DOMINANCE-BASED ROUGH SET APPROACH TO NETWORK BRIDGE MANAGEMENT

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Abstract. Bridge management is essential for optimizing available resources. Prioritising maintenance and assessing the effectiveness of different maintenance strategies are based on the conventional condition ratings of bridge components. This paper presents the development of a Decision Support System for maintenance management at the Network Bridge Level, based on the theory of the Dominance-Based Rough Set Approach. Dominance-Based Rough Set Approach produces a decision model expressed in terms of easily understandable "if…, then…" decision rules which control the decision process and avoid the "black box" effects of many alternative decision support methods. In the proposed Decision Support System, bridges are described by a set of 16 parameters that describe the state of degradation, structure, territory, traffic and network characteristics. To illustrate this Decision Support System, a case study involving Italian roads is presented. The proposed Decision Support System is a flexible tool because it allows updating parameters periodically as a consequence of practice, expertise and management policies.

Keywords: decision support systems, multiple criteria decision analysis, dominance-based rough sets, decision-making, bridge management, network bridge level.

1. Introduction

Bridges are subject to decline due to changes in both physical and mechanical properties of the materials used, but also due to traffic volume and speed, as well as to environmental aggression (e.g. exposure to chlorides, freezing, thawing cycles, etc.) and catastrophic events (floods, earthquakes, landslides, etc.).

In order to optimize the available budget, it is useful for bridge authorities to implement a management system for identifying the structures requiring maintenance and the substantial interventions at an early stage.

It is worth emphasizing the following two aspects:

– timely maintenance intervention leads to a longer bridge lifetime;

– maintenance costs tend to rise quickly after the deterioration process has started (Rens *et al.* 2005).

A Bridge Management System (BMS) is a decision-making process used to select and prioritize the tasks needed to keep the structure's functional parameters within acceptable limits given its lifetime cycle. The priority-setting process requires considerable attention to minimize costs and disruption of road traffic (Godart, Vassie 2001). An effective management system needs to carry out an individual analysis of the structure (Project Level Bridge Management) and to analyze the level of the network (Network Level Bridge Management). Therefore, decisions need to be considered at two levels: network and project. The one at network level defines when it is necessary to intervene while the one at project level defines the kind of intervention required. Integrating the Project Level with the Network Level is rather complex. This paper proposes a Decision Support System (DSS) for managing the Network Level based on the theory of the Dominance-Based Rough Set Approach (DRSA). This methodology produces a decision model expressed in terms of easily understandable "if….then…" decision rules which reflects the decision-maker's choices.

The parameters used in the decisional process take into account not only the state assessment of the bridge but also the characteristics of the environment, traffic and road conditions since the network is an integral part of the territory. The goal is not to define any maintenance activities for a bridge, but to determine the priorities on which to base a first approximation of the maintenance program for a bridge network in a way that the available resources are managed optimally.

2. Maintenance management of bridges

BMS includes all the activities which help maintain a bridge network efficiently, assuring safety and usability through the design, construction and operational phases (Godart *et al.* 2001). Several procedures have been developed to optimize bridge network maintenance where the funds available are limited. Some are based on inspecting and evaluating bridge condition, whereas more up-to-date procedures use statistical and mathematical tools to completely design maintenance plans at the network and project level (Valenzuela *et al.* 2010).

A complete management system includes:
– database;
– bridge evaluation;
– deterioration prediction and future conditions;
– maintenance alternatives evaluation and their cost;
– optimized maintenance plans.

The database is the sum of information on the bridge network (data, maintenance activities, state of damage, etc). The database is periodically updated as a result of inspections.

The bridge evaluation identifies deterioration processes and the causes of such processes by means of inspections (visual, instrumental, etc). Subsequently, bridges are classified into several categories. These are classified by severity which determines the urgency of the intervention (Bevc *et al.* 2001).

The condition data are then used to assess load capacity in terms of structure longevity and towards maximizing the safety and stability over that period (Bevc *et al.* 1999).

Future conditions are predicted by means of algorithms based either on standard statistical methods or on artificial intelligence techniques (neural networks, genetic algorithms).

Once these bridges are classified, by maintenance alternatives the optimization procedures are formulated.

Optimization means the best maintenance at the minimum cost, while maintaining adequate service levels. Optimization is performed over the period of maintenance interventions; this can vary from a few years to the entire life-time of the bridge. Several optimization procedures are available: classical mathematical formulations (scalar, unconstrained or constrained minimization, linear or quadratic programming, etc.) and artificial intelligence techniques (neural networks, genetic algorithms) (Bevc *et al.* 2001).

BMS is a difficult process with different mathematically complex stages. There are many studies on this subject in the literature. For example, Hai (2008) proposes a computer database for maintenance and management for highway bridges in Vietnam. It includes several assessment techniques such as lifespan estimation, deterioration prediction, life cycle cost analysis, cost-benefit analysis and priority maintenance index (PMI). Miyamoto *et al.* (2001) have proposed a concrete bridge management system based on visual inspection and Concrete Bridge Rating Expert System with machine learning for evaluated bridge performance, and on genetic algorithms for researching optimized maintenance.

Other authors have devoted their attention to the assessment of bridges; for example Valenzuela *et al.* (2010) proposed an integrated bridge index (IBI) which depends on four factors: the 'BCI Index' which reflects bridge damage levels, the "SI Index" which reflects the importance of the bridge in the road network, the "HV Index" which reflects hydraulic vulnerability and the "SR Index" which reflects seismic risk. The index was calibrated using visual inspection, expert surveys, and regression analysis.

Other authors have devoted their attention to predicting deterioration ratio and optional maintenance plans. Frangopol *et al.* (2001) showed that bridge management system based on Markovian deterioration modeling has several limitations that overcome using a reliability-based approach. Neves *et al.* (2004) proposed a model for predicting the performance of deteriorating structures by measuring it in terms of condition, safety and maintenance cost. This model considers the interaction between condition and safety by correlating the random variables of the two associated profiles and their relationship. Liu, Frangopol (2004) proposed a multi-objective genetic algorithm for optimal life-cycle maintenance planning of deteriorating bridges where condition index, safety index, and cumulative life-cycle maintenance costs were simultaneously considered in the optimization process. Liu, Frangopol (2005) proposed a multi-objective genetic algorithm in which structure condition, safety, and cumulative lifecycle maintenance costs have been considered as separate objective functions subject to simultaneous optimization.

This study presents an automated procedure with a large pool of alternative maintenance solutions establishing optimized tradeoffs between the competing meritorious objectives. Neves *et al.* (2006) proposed a probabilistic multi-objective approach to bridge maintenance using genetic algorithms which considers single maintenance types. In this study, the condition index (by visual inspections) and the safety index (by structural analysis) are used as indicators of the deteriorating performance of structures. The decision maker choices are the best possible compromise between available funds, safety and condition parameters and acceptable levels of deterioration, depending on the specific situation, the bridge manager preferences and the on-going maintenance policy. Liu, Frangopol (2006) proposed a comprehensive mathematical model for probability-based bridge network performance evaluation using network theories. Elbehairy *et al.* (2009) proposed multiple-element bridge management that optimizes repair decisions. In this study, the proposed system uniquely segments the problem into smaller sequential optimizations which are solved using the genetic algorithms technique. Orcesi, Cremona (2011) proposed optimized maintenance strategies for managing bridges across France based on Markov chains fitted to condition

data. This study evaluated prediction models for cost analysis and different maintenance strategies.

There is some specific software for managing bridges. Pontis is the most popular, developed by AASHTO in collaboration with the Federal Highway Administration (FHWA). Pontis is currently used in more than 40 agencies in the US and is widely adopted in other countries. Pontis allows both network- and project-level planning where bridges are represented as an assemblage of structural elements each is being classified by visual inspections every two years, in condition-state classes (Estes, Frangopol 2003). Pontis provides optimal maintenance policies for each state and for each type of element and environmental condition. Pontis generates simulated scenarios to determine current and future requirements, predict future performance levels and provide recommendations. The optimized policies at the network-level are selected by the software based on minimizing costs over the life-time of the bridge (Woodward *et al.* 2001).

3. Proposed methodology

One of the BMS phases is classifying bridges by assigning an intervention priority level. Accordingly, a first schedule of maintenance interventions is provided which is often based on linear equations that combine all the selected parameters, each having a weight assigned by expertise or literature data.

The aim of this paper is to define maintenance activity priorities by means of a decision-maker support system, taking into account the different roles involved in decision-making, each with its own objective. Thus, a methodology based on decisional rules obtained by the rough-set theory, the DRSA, has been applied. DRSA highlights both the methodological and operational point of view.

Using this methodology it is possible to derive a logical behavior model by observing actions through an inductive learning process (Greco *et al.* 2002a, 2002b).

The advantages of this methodology are:
– ability to manage vague or inaccurate data;
– ability to manage qualitative data;
– no need to assign a weight to each criterion;
– it is possible to highlight cause-effect relationships between the available data, separating the most relevant and strategic information from the inessential;
– construction of a priority model based on decisional rules such as "if … then …";
– identification of rules which support each decision;
– facility for the decision-makers to understand how the rules influence their decisions.

The proposed decision-making support system is a flexible tool. In fact, it is possible to evaluate and update it periodically as a consequence of practice, expertise and managing authority's different policies.

In the first part of the paper, the parameters required to describe the phenomenon are defined. In the second part, the form of on-field data gathering is defined. In the third part the proposed decision-making support system is presented. Finally, the proposed methodology is applied to a bridge network.

Bridge characteristics are defined by a set of attributes that describe the state of degradation, the structure, territory, and traffic and network characteristics. The attributes are divided into condition attributes, also called criteria (A_1 to A_{15}) and decision attributes (A_{16}). The value assigned to each criterion increases as conditions worsen. Table 1 describes the attributes and the values in more detail.

3.1. Visual inspection and data acquisition

The data required for implementing the DSS are:
– project;
– thematic maps (hydro-geological risk maps, seismic vulnerability maps, etc);
– inspections:
a) to identify and classify the various types of structures;
b) to identify the damage and its causes;
c) to prevent collapse.

Each damage depends on several factors: material deterioration, increase in traffic volume, increase in traffic load, lifespan reduction, natural disasters, etc.

There are many kinds of inspections: superficial, general, principal and special. Each country adopts different procedures for such inspections. Usually, general inspections are carried out every 2 or 3 years.

It is possible to do visual inspections or inspections using instruments. Visual inspections are performed in the field by compiling a form that can help identify and classify the damage and the damaged components. Photographs, sketches and notes are useful. The literature reports more evolved data gathering, recording and presentation, such as 3D images and virtual reality.

To investigate the causes and magnitude of the damage in detail, instrument inspections are necessary after visual inspection.

In this study, visual inspections were carried out using 1st level sheets for the damage state survey. These sheets are classified by bridge type (masonry arch bridges, reinforced concrete girder bridges, reinforced concrete arch bridges, steel girder bridges, pre-stressed r. c. girder bridges). This information facilitates the operator compile the sheets to obtain an objective description.

Each form contains 6 sections:
– section 1 – identifying the bridge, location, road type etc.;
– section 2 – geomorphological data, foundation soil;
– section 3 – components: slabs, arches, piers, abutments etc.;
– section 4 – simplified representation of the bridge, accessibility, images;
– section 5 – survey of structural component damage;
– section 6 – survey of non-structural component damage.

The damage to each bridge component has been identified by means of these sheets corroborated by photographs.

Table 1. Attributes description

Attribute	Characteristic	Refer	Value			Remarks
A_1	Bridge age	Approval date of the design project	1	low	The bridge was designed in the 90s	The more recent the project, the more probable anti-seismic design rules have been applied
			2	moderate	The bridge was designed 70s–90s	
			3	high	The bridge was designed earlier in the 70s	
A_2	Material	Used in constructing the bridge	1	low	Bridge is made of masonry	The better the materials' mechanical characteristics, the lower the vulnerability of the bridge
			2	moderate	Bridge is made of steel	
			3	high	Bridge is made of reinforced concrete	
			4	very high	For pre-stressed concrete	
A_3	Environmental conditions	Structural exposure	1	low	Non-aggressive environment	The higher the value, the worse the bridge conditions
			2	moderate	Moderately aggressive environment	
			3	high	Very aggressive environment	
A_4	Foundation soil	Physical and mechanical characteristics of soil	1	low	Rocky soil	The characteristics of the soil influence bridge stability and longevity
			2	moderate	Granular (uncohesive) soil	
			3	high	Limey or clayey (cohesive) soil	
A_5	Damage type	Structure degradation	1	low	No damage or superficial damage	The higher the value, the worse the bridge conditions
			2	moderate	Cracks, corrosion or imperfect bearings	
			3	high	Large deformations, ruptures, displacements, instability, pre-stressed cable damage, minimum hydraulic clearance is not met.	
A_6	Damaged surface	Damage extent	1	low	Little or no surface damage (i.e. < 10% of the total component surface)	The higher the value, the worse the bridge conditions
			2	moderate	Damaged surface is between 10% and 60% of the total component surface	
			3	high	Damage dsurface is more than 60% of the total component surface	
A_7	Damaged components	Importance of the damaged component in the overall structure	1	low	No damaged components	The higher the value, the worse the bridge conditions
			2	moderate	Damaged components are non-structural or secondary structural	
			3	high	Damaged components are main structural components (piers, spandrels, spans)	
A_8	Seismic zone	Peak ground acceleration (PGA) which a measure of seismic risk at the site	1	low	PGA less than 0.15 g	The higher the value, the greater the seismic risk
			2	moderate	PGA between 0.15 g and 0.25 g	
			3	high	PGA greater than 0.25 g	
A_9	Anti-seismic devices	Dampers or isolators which dissipate earthquake energy	1	yes	Anti-seismic devices are present	The presence of anti-seismic device increases the bridge characteristics
			2	no	Anti-seismic devices are not present	

Continued Table 1

Attribute	Characteristic	Refer			Value	Remarks
A_{10}	Static scheme	Structural components configuration	1	low	Arch bridge	The higher the value the worse the ability of the structure to respond to an earthquake
			2	moderate	Continuous beam	
			3	high	Series of simply-supported beams	
A_{11}	Hydrological instability	Morphological processes caused by soil and soil and sub-soil water flow which produce superficial degradation or even a landslide	1	low	No hydrological instability	The higher the value the greater the risk
			2	moderate	Moderate hydrological risk	
			3	high	High hydrological risk	
A_{12}	Average daily traffic (*ADT*)	Level of daily traffic over the bridge	1	low	*ADT* < 6000 vpd	The higher the value the greater the stress suffered by the structure
			2	moderate	*ADT* < 6000 < 20 000 vpd	
			3	high	*ADT* > 20 000 vpd	
A_{13}	Heavy traffic rate	Percentage of heavy vehicles with respect to total traffic over the bridge	1	low	Less than 10%	The higher the value the greater the stress suffered by the structure
			2	moderate	Between 10% and 20%	
			3	high	More than 20%	
A_{14}	Strategic viability	The bridge is on a main itinerary subject to emergency vehicle transit	1	yes	Bridge is strategic	
			2	no	Bridge is not strategic	
A_{15}	Alternative viability availability	Presence of an alternative itinerary to the bridge without an excessive increase in the travel time	1	yes	An alternative itinerary is present	
			2	no	An alternative itinerary is not present	
A_{15}	Decision attribute	Level of maintenance urgency	1	low	No intervention is required. It is possible to have next inspection within the scheduled time	
			2	moderate	No intervention is required. It is necessary to anticipate the next inspection (prior inspection)	
			3	high	Urgent intervention is required	
			4	very high	It is necessary to close the bridge or to reduce the traffic on he bridge	

4. Dominance-based rough set approach to prioritize maintenance

To prioritise bridge maintenance for a road system, a multi-criterion model based on the DRSA theory (Greco *et al.* 1999, 2001, 2002a, 2005; Slowinski *et al.* 2005) has been adopted. This is an upgrade of the Classical Rough Set Approach (CRSA) developed by Pawlak (1991) which is applicable to multi-criterion issues. The DRSA does not only allow the representation and analysis of decision-making but, more generally, of all the phenomena involving monotonicity. DRSA theory grew out of research in the field of multi-criterion decision-making within AI techniques.

4.1. Information table and dominance relation

The rough set philosophy assumes that every object in the universe is described by a set of attributes. This requires inputting a set of examples representing preferential information by decision makers, while the analysis output is the model of preferences in terms of decision rules.

For algorithmic reasons, object information is supplied by a "data table" whose rows refer to distinct objects and whose columns refer to different attributes. Each table cell indicates an evaluation (quantitative or qualitative) of the object located in that row by an attribute in the corresponding column.

In this case study the decision support system was collated from a set of 100 bridges whose features represent most of the bridges found on Italian roads.

The row objects are bridges and the columns are the criteria which characterize the bridges, as shown in Table 2.

Formally, a data table in the 4-tuple $S = (U, Q, V, f)$ where U is a finite set of objects (universe), $Q = \{q_1, q_2, ... , q_n\}$ is

Table 2. Data table

Bridge	Criteria															Decision attribute
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	
1	3	1	1	1	1	1	1	1	2	1	3	1	3	1	1	2
2	3	1	2	2	1	1	1	2	2	1	2	2	3	2	1	1
3	3	1	3	1	1	1	1	1	2	1	1	1	2	1	2	1
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
98	2	4	1	1	3	2	3	3	2	3	1	2	2	2	2	4
99	2	4	2	3	3	3	3	2	2	2	2	1	2	1	2	4
100	1	4	2	1	3	1	3	3	1	2	2	2	2	2	1	3

a finite set of attributes, V_q is the domain of attribute q, $V = U_{q \in Q} V_q$ and $f{:}UxQ \rightarrow V$ in a total function such as $f(x, q) \in V_q$ for each $q \in Q$, $x \in U$, called "information function". The set Q is, in general, divided into set C of condition attributes and a decision attribute d. In multi-criteria classification condition attributes are "criteria". The notion of criterion involves a preference order in its domain while the domains of attributes are not preference-ordered.

In this case, all the condition attributes are criteria because it is possible to order them according to increasing preference of maintenance activity.

Furthermore, decision attribute d makes a partition of U into a finite number of classes $Cl = \{Cl_t, t \in T\}$, $T = \{1, ..., n\}$. Each $x \in U$ belongs to one and only one class $Cl_t \in Cl$. The classes from Cl are preference-ordered according to the increasing order of class indices, i.e. for all $r, s \in T$, such as $r > s$, the objects from Cl_r are preferred to the objects from Cl_s.

In the presented case, the set of decision D attributes is a singleton given by the attribute "degree of urgency of the maintenance activity" which divides the set U of 100 bridges into four classes:

– Cl_1: bridges that keep to the inspection schedule;
– Cl_2: bridges requiring prior inspection;
– Cl_3: bridges requiring urgent intervention;
– Cl_4: bridges requiring partial or total closure.

4.2. Dominance-based approximation

In multi-criteria classification, due to the preference order in Cl classes the sets requiring approximation are not particular classes but upward unions (1) and downward unions (2) of classes, respectively:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, \qquad (1)$$

$$Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s, \qquad (2)$$

Union $Cl_t^{\geq}$ is the set of objects belonging to class Cl_t or to a more preferred class, while $Cl_t^{\leq}$ is the set of objects belonging to class Cl_t or to a less preferred class.

Note, that for $t = 2, ..., n$ $Cl_t^{\geq} = U - Cl_{t-1}^{\leq}$, i.e. all the objects not belonging to class Cl_t or better, belong to class $Cl_t - 1$ or worse.

It the case study the upward union classes are:
– the union upward $Cl_1^{\geq}$ is formed by bridges with necessity of inspection visual "at least" not advance;
– the union upward $Cl_2^{\geq}$ is formed by bridges with necessity of inspection visual "at least" advance;
– the union upward $Cl_3^{\geq}$ is formed by bridges with necessity "at least" of programming for urgent action;
– the union upward $Cl_4^{\geq}$ is formed by bridges that need "at least" immediate closure or the traffic reduced.

The downward union classes are:
– the union upward $Cl_1^{\leq}$ is formed by bridges with necessity of inspection visual "at most" not advance;
– the union upward $Cl_2^{\leq}$ is formed by bridges with necessity of inspection visual "at most" advance;
– the union upward $Cl_3^{\leq}$ is formed by bridges with necessity "at most" of programming for urgent action.
– the union upward $Cl_4^{\leq}$ is formed by bridges that need "at most" immediate closure or the traffic reduced.

In this application, the upward union classes $Cl_1^{\geq}$ and the downward union classes $Cl_4^{\leq}$ contain all 100 bridges considered: in fact for all these bridges degree of maintenance urgency is always at least scheduled and at most immediate partial or total closure. Usually, classification issues concern data collection for a given class Cl_t dividing the universe U into class Cl_t (set of positive examples) and its complement $U - Cl_t$ (set of negative examples), $t = 1, ..., n$. However, such bipartitions do not take into account the preference order among classes. Thus, multi-criteria classification requires another type of bipartition which divides the universe into upward and downward unions of classes and $Cl_t^{\geq}$ and $Cl_{t-1}^{\leq}$, $t = 1, ..., n$. Each object from the upward union $Cl_t^{\geq}$ is preferred to each object from the downward union $Cl_{t-1}^{\leq}$. Data collection for upward union $Cl_t^{\geq}$ considers all objects positive and all objects belonging to $Cl_{t-1}^{\leq}$ as negative. Analogously, data collection for downward union $Cl_{t-1}^{\leq}$ consider all objects belonging to $Cl_{t-1}^{\leq}$ as positive and all objects belonging to $Cl_t^{\geq}$ as negative.

In this approach to data collection, the dominance principle is applied as follows.

Let $\geq_q$ be a weak preference relation of U (often called outranking) representing a preference applied to the set of objects associated with criterion q; $x \geq_q y$ means 'x is at

least as good as y with respect to criterion q'. If $x \geq_q y$ for all $q \in P$, then *x dominates y* with respect to $P \subseteq C$ (for short *x P-dominates y*) denoted by $xD_P y$. Assuming, without loss of generality, that domains of all criteria are ordered such that preference increases with the value, $xD_P y$ is equivalent to: $f(x, q) \geq f(y, q)$ for all $q \in P$. Observe that for each $x \in U$, $xD_P x$, that is, *P*-dominance is reflexive.

Given $P \subseteq C$ and $x \in U$, the "granules of knowledge" used in DRSA for approximation of the unions $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are:

– a set of objects dominating *x*, called *P*-dominating set (3):

$$D_P^+(x) = \left\{ y \in U : yD_P x \right\}, \quad (3)$$

– a set of objects dominated by *x*, called *P*-dominated set (4):

$$D_P^-(x) = \left\{ y \in U : xD_P y \right\}. \quad (4)$$

In the case study, for example $D_P^+(x)$ is composed of all bridges with a degree of inspection urgency "at least" equal to *x*, while $D_P^-(x)$ is composed of all bridges that have a degree of inspection urgency "more than" equal to *x*. For example, if the criteria were "type of damage" and "seismic zone", both evaluated on three scales of high, moderate and low, and bridge *x* is evaluated as moderate regarding "type of damage" as well as "seismic zone", then:

$D_P^+(x)$ is composed of all moderate or low bridges regarding type of damage and seismic zone, and $D_P^-(x)$ is composed of all moderate or high bridges regarding type of damage and seismic zone.

Given the set of criteria $P \subseteq C$, the inclusion of object $x \in U$ in the upward union of classes $Cl_t^{\geq}$, $t = 2, …, n$, creates an inconsistency in the dominance principle if one of the following conditions holds:

– *x* belongs to class Cl_t or better but it is *P*-dominated by object y belonging to a class worse than Cl_t,

– *x* belongs to a worse class than Cl_t but it *P*-dominates object y belonging to class Cl_t or better.

If, given the set of criteria $P \subseteq C$, the inclusion of $x \in U$ in $Cl_t^{\geq}$, $t = 2, …, n$, creates an inconsistency in the dominance principle, we say that *x* belongs *to $Cl_t^{\geq}$, with some ambiguity*. Thus, *x* belongs to *Clt³ without any ambiguity* with respect to $P \subseteq C$, if $x \in Cl_t^{\geq}$, and there is no inconsistency in the dominance principle. This means that all objects *P*-dominating x belong to $Cl_t^{\geq}$.

It is possible that $y \in U$ belongs to class $Cl_t^{\geq}$, with eventually some ambiguity, if one object $x \in Cl_t^{\geq}$ exists such as *y* dominates *x* with respect to the set $P \subseteq C$, or $y \in D_P^+(x)$. For example, if "*bridge y dominates bridge x*", with the latter belonging to the ascending union of classes $Cl_3^{\geq}$ of bridges with not less than urgent intervention, it is possible that *y* belongs to the ascending union of classes $Cl_3^{\geq}$ too, if with some ambiguity. In simpler words, if bridge *y* is no worse than bridge *x* for all criteria (i.e. *y* dominates *x*) then the

maintenance urgency of y should be no less than that of *x*. Some ambiguity is possible if *y* or some other bridge that dominates *x* has lower maintenance urgency for specific reasons not taken into account (for example criteria not considered in the general case).

Saying that $y \in U$ belongs to $Cl_t^{\geq}$ does not necessarily mean that it actually belongs to this class. In the previous example, it is possible that y belongs to $Cl_3^{\geq}$ but, if its maintenance urgency is 2 (prior inspections), *y* belongs to class $Cl_2^{\geq}$. This is due to the ambiguity between *x* and *y* with respect to criteria set *P*.

For $P \subseteq C$, the set of all objects belonging to $Cl_t^{\geq}$ without any ambiguity constitutes the *P-lower approximation* (5) of $Cl_t^{\geq}$, denoted by $\underline{P}(Cl_t^{\geq})$, and the set of all objects that possibly belong to $Cl_t^{\geq}$ constitutes the *P-upper approximation* (6) of $Cl_t^{\geq}$, denoted by $\overline{P}(Cl_t^{\geq})$:

$$\underline{P}(Cl_t^{\geq}) = \left\{ x \in U : D_P^+(x) \subseteq Cl_t^{\geq} \right\}, \text{ for } t = 1, …, n, \quad (5)$$

$$\overline{P}(Cl_t^{\geq}) = \left\{ x \in U : D_P^-(x) \cap Cl_t^{\geq} \neq \varnothing \right\}, \text{ for } t = 1, …, n. \quad (6)$$

Analogously, it is possible to define *P-lower approximation* (7) and *P-upper approximation* (8) of $Cl_t^{\geq}$ as follows:

$$\underline{P}(Cl_t^{\leq}) = \left\{ x \in U : D_P^-(x) \subseteq Cl_t^{\leq} \right\}, \text{ for } t = 1, …, n, \quad (7)$$

$$\overline{P}(Cl_t^{\leq}) = \left\{ x \in U : D_P^+(x) \cap Cl_t^{\leq} \neq \varnothing \right\}, \text{ for } t = 1, …, n. \quad (8)$$

All the objects belonging to $Cl_t^{\geq}$ and $Cl_t^{\leq}$ with some ambiguity constitute the *P-boundary* (9, 10) of $Cl_t^{\geq}$ and $Cl_t^{\leq}$, denoted by $Bn_P(Cl_t^{\geq})$ and $Bn_P(Cl_t^{\geq})$, respectively. It is possible to represent them in terms of upper and lower approximations as follows:

$$Bn_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq}), \text{ for } t = 1, …, n, \quad (9)$$

$$Bn_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq}), \text{ for } t = 1, …, n. \quad (10)$$

From a data collection point of view, *P*-lower approximations of unions of classes represent *certain knowledge* provided by criteria from $P \subseteq C$, while *P*-upper approximations represent *possible knowledge* and the *P*-boundaries contain *doubtful knowledge* (Greco *et al.* 2002b).

4.3. Quality of sorting and reducts

For every $P \subseteq C$ and $t \in T$, the quality of approximation of partition *Cl* by set of attributes *P*, or in short, *quality of sorting* was defined (11):

$$\gamma_P(Cl) = \frac{card\left(U - \left(\bigcup_{t \in T} Bn_P\left(Cl_t^{\leq} \right) \right) \right)}{card(U)} =$$

$$\frac{card\left(U - \left(\bigcup_{t \in T} Bn_P\left(Cl_t^{\geq} \right) \right) \right)}{card(u)}. \quad (11)$$

This quality expresses the ratio of all *P*-correctly sorted objects in the information table. Note, that enlarging the set of criteria *P*, the quality of sorting not increase but decrease. In fact, any new criteria that were ambiguous before become non-ambiguous.

In the case study, for example, sorting quality is 0.98; meaning the information table contains "ambiguous objects". In Table 3 the bridge 25 has the same characteristics as bridge 46 but it is made of better material which requires higher maintenance than bridge 46. This means that the conditional criteria alone do not explain the decision maker's choices.

Each minimal subset $P \subseteq C$ such as $\gamma_P(Cl) = \gamma_C(Cl)$ is called a reduct of *Cl* and is denoted by RED_{Cl}. Again, it is possible that a data table has more than one reduct. The intersection of all the reducts is known as the core, denoted by $CORE_{Cl}$ (12):

$$Core_\gamma(P) = \bigcap \mathrm{Re}\, d_\gamma(P). \qquad (12)$$

Is not possible to remove from the data sample set criteria from $CORE_{Cl}$ without impairing the knowledge to be discovered. This means that in set *C* there are *indispensable* criteria included in the core, *exchangeable* criteria included in some reducts but not in the core, and *redundant* criteria being neither indispensable nor exchangeable, thus not included in any reduct.

In the case study there are 13 reducts. Table 4 shows the criteria included in the reducts and the core which shows that there are 5 indispensable criteria (material, type of damage, damaged surface, seismic zone, hydrogeological instability), 8 exchangeable criteria and 0 redundant criteria.

4.4. Extraction of decision rules

The dominance-based rough approximations of upward and downward unions of classes allow to induce a generalized description of objects contained in the data matrix in terms of "*if..., then...*" decision rules.

For a given upward or downward union of classes, $Cl_t^\geq$ or $Cl_S^\leq$, the decision rules induced under a hypothesis that objects belonging to $\underline{P}(Cl_t^\geq)$ or $\underline{P}(Cl_S^\leq)$ are *positive* and all the others *negative*, suggest an assignment to "class Cl_t or better", or to "class Cl_S or worse", respectively. On the other hand, the decision rules induced under a hypothesis that objects belonging to the intersection $\overline{P}(Cl_S^\leq) \cap \overline{P}(Cl_t^\geq)$

are *positive* and all the others *negative* are suggesting an assignment to some classes between Cl_S and Cl_t ($s < t$).

In multi-criteria classification, it is meaningful to consider the following three types of decision rules:

– certain $D \geq$-decision rules, providing lower profile descriptions for objects without ambiguity: if $f(x, q_1) \geq rq_1$ and $f(x, q_2) \geq rq_2$ and …$f(x, q_p) \geq rq_p$, then $x \in Cl_t^\geq$, for example:

if "Bridge Age" is ≤2, "rate of heavy traffic" is ≤1 and "strategic viability" is ≤1, then the recommended degree maintenance urgency is at least 2 (prior inspection), i.e. bridge $x \in Cl_2^\geq$;

– certain $D \leq$-decision rules providing upper profile descriptions for objects without ambiguity: if $f(x, q_1) \leq rq_1$ and $f(x, q_2) \leq rq_2$ and ... $f(x, q_p) \leq rq_p$, then $x \in Cl_t^\leq$, for example:

if "damaged surface" is ≥2, "hydrogeological instability" is ≥2 and "average daily traffic" is ≥2, then the recommended degree of maintenance urgency is at most 2 (prior inspection), i.e. bridge $x \in Cl_2^\leq$;

– approximate $D \geq\leq$-decision rules, providing simultaneously lower and upper profile descriptions for objects belonging to $Cl_s \cup Cl_s+1 \cup … \cup Cl_t$ without being able to discern class: if $f(x, q_1) \geq rq_1$ and $f(x, q_2) \geq rq_2$ and … $f(x, q_k) \geq rq_k$ and $f(x, q_{k+1}) \leq rq_{k+1}$ and ... $f(x, q_p) \leq rq_p$, then $x \in Cl_s \cup Cl_{s+1} \cup … \cup Cl_p$ for example:

if "material" is ≤3, "environmental condition" is ≥2, "seismic zone" is ≤2, "static scheme" is ≥3, "hydrogeological instability" is ≥2, then the recommended degree of maintenance urgency must be between 2 (prior inspection) and 1 (scheduled inspection) i.e. bridge $x \in Cl_1 \cup Cl_2$.

On the left side of $D \geq\leq$-decision rule it is possible to have $f(x, q) \geq rq$ and $f(x, q) \leq r'q$, where $rq \leq r'q$, for the same $q \in C$. Moreover, if $rq = r'q$ the two conditions boil down to '$f(x, q) = rq$'.

An object $x \in U$ *supports* decision rule *r* if its description matches both the condition and decision part of the rule. Decision rule *r covers* object *x* if it matches the condition part of the rule. Each decision rule is characterized by its *strength* defined as the number of objects supporting the rule. In the case of approximate rules, strength is calculated for each possible decision class separately. If a univocal rule is supported by objects from the lower approximation of the corresponding decision class only, then the rule is called *certain* or

Table 3. Ambiguous object

Bridge	Criteria															Decision attribute
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
25	3	2	2	2	1	1	1	1	2	3	2	2	3	2	1	2
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
46	3	3	2	2	1	1	1	2	2	3	2	2	3	2	1	1
….	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

Table 4. Reducts and core

Criteria	Reduct													Core
	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	
A_1	×	×		×	×	×	×	×	×		×		×	
A_2	×	×	×	×	×	×	×	×	×	×	×	×	×	×
A_3				×		×	×		×	×				
A_4					×			×			×	×	×	
A_5	×	×	×	×	×	×	×	×	×	×	×	×	×	×
A_6	×	×	×	×	×	×	×	×	×	×	×	×	×	×
A_7													×	
A_8	×	×	×	×	×	×	×	×	×	×	×	×	×	×
A_9										×		×		
A_{10}				×	×	×							×	
A_{11}	×	×	×	×	×	×	×	×	×	×	×	×	×	×
A_{12}	×		×	×	×				×	×	×	×		
A_{13}			×				×	×	×	×	×	×		
A_{14}		×				×	×	×					×	
A_{15}	×	×	×											

Table 5. Decision rules

Rules	If…	Then…	Support	Cases supported
1	(Environmental conditions ≥3) & (Damage type ≥3)	Urgency at most 4	3	22, 44, 67
…				
538	(Material ≥4) &(Foundation soil ≥3)	Urgency at most 2	4	82, 88, 95, 99
…				
713	(Bridge age ≤1) & (Material ≤2) & (Environmental conditions ≤1)	Urgency at least 1	2	17, 40
…				
900	(Bridge age ≤1) & (Material ≤3) & (Environmental conditions ≤1)	Urgency at least 2	4	17, 40, 61, 70
…				

deterministic. If, however, a univocal rule is supported by objects from the upper approximation of the corresponding decision class only, then the rule is called *possible* or *probabilistic*. Approximate rules are supported, in turn, only by objects from the boundaries of the corresponding decision classes. Generating decision rules from decision tables is a complex task and a number of procedures have been proposed to simplify it. Existing induction algorithms use one of the following strategies (Stefanowski 1998):

– generate a *minimal* set of rules covering all objects from a decision table,

– generate an *exhaustive* set of rules consisting of all possible rules for a decision table,

– generate a set of "strong" decision rules, called a *satisfactory* set of rules, each of which apply to many objects but not necessarily to all the objects in the decision table.

For the induction of decision rules free software is also available called **4eMka2** [http://idss.cs.put.poznan.pl/site/4emka.html]. This software solves multi-criteria sorting problems using rough set theory and decision rule induction and is freely available on the internet.

In this case, 1183 "strong" decisional rules were generated, as follows:

– 187 recommend a maintenance urgency degree ≤1 (scheduled inspection);

– 188 recommend a maintenance urgency degree ≤2 (prior inspection);

– 96 recommend a maintenance urgency degree ≤3 (urgent intervention);

– 214 recommend a maintenance urgency degree ≥4 (reduced traffic or bridge closure);

– 323 recommend a maintenance urgency degree ≥3 (scheduled urgent intervention);

– 175 recommend a maintenance urgency degree ≥2 (prior inspection).

For each rule, the number and identity of data table objects supporting that rule are known. They are given in Table 5.

5. Applying SSDs to the bridges of a secondary suburban road

This study focused on applying current methodology to the bridges of the Italian national road, owned by the National Road Agency (ANAS), to estimate maintenance urgency. There are 14 bridges whose main characteristics are reported in Table 6.

5.1. Data acquisition

For each bridge, the data on construction year, static scheme, average daily traffic and heavy traffic ratio were provided by the ANAS database. The foundation soil characteristics and hydrogeological instability data were obtained from thematic maps. The PGA values were obtained by the Italian Technical Code. A GIS was used to locate the bridges on the net and to process alternative routes.

　　Damage typology, damaged surface, damaged components and the presence or not of anti-seismic devices were assessed by visual inspections (section 5.2).

5.2. Bridge assessment by visual inspection

In this study, visual inspections were carried out using the First Level Sheets for the Degradation Survey described in section 3.2. For example, for bridge 1 the following phenomena were detected: corrosion of reinforcement and spalling both in the deck and in beam-abutment connection, joint deterioration and intrusive vegetation. Therefore the "type of damage" criterion corresponds to 2, the "damaged surface" criterion is 1 and the "damaged components" criterion is 3.

5.3. Determination of the urgency degree of maintenance activities

Once the condition criteria values were known, the following Table 7 was compiled:

Table 6. Main bridge characteristics

Bridge	Length, m	Material
1	70.00	Reinforced concrete
2	106.00	Masonry + Reinforced concrete
3	12.00	Masonry +Reinforced concrete
4	23.00	Masonry +Reinforced concrete
5	10.00	Reinforced concrete
6	14.00	Reinforced concrete
7	14.00	Reinforced concrete
8	14.00	Reinforced concrete
9	26.00	Reinforced concrete
10	14.00	Reinforced concrete
11	26.00	Reinforced concrete
12	29.00	Reinforced concrete
13	35.00	Reinforced concrete
14	12.00	Masonry

　　The decision support system above was used. At this stage of the research, the proposed DSS makes use of simple software to receive the DRSA output. Table data are inputted and the recommended urgency degree for inspections and decisional rules behind them are outputted. For example, let us evaluate the maintenance urgency of bridge No. 1 which has the characteristics described in Table 8.

　　From the rules above, the DSS suggests a maintenance urgency of 2 (prior inspection) returning 4 rules which recommend a degree ≥2 (prior inspection), 37 rules for a degree ≤2 (prior inspection) and 46 rules for a degree ≤3 (urgent intervention). The inspection urgency degree returned by the DSS is that which satisfies all the decisional

Table 7. Data table

Bridge	Criteria														
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
1	2	3	1	1	2	1	3	3	2	3	1	1	2	1	1
2	2	3	1	1	2	1	3	3	2	3	1	1	2	1	1
3	3	3	1	1	2	3	3	2	2	1	1	1	2	1	1
4	3	3	1	1	2	3	3	2	2	1	1	1	2	1	1
5	3	1	1	1	3	1	1	2	2	1	3	1	2	1	1
6	2	3	1	1	1	1	1	2	2	3	1	1	2	1	1
7	2	3	1	1	1	1	1	2	2	3	1	1	2	1	1
8	2	3	1	1	1	1	1	2	2	3	1	1	2	1	1
9	2	3	1	3	2	1	2	2	2	3	1	1	2	1	1
10	2	3	1	2	2	1	2	2	2	3	1	1	2	1	1
11	2	3	1	2	2	1	2	2	2	3	3	1	2	1	1
12	2	3	1	1	2	2	2	2	2	3	3	1	2	1	1
13	2	3	1	3	2	1	3	3	2	3	1	1	2	1	1

Table 8. Characteristics of bridge No. 1

Criteria	Bridge characteristics
Bridge age	designed between the 70s and 90s
Material	reinforced concrete
Environmental conditions	non-aggressive environment
Foundation soil	rocks
Damage type	corrosion
Damaged surface	<10% total component surface
Damaged components	structural component (deck)
Seismic zone	0.252 g
Anti-seismic devices	no
Static scheme	series of simply-supported beams
Hydrogeological instability	no risk
Average Daily Traffic	<60 00 vpd
Heavy traffic volume	between 10% and 20%
Strategic viability	no
Alternative viability	yes

rules, being 2 in this case. If it is impossible to satisfy all the rules returned by the DRSA, the rules supported by a larger number of "objects" in the decision table are considered, until the rule set allows for a unique urgency value which satisfies all the decisional rules.

The decisional rules allow the decision maker to understand the DRSA's recommended urgency.

Obviously, it is unreasonable to give the decision maker a large number of rules (88 for bridge 1), thus, for each class only the most supported rules have been reported. For bridge 1 these are:

– if "type of damage" ≤2 then urgency at least 3 (support 80);

– if "type of damage" ≤2 and "damaged surface" ≤1 then urgency at least 2 (support 32);

– if "seismic zone" ≥3 then urgency at most 2 (support 34);

– if "age of the bridge" ≥2 and "damaged elements" ≥3 then urgency at most 2 (support 34).

From these rules, it is clear that the decision criteria are those related to structure damage (type of damage, damaged surface, damaged components), seismic zone (PGA value) and bridge age.

It is possible that the decision maker take into account the suggestion given by the DSS or that he prefers to carry out the maintenance to improve the bridge and reduce urgency.

Proceeding on the same for all other bridge is possible to classify the bridges of the network depending on the degree of urgency of maintenance activities.

6. Conclusion

1. In this paper a Decision Support System for bridges maintenance management at the network level, based on Dominance Rough Set Approach, is proposed. It allows to set the order of bridges according to their maintenance urgency, on the basis of parameters related to bridge damage, characteristics of the territory, traffic and the network.

2. Using the Dominance Rough Set Approach, a decision model expressed in terms of easily understandable "if…then…" rules has been generated. The decision rules allow to control the decision process and to avoid the "black box" effects of many alternative methods. The starting point of the methodology is represented by the "exemplary decisions" with which the decision maker expresses his preferences.

3. This methodology is like a "glass box", since it is possible to map out from where each rule is derived. The model generated is flexible and could be updated by varying the exemplary decisions set required to calibrate the model.

4. A sample application of the proposed model is also reported. Putting as input the characteristics of some bridges of a rural Italian road, the built Decision Support System gives back the intervention urgency for each bridge and only the more important decision rules that can help decision maker to understand the reasons of the suggestions.

5. This is the first application of the Dominance Rough Set Approach to this type of issue and other new developments will be presented in future.

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