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BUCKLING OF THE STEEL LINERS OF UNDERGROUND ROAD STRUCTURES: THE SENSITIVITY ANALYSIS OF GEOMETRICAL PARAMETERS

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Abstract. Designing a suitable, applicable and efficient support system for underground road structures have always been one of the most important engineering tasks for tunnel engineers. There are some different support systems applied to making underground structures safe against overburden and lateral pressures. Among these systems, permanent or temporary steel frames, wire meshes, rock bolts and shotcretes have been commonly used for suffering the exerted burdens and making the structure a safe place. This paper proposes a numerical analysis of the geometrical instability of steel-arch shells as one of the main bodies of underground road structure liners by means of calculating their buckling load and utilizing the finite element method. In this regard, a considerable number of structures (84) having different geometrical parameters have been modelled and their buckling loads have been calculated. For this purpose, the thickness, internal angle and radius of the periphery cylinder of the arch-shell system were considered taking into account geometrical parameters. Moreover, to accurately model the buckling load using the proposed algorithm, the weight of the structure has also been included in the made calculations. Finally, as the main scope is based on the Cosine Amplitude Method, sensitivity analysis is carried out to investigate the strength of the relationship between each input geometrical parameter and their buckling load. Based on the obtained relationships, the thickness of the structure is reported as the most affective geometrical parameter on buckling steel arch-shell support systems. In addition, the internal angle of arch supports is the least influential parameter.

Keywords: buckling, Finite Element Method (FEM), steel arch-shell support system, sensitivity analysis, underground road structures, geometrical parameters, Cosine Amplitude Method (CAM).

1. Introduction

To counteract the threats imposed as a consequence of road structure falls, understanding the most important cause of their instability and designing a powerful support system will be significantly helpful. Wide variety in overburden shapes, thickness and characteristics that are encountered in underground road projects mean that each underground structure presents a unique design challenge. There are a number of support systems that can be applied to resisting against the instability of underground openings such as rock bolting, wire meshes, shotcrete, steel frames, liners, etc. Also, in many practical cases, two or three of the introduced support systems are used together for designing a suitable support system, which is therefore completely dependent on the geological conditions of the project site, overburden pressure, water level and overburden type (Hoek *et al.* 2000).

Tunnel liners are recognized as one of the most demanding tools used for supporting underground excavations, including underground road and mining tunnels (Berti *et al.* 1998; Hashash *et al.* 2005; Carranza-Torres, Diederichs 2009). Tunnel liners are generally made up of steel or concrete. Also, in some cases, both steel and concrete liners are used for supporting the structure.

In the case of steel liners, buckling is one of the possible types of instability. Since a steel liner is indeed a type of arch-shell steel, the point is solving a buckling problem and computing its buckling load. Then, knowing the buckling load, their concerning parameters and the weight of the relationship between each of the affecting parameters

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and the buckling load of the structure will considerably help the engineers in creating economical and safe design.

There are plenty of studies on the concept of stability and instability of structures and structural membranes (Bai *et al.* 2011; Hasanzadehshooiili *et al.* 2012a, 2012b; Šapalas 2004). This issue has been always one of the most important problems controlling the design phase in engineering projects. The buckling of structures, which is one of the common types of instabilities, has been critically studied from different points of view (Batoz 1979; Xue, Fatt 2002). However, the existent complexities of partial differential equations governing their behaviour have made it a difficult problem from the theoretical viewpoint.

A cylindrical shell was the first buckling problem solved by Timoshenko under axial loading. This issue was then critically perused and investigated taking into account different standpoints. In this regard, all concerning parameters such as boundary conditions, pre-buckling deformations, geometric imperfections, and load eccentricities were studied (Almroth 1966; Hoff 1966). Despite of this bulk of studies, due to sensitivity to small geometric imperfections in arch-shell liners and complicated nonlinear partial differential equations governing them, solving a meaningful number of arch-shell buckling problems is theoretically difficult. Moreover, design methods strictly depend on extensive experimental data. Nevertheless, these data are available only for limited cases (Beedle 1991). Nevertheless, the use of new numerical methods such as the finite element (FE) and boundary element method has effectively facilitated their solution and design (Barla *et al.* 2011; Bushnell 1985; Yang *et al.* 1990).

To cope with this difficulty, numerical techniques considering all imposed considerations are widely applied (Teng 1995). Eigenvalue buckling analysis, that determines the bifurcation points of the structure and is widely and commonly used by engineers in a variety of FE software, is used for gaining the buckling modes of steel arch-shells and the buckling load of each structure (Brendel, Ramm 1980).

Then, to obtain a comprehensive model for predicting the buckling load of arch-shells, which encompasses all engineering and economic plausible geometric forms of arch-shells, FE modelling is used for preparing precise datasets of sensitivity analysis.

Since the main scope of this paper is obtaining the most and the least geometrical influential parameters affecting the liner buckling load, first, 84 arch-shells are numerically modelled. Then, conducting buckling analysis based on the Finite Element Method (FEM) and their buckling loads is calculated. The performed analysis considers the radius of the periphery cylinder, the thickness of the shell and its internal angle, as then input variables and the buckling load of different models having different input parameters are gained. Finally, with reference to the Cosine Amplitude Method (CAM), the strength of the relationship between input parameters and the buckling load of the shells has been attained.

Next, using sensitivity analysis, the parameter highly influential in the buckling load has been investigated.

2. Background and procedure for numerical analysis

Linear buckling analysis is based on a classic eigenvalue problem for solving which, first, the load-displacement relationship of a linear elastic pre-buckling load state, $\{F_0\}$ should be solved (Brendel, Ramm 1980).

$$\{F_0\} = [K_e]\{u_0\}, \tag{1}$$

where $\{u_0\}$ – displacement resulting from the applied load, $\{F_0\}$; $\{\sigma_0\}$ – resulting stress from$\{u_0\}$. At an arbitrary state ($\{F\}$,$\{u\}$ and $\{\sigma\}$), incremental equilibrium equations, presuming that pre-buckling displacements are not large, are presented by Brendel, Ramm (1980):

$$\{\Delta F\} = \left[[K_e] + [K(\sigma)_\sigma]\right]\{\Delta u\}, \tag{2}$$

where $[K_e]$ and $[K(s)_\sigma]$ – the elastic stiffness matrix and initial stress matrix evaluated at the stress state $\{\sigma\}$ respectively.

Assuming that the pre-buckling load is a linear function of the applied load, $\{F_0\}$, it can be expressed like (Brendel, Ramm 1980).

$$\{F\} = \lambda\{F_0\}, \tag{3}$$

$$\{u\} = \lambda\{u_0\}, \tag{4}$$

$$\{\sigma\} = \lambda\{\sigma_0\}. \tag{5}$$

Considering that λ is a buckling load multiplier, it was accepted that (Brendel, Ramm 1980).

$$\left[K(\sigma)_\sigma\right] = \lambda\left[K(\sigma_0)_\sigma\right]. \tag{6}$$

Then, the stated equilibrium equation for the entire pre-buckling range becomes (Brendel, Ramm 1980).

$$\{\Delta F\} = \left[\left[K_e\right] + \lambda\left[K(\sigma_0)_\sigma\right]\right]\{\Delta u\}. \tag{7}$$

At the buckling load state, $\{F_{Cr}\}$, although in some cases there are not any changes in the load amplitude, $\{\Delta F\}= 0$, the structure exhibit a change in deformation, $\{\Delta u\}$.

Thus, by substituting $\{\Delta F\}= 0$, with Eq (2), and considering that to satisfy this condition, Eq (6) was solved, Eq (8) was satisfied (Brendel, Ramm 1980):

$$Det\left[[K_e] + \lambda[K(\sigma_0)_\sigma]\right] = 0. \tag{8}$$

In the FEM with n degrees of freedom, Eq (8) yields the n^{th} order polynomial in λ, the eigenvalues. Then, the elastic critical load, $\{F_{Cr}\}$, is given by the lowest value of the calculated λ. Thus, as for the described FEM, the minimum calculated value of λ will be used for calculating its buckling load. The FE-based software neglects the weight of the structure, and therefore this method is used for gaining the

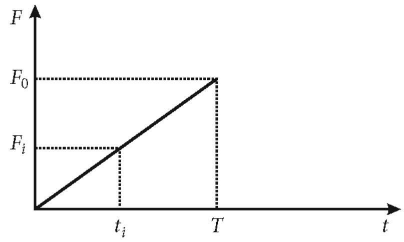

Fig. 1. The manner of imposing external ramping force upon the structure in FE software during its first loading sub-step

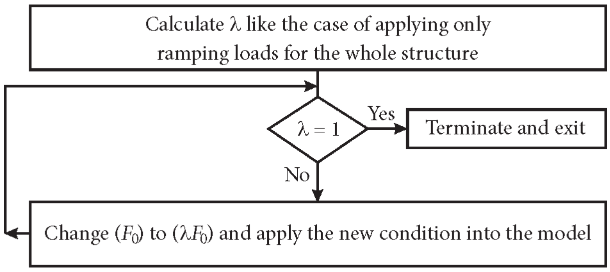

Fig. 2. The algorithm applied for considering the weight of the structure as a constant load (Hasanzadehshooiili *et al.* 2012b)

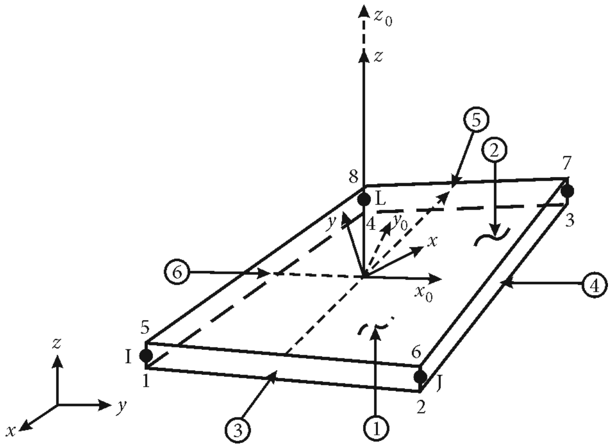

Fig. 3. Degrees of freedom of element Shell 181

buckling load of structures (Brendel, Ramm 1980). However, in order to take the weight of the structure into consideration, the following elaborated algorithm is proposed.

As regards FE software, the buckling load is considered as a ramping load, and the applied load will increase to reach the bifurcation point, the one the structure of which starts buckling (Singer *et al.* 1998). This ramping load is applied to the structure in the way shown in Fig. 1.

In Fig. 1 T – time consumed during the first sub-step. Thus,

$$F_i = \left(\frac{F_0}{T}\right) T_i \tag{9}$$

and

$$F_{Cr} = \lambda F_0. \tag{10}$$

Hence, the buckling load of the structure is calculated using Eq (10) and considering the minimum obtained value λ. However, as mentioned above, due to the fact that the pre-buckling load is known as a linear function of the applied load and cannot contain the weight of the structure as a constant load, the algorithm depicted in Fig. 2 is applied for considering the weight of the shell.

Table 1. Mechanical parameters of steel

Parameter	Description
Young Modulus	$207 \cdot 10^9$ Pa
Poison Ratio	0.3
Density	7890 kg/m^3

Table 2. Geometric parameters of the studied arch-shells

Parameter	Description
Radius of the periphery cylinder of arch shell	4–6 m
Thickness of shell	3–6 mm
Internal angle of shell	40–70 °

Also, to comprehensively study the buckling behaviour of arch-shells, the mechanical and geometric parameters of the studied structures are listed in Tables 1 and 2, respectively.

Figs 4 and 5 show the modelled structure and the shapes of the buckling mode of the analysed structure respectively.

Also, the element used for modelling thin shells, for example, Shell 181 that is a 4 node element, is shown in Fig. 3. This element has 6 degrees of freedom at each node, and the structure is supported by the fixed boundaries.

3. Sensitivity analysis

After obtaining the buckling load of structures, the sensitivity analysis of the strength of the relationship between input parameters and output is accomplished. To conduct sensitivity analysis, all input parameters and the buckling load of the structure as output were normalized in a scale of 0–1 using Eq (11). Thus, the values used in Fig. 6 are dimensionless.

$$\text{Scaled value} = \frac{\text{unscaled value} - \text{min value}}{\text{max value} - \text{min value}}. \tag{11}$$

The strength of relations between input parameters and the buckling load of arch-shells is determined by the CAM.

To determine the most sensitive parameters affecting the buckling load, all data pairs are arranged in X-space. The data pairs used for constructing data array X is defined as

$$X = \{x_1,\ x_2,\ x_3,\ x_4,\ \ldots,\ x_i,\ \ldots,\ x_n\}. \tag{12}$$

Each of the elements, x_i, in data array X is a vector of the lengths of k, i.e.

$$x_i = \{x_{i_1},\ x_{i_2},\ x_{i_3},\ \ldots,\ x_{i_k}\}. \tag{13}$$

All in all, each of the above pairs was considered as a point in m-dimensional space, in which, to specify each

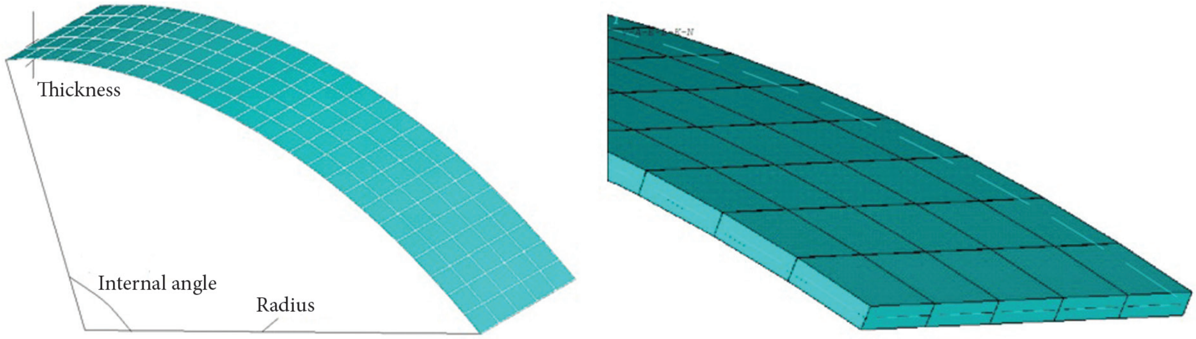

Fig. 4. An isometric view of the structure and the shape of the element

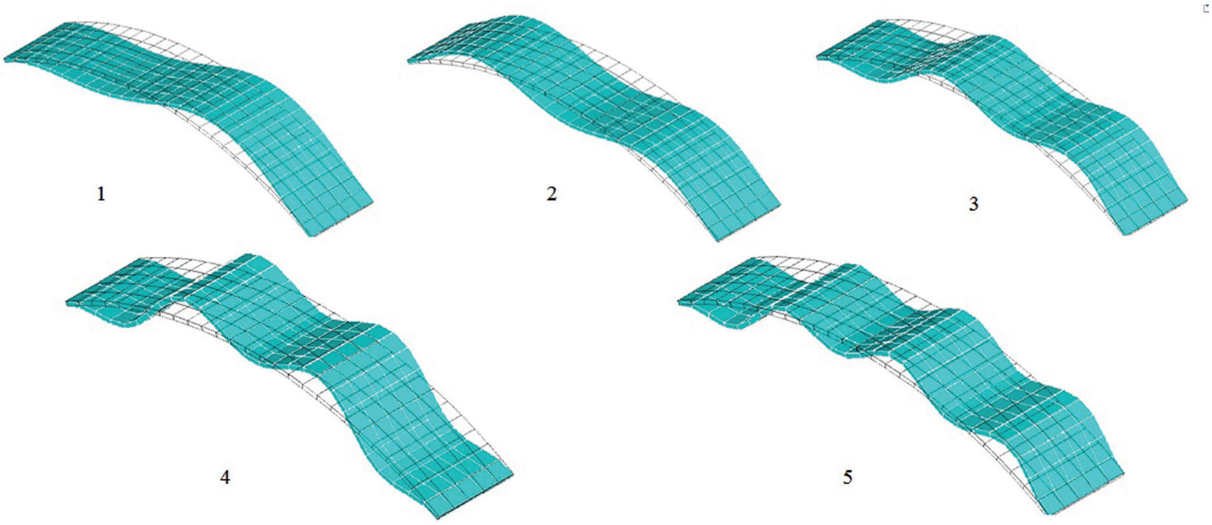

Fig. 5. The buckling mode shapes of arch-shells from the 1st to the 5th

point, k coordinates is defined. Each point in space is connected to the result in a pair wise comparison (Hasanzadehshooiili *et al.* 2012a). Datasets, x_i and x_j are in relation with strength mentioned in Eq (14).

$$r_{ij} = \frac{\sum_{m=1}^{k} x_{i_m} x_{j_m}}{\sqrt{\sum_{m=1}^{k} x_{i_m}^2 \sum_{m=1}^{k} x_{j_k}^2}}. \qquad (14)$$

According to the gained strength values derived from the application of CAM, Fig. 6, thickness is the most sensitive geometric parameter influencing the buckling load of steel arch-shells.

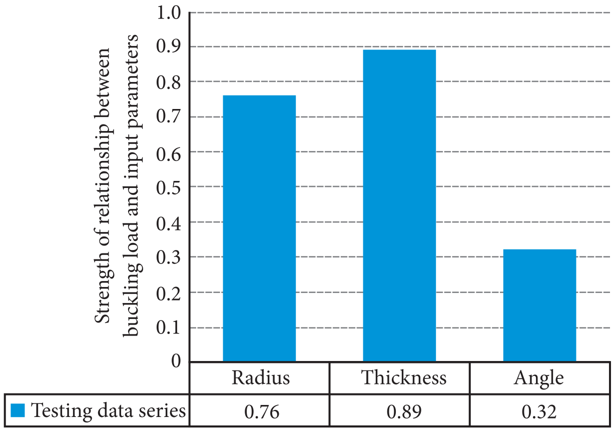

Fig. 6. The sensitivity analysis of the buckling load and each input parameter

4. Conclusion

1. As a matter of fact, arch-shell steel liners are broadly used for supporting underground road and mining tunnels. Furthermore, one of the instability mechanisms is that these structures are dealing with their buckling. Thus, knowing the most influential parameter of their buckling load will considerably decrease design expenses and will increase their safety. Regarding the existing complexities of theoretical solutions to buckling problems, numerical methods such as the FEM are widely applied for solving this issue. Hence, this paper numerically calculates the buckling load of steel arch shells used as underground road structure liners taking into account different geometrical parameters of the model. It should be noted that to better model structure buckling, in addition of ramping loads, the weight of the structure as a constant load has been considered using a simple proposed algorithm. Next, with reference to 84 prepared datasets, sensitivity analysis

is carried out to achieve the weightiest geometrical parameter on the buckling load of the liners.

2. The sensitivity of the buckling load to variation in input parameters is assessed using the CAM. Based on the obtained results, the influence of the internal angle of the shell on the buckling load is low. On the other hand, the influence of the radius of the periphery cylinder and structure thickness on the buckling load of arch-shells is almost the same. Furthermore, it should be noted that the thickness of arch-shells was gained as the geometrical parameter significantly affecting their buckling load. Moreover, neglecting the existing strength of the relationship between thickness and the buckling load will be conducive to overestimation or underestimation during the design phase.

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