_m l_m)$ shall be calculated. Further, calculation shall be carried out applying gradual approximating with the help of Eqs (15)–(17). Conversion condition could be expressed as follows:

$$\Delta s_{el,m} - \Delta s_{gm} \leq \varepsilon, \quad (18)$$

where $\Delta s_{gm} = s_m - s_{0m}$

It shall be noted that calculation has been complicated by the indeterminate horizontal displacement of supports Δl , which could be determined after the calculation of parallel side spans.

Final values $\Delta f_{fic,m}$ and H_m shall be used for calculation of “rigid” cable and stiffening girder displacements $w_m(x_m)$ their bending moments $m_{cm}(x_m)$ and $m_{bm}(x_m)$. It shall be highlighted that fictive displacement enables minimization of iterative number and simplifies the calculation itself.

2.3.2. Analysis of a side span of suspension bridge

Behaviour of side spans of suspension bridge is identical to the behaviour of the mid span structure. The distinctive feature includes the fact that the “rigid” cable of the side span is inclined (Fig. 1). Improved analysis of inclined “rigid” cables (local reference system) is more complex. Consequently, in order to simplify calculation this inclined cable will be analyzed by applying the global reference system, and the accuracy of calculation will be enhanced by applying the cable flexibility $k_a l_a$ adjustment coefficient $\eta_a = \cos^2 \varphi$.

Further, behaviour of the right span structures of suspension bridge will be discussed. Deformed diagram of individual structures is shown in Figs 1a, 1b. Suspension cable the same as mid span structure will be subjected to the dead load and sharing part of temporary load ($g + v_c$), and stiffening

girder to sharing part of temporary load (v_b). Equilibrium condition of the side span stiffening girder will be as follows:

$$-E_b J_b w_a''(x_a) + M_{b,a}(x_a) = 0, \quad (19)$$

where $w_a''(x_a)$ – the second expression of a described span stiffening girder displacement;

$$M_{b,a}(x_a) = 0.125 v_b l_a^2 \left(\frac{4x_a}{l_a} - \frac{4x_a^2}{l_a^2} \right) - \text{moment caused by}$$

the temporary load (v_b).

Equilibrium equation of the inclined „rigid“ cable:

$$H_a [z_{a0}(x_a) + w_a(x_a)] - E_c J_c w_a''(x_a) + M_{c,a}(x_a) = 0, \quad (20)$$

where $E_c J_c w_a''(x_a)$ – bending moment inside the inclined “rigid” cable; $M_{c,a}(x_a)$ – moment caused by the dead load g and temporary load v_c inside the analogous span girder.

After the translation of Eq (20) and application of fictive bending concept, the solution analogous to the mid span solution could be drawn:

$$w_a(x_a) = \Delta f_{fic,a} \left[\frac{4x_a}{l_a} - \frac{4x_a^2}{l_a^2} + \frac{8}{k_a^2 l_a^2} \beta_a \right], \quad (21)$$

where

$$\beta_m = ch k_m x_m - \frac{1 - ch k_m l_m}{sh k_m l_m} sh k_m x_m - 1;$$

$\Delta f_{fic,a} = z_{fic,a}(x_a^*) - z_{0a}(x_a^*)$ – fictive displacement of “rigid” cable at the center of the span ($x_a^* = 0.5l_a$), shall be calculated starting from the straight line connecting the upper and the bottom supports (Fig. 1b).

Horizontal element of inclined “rigid” cable tension force shall be calculated as follows:

$$H_a = \frac{(g + v_c) l_a^2}{8(f_{a0} + \Delta f_{fic,a})}. \quad (22)$$

Iterative calculation shall be performed applying the abovementioned equation:

$$s_a = s_{0a} + \Delta s_{el,a}, \quad (23)$$

Initial length of the inclined cable under the square parabola shape shall be calculated applying global reference system and previously drawn equation:

$$s_{0a} = \frac{l_a}{\cos \varphi} + \frac{8f_{a0}^2}{3l_a} \cos^3 \varphi. \quad (24)$$

Length of the inclined “rigid” cable after the deformation, considering the displacements of its supports Δl , will be equal to:

$$s_m = \frac{l_m + \Delta l}{\cos \varphi} + \frac{8(f_{a0} + \Delta f_{fic,a})^2}{3(l_a + \Delta l)} \phi(k_a l_a \eta) \cos^3 \varphi, \quad (25)$$

where $\phi(k_a l_a \eta)$ – function, describing the impact of inclined “rigid” cable bending stiffness on the deformation.

Iterative calculation of the side suspension bridge span structure shall be performed applying the sequence analogous to the sequence used for main span. If accepted values of values of the initial main indeterminates $\Delta f_{fic,a}$ and H_a shall be used for gradual approximating with the help of Eqs (22)–(25). Conversion condition is analogous to the mid span condition and could be expressed as follows $\Delta s_{el,a} - \Delta s_{ga} \leq \varepsilon$. It shall be noted that the determination of the displacement of side span inclined cable, stiffening girder and internal forces the following horizontal displacements of supports shall be known Δl . For this purpose the additional iteration of the second level shall be drawn, it foresees equilibrium condition of the mid and side span tension force and suspension bridge cable deformation equation. Analysis could be simplified by preliminary calculation of cable kinematic displacement performed at the very beginning of iteration.

3. Concluding remarks

The work describes the modern suspension three-span bridge with stiff in bending cables, the initial shape of which is stabilized by so-called “rigid” cables. Initial and operation stages of that bridge are being described in terms of erection sequence of its “rigid” cables. Analysis formulas of internal forces and displacements of individual components are provided in terms of horizontal displacements of supports. Analysis of inclined “rigid” cable applying global reference system has been described as well. It has been noted that calculation expressions used for modern three-span suspension bridge describe the general cases used for calculations of the above bridges, i.e., ignoring the impact of stiffening girders could help to determine appropriate formulas for individual “rigid” cables. In case of “rigid” cable transformation into the flexible one – expressions for the calculation of standard suspension bridge could be delivered. It has been determined that modern suspension bridge enables the adjustment of structural tension force deformation state by changing the ratio between the cable and stiffening girder bending stiffness. Application of the cable’s fictive displacement concept could be used in order to significantly simplify the iterative calculation of the total system.

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