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TRANSITION CURVES IN VERTICAL ALIGNMENT AS A METHOD FOR REDUCING FUEL CONSUMPTION

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Abstract. This paper presents a new approach to the fuel consumption reduction. This approach is based on vertical curve designs based on transition curves. The work presents selected curves, which are used for this purpose. Moreover, the results of comparative studies are shown for these curves, carried out in order to compare the possible reduction of fuel consumption in vertical curves designed using both conventional and proposed methods. Numerical studies were based on German engineering guidelines. In the case of vertical arcs formed by transition curves, fuel consumption indexes prove to be significantly lower than in curves based on a grade line consisting of straight lines with parabolic arcs. The obtained results indicate the advisability of forming vertical curves using transition curves.

Keywords: vertical alignment, vertical curves, fuel consumption reduction.

1. Introduction

A very important problem is the reduction of fuel consumption and exhaust emission. This is the object of consideration in many works, including the following papers (Ahn *et al.* 2002; Bastani *et al.* 2012; Jakimavičius, Burinskienė 2014; Mickūnaitis *et al.* 2007; Oprešnik *et al.* 2011; Várhelyi 2002). The present work is also linked to the problems of fuel consumption reduction. This paper proposes an approach that relies on a suitable forming of vertical curves geometry.

The vertical alignment is composed of a series of straight-line gradients connected by curves, normally quadratic parabola in form. However, in certain circumstances this procedure proves to be not the most appropriate. This is because when using conventional geometrical elements including straight line and parabola it is harder to obtain optimal gradient adjustment to the vertical layout of the land (Fulczik 1975). As a result of this, there are suggestions and suitable solutions for grade line development using polynomial spline functions, making it easier to adjust its run to predetermined fixed points and it is particularly useful during the reconstruction of old routes (Fulczik 1975, 1977a, 1977b).

According to the author of this paper, a change in the principles used in vertical alignment is also justified in the case of conventional design methods. This is due to economic criteria connected with fuel consumption reduction.

The gradient is one of the factors affecting the increase in fuel consumption. Fuel consumption reduction is possible by means of reducing gradients, which means either the necessity to extend the distance between specified points, or (for a fixed route run in a general layout) departure from the recommended grade line adjustment to the vertical layout of the land. In the latter case, a possible direct consequence of this procedure would be the increased scale of necessary earthworks, the need to build tunnels or flyovers, which would increase the route construction costs. However, the issues related to environmental protection are equally important. It is possible that in the long run a suitable economic calculation indicates the predominance of advantages resulting from reduced fuel consumption, exhaust gas emission, etc.

A lower average a lower average ascending route gradient is possible by using a grade line formed by two transition curves (an equivalent of bi-clothoid) instead of two straight lines with circular arc. Therefore, in the case of vertical arcs formed by transition curves one is expect the corresponding reduction of fuel consumed by vehicles in ascending routes. A review of the world and Polish scientific achievements in this area has brought to light the fact that this issue has not been discussed in literature, which has inclined the author to address the matter. This will be the subject of discussion in this work.

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2. The concept of vertical alignment from the point of view of a fuel consumption reduction

As previously mentioned, fuel consumption for moving vehicles depends, among other things, on the road gradients. Therefore, fuel consumption reduction is achievable by means of changing the approach to the vertical alignment. Generally speaking, this would involve reducing the mean values of gradients in ascending routes – that is in sections where fuel consumption rises significantly. Excluding the possibility of applying lower inclination values along straight sections in the grade line design process, a changed approach is analysed to the rounding of changes in grade line designed in the form of a series of straight-line gradients.

It is proposed that the rounding of crest or sag (Fig. 1) changes of gradient be carried out using vertex transition curves (variant B) instead of the conventional solution employing 2nd degree parabolic curves (variant A). For the same value of the minimum curvature radius R, the grade line section defined by points PP'S'K'K in the conventional approach (variant A) would be replaced with the PSK section in the proposed approach (variant B).

Compared to the circular arc with the specified value of curvature radius R or the square parabola arc with the minimum curvature radius R, transition curves are characterised by the following – for the identical change of gradient between the beginning and the end of the arc, the transition to the same value of the curvature radius R takes place along the greater curve length. As a result of this, the average value of the gradient of the tangent to the curve also decreases (the gradient is calculated relative to the tangent in arc starting point).

If one compares the above grade line sections in both variants, the mean grade line inclination for the ascending or descending route in variant B is unquestionably smaller than in the conventional variant A. As a result of this, it is expected a corresponding reduction in fuel consumption for moving vehicles, while the extent of this reduction would be certainly dependent on the type of vehicle. It is only a geometric concept of reducing fuel consumption. The real reduction will depend on many factors, such as travel-related factors: driver behavior, roadway characteristics, traffic volumes and its composition, pavement type, etc. The limited volume of the paper does not allow for analysis of these aspects.

3. Analysed transition curves

The scope of the research covered curves, which deserve attention due to their original geometrical characteristics, allowing a multitude of practical applications owing to the freedom in forming their geometry. Their usefulness also results from the relative simplicity of their mathematical description, which is convenient from the point of view of designing a road route grade line. These are the curves described by the polynomial function $y = f(x)$ (Kobryń 2002, 2009, 2011b, 2014).

Among many different transition curve solutions described by the function $y = f(x)$ those curve families will be examined, which give many possibilities as regards the forming curvature and geometrical shape of curves (Kobryń 2002). It is important from the point of view of optimising solutions used in design practice. Among these curves are:

– parabolic transition curves (Kobryń 2002)

$$y=\frac{x_K \tan u_K}{n}t^n, \qquad (1)$$

where $t=\frac{x}{x_K}$, $x\in <0; x_K>$, $n \geq 2$, while x_K – end point abscissa;

– polynomial transition curves with a smooth curvature diagram (Kobryń 2002, 2011b)

$$y=\frac{x_K \tan u_P}{C}\left[Ct+\frac{2-5C}{2}t^4-\frac{7-15C}{5}t^5+\frac{1-2C}{2}t^6\right], \qquad (2)$$

where $t=\frac{x}{x_K}$, $x\in <0; x_K>$ (Fig. 2), while x_K indicates abscissa of the end point K of the curve, which is matched by the maximum curvature of $-\frac{1}{R_K}$; u_P – a tangent turn angle at the starting point P. Parameter C is described by the following relation:

$$C=\frac{R_K \tan u_P}{x_K}, \qquad (3)$$

while C is the value from the interval $C\in <(4/10); (6/10)>$;

– polynomial transition curves with non-smooth curvature diagram (Kobryń 2002, 2011b)

$$y=\frac{x_K \tan u_P}{C}\left[Ct+\frac{1-3C}{3}t^3-\frac{1-2C}{4}t^4\right], \qquad (4)$$

In the case of curves (10), parameter C is the value from the interval $C\in <(1/3); (2/3)>$.

– polynomial transition curves with a horizontal main tangent and non-smooth curvature diagram (Kobryń 2002, 2009)

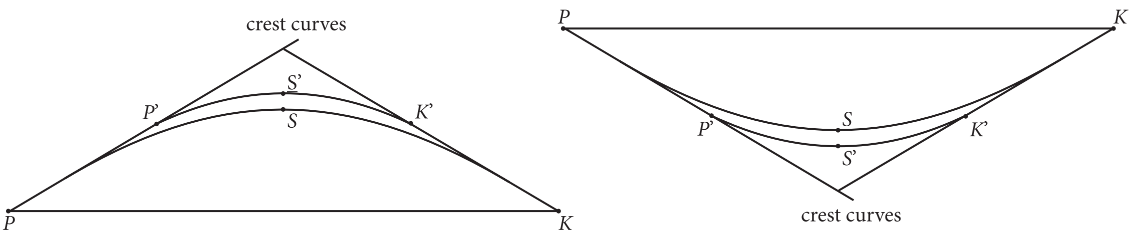

Fig. 1. Two types of vertical arcs: parabolic (P'S'K') and formed by transition curves (PSK) (Kobryń 2011a)

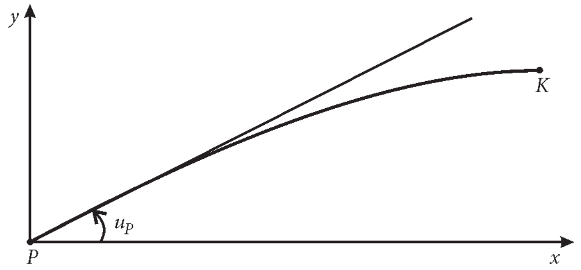

Fig. 2. Transition curves with sloping main tangent in Cartesian coordinate system (Kobryń 2002)

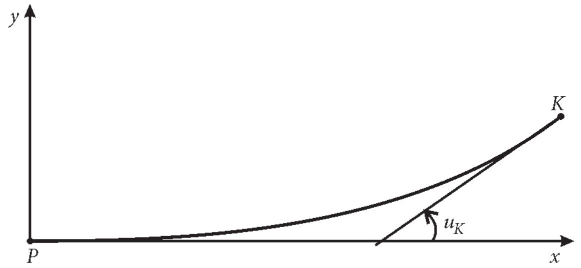

Fig. 3. Transition curves with horizontal main tangent in Cartesian coordinate system (Kobryń 2002)

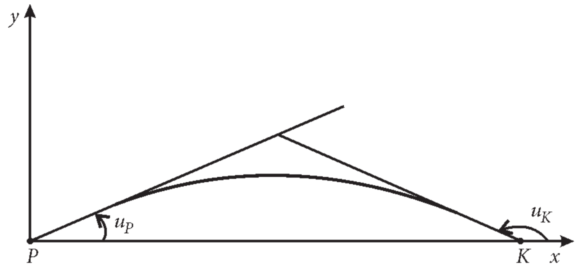

Fig. 4. General transition curves in Cartesian coordinate system (Kobryń 2002)

Table 1. Necessary extensions of individual tangents with reference to circular arc tangent

Curve		Change in tangent inclination between arc beginning and centre		
		4%	8%	12%
(2)		2.000	1.999	1.997
(1)	$n = 3$	1.996	1.985	1.967
(2)	$C = 0.4$	2.502	2.508	2.518
	$C = 0.5$	2.002	2.006	2.014
	$C = 0.6$	1.668	1.672	1.679
(4)	$C = 1/3$	3.002	3.010	3.022
	$C = 1/2$	2.002	2.006	2.014
	$C = 2/3$	1.501	1.505	1.511
(7)		1.876	1.881	1.888
(8)		1.501	1.505	1.511

$$y = \frac{x_K \tan u_K}{D}\left[\frac{3D-1}{3}t^3 - \frac{2D-1}{4}t^4\right], \quad (5)$$

where $t = \dfrac{x}{x_K}$, $x \in <0; x_K>$ (Fig. 3). According to the work (Kobryń 2002), parameter D is defined by the following dependence:

$$D = \frac{R \tan u_K}{x_K\left(1+\tan^2 u_K\right)^{\frac{3}{2}}}, \quad (6)$$

and is the value from the interval $D \in <1/3; 2/3>$.

– general polynomial transition curves with smooth curvature diagram (Grabowski 1984; Kobryń 2011)

$$y = x_K\left(G_1 \tan u_P + G_2 \tan u_K\right), \quad (7)$$

where: $G_1 = t - 20t^4 + 45t^5 - 36t^6 + 10t^7$,
$G_2 = -15t^4 + 39t^5 - 34t^6 + 10t^7$,
while $t = \dfrac{x}{x_K}$, $x \in <0, x_K>$ and according to Fig. 4: x_K – abscissa of the end point K; u_P and u_K – turn angles of the tangent to the curve – respectively at the starting point P and end point K. The following inequality is the basic design condition for curves (7), which guarantees the existence of only one curvature maximum and function (7) concavity in $(0; x_K)$ range:

$$-\frac{4}{3} \le \frac{\tan u_P}{\tan u_K} \le -\frac{3}{4}. \quad (8)$$

This makes it possible to design asymmetric curvilinear transitions, in which the position of the point corresponding to the curvature maximum is changed within abscissa x range from 0.4 x_K to 0.6 x_K.

– general polynomial transition curves with a non-smooth curvature diagram (Grabowski 1984; Kobryń 2011)

$$y = x_K\left(M_1 \tan u_P + M_2 \tan u_K\right), \quad (9)$$

where $M_1 = t - 6t^3 + 8t^4 - 3t^5$, $M_2 = -4t^3 + 7t^4 - 3t^5$ and $t = \dfrac{x}{x_K}$, $x \in <0; x_K>$. Function (9) has only one curvature maximum and is concave in $(0; x_K)$ range, provided that the following condition is met:

$$-\frac{3}{2} \le \frac{\tan u_P}{\tan u_K} \le -\frac{2}{3}, \quad (10)$$

as a result of which the point with the maximum curvature value changes the position within abscissa x range from $\dfrac{1}{3x_K}$ to $\dfrac{2}{3x_K}$.

Using vertex transition curves instead of the conventional rounding of grade line bends with the help of circular or 2$^{\mathrm{nd}}$ degree parabolic arcs would require the appropriate increase in the length of tangents to arcs – that is also the distance between the starting and end point of the arc. Adequate tangent extensions, specified for different values of change in a tangent inclination between the starting and end point of the curve, are compared in Table 1 (according to Kobryń (1999)).

It is worth adding here that, for traffic safety reasons, both in the conventional and the proposed approach it is very important to keep required sight distances. The work (Kobryń 1999) describes research on the sight distances on vertical curves created by the transition curves. These distances were calculated in accordance with Fig. 5 and Fig. 6. According to these research, in the proposed approach to the vertical curve design, applying radius values recommended in the conventional approach is enough. Suitable values illustrating this are specified in Tables 2 and 3 (Kobryń 1999).

The above curves (described by (1), (2), (4) (7) and (9)) are analyzed in the proposed approach to reduce fuel consumption.

4. The impact of longitudinal gradient on fuel consumption in the light of norms and regulations

In various norms and codes of practice there are no up-to-date source materials which would constitute a starting point for the analyses aimed at determining the relation between fuel consumption and the longitudinal road gradient, and as a result – geometrical grade line shape. Moreover, in some cases fuel consumption standards for automotive vehicles contain rather general statements. For example, in Poland for 5% grade line inclination an increase of 60% in fuel consumption for trucks and on average 40% for other automotive vehicles is assumed. This problem has been dealt with in applicable regulations issued over the last decades. For example, in Germany these

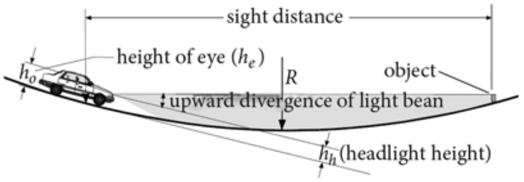

Fig. 5. Sight distance on sag curves

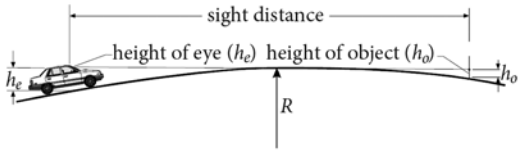

Fig. 6. Sight distance on crest curves

Table 2. Minimum sight distances, m, in symmetric sag arcs characterised by variable curvature for selected radii *R*, m

Gradient in vertical curve central point (relative to initial gradient)	Curve	Minimum curvature radius			
		R = 10 000 m	*R* = 5000 m	*R* = 1000 m	*R* = 500 m
	Circular arc	154.9	109.5	49.0	34.6
10%	(1) $n = 3$	156.1	110.7	50.2	35.9
	(2) $C = 1/2$	155.0	109.7	49.2	35.0
	(4) $C = 1/2$	156.1	110.8	50.3	36.0
	(7)	155.0	109.6	49.2	34.9
	(9)	155.0	109.6	49.2	34.9
4%	(1) $n = 3$	158.0	112.7	52.5	38.4
	(2) $C = 1/2$	155.4	110.2	50.5	36.8
	(4) $C = 1/2$	158.0	112.7	52.5	38.4
	(7)	155.3	110.1	50.2	36.5
	(9)	155.2	110.0	50.0	36.2

Table 3. Minimum sight distances, m in symmetric crest arcs characterised by variable curvature for selected radii *R*, m

Gradient in vertical curve central point (relative to initial gradient)	Curve	Minimum curvature radius			
		R = 10 000 m	*R* = 5000 m	*R* = 1000 m	*R* = 500 m
	Circular arc	387.8	210.2	59.9	37.5
10%	(1) $n = 3$	401.5	217.8	62.6	39.4
	(2) $C = 1/2$	389.9	211.5	60.5	38.0
	(4) $C = 1/2$	402.1	218.2	62.7	39.5
	(7)	389.5	211.2	60.4	37.9
	(9)	389.1	211.0	60.3	37.8
4%	(1) $n = 3$	428.7	233.1	68.2	43.9
	(2) $C = 1/2$	401.4	218.4	63.9	41.2
	(4) $C = 1/2$	429.0	233.3	68.3	43.9
	(7)	399.0	217.0	63.4	40.8
	(9)	396.5	215.5	62.6	40.1

guidelines are *RAS/RAS-W:1986 Richtlinien für Anlage von Straßen. Teil: Wirtschaftlichkeits-untersuchungen* and *Aktualisierung RAS-W 86:1997 Empfehlung für Wirtschaftlichkeitsuntersuchungen an Straßen*). Because of this, the researchers have used German guidelines *Aktualisierung RAS-W 86:1997*, representing a complex approach to fuel consumption issues and, connected with this, the impact of emitted exhaust gas on the natural environment.

These guidelines provide for the division of road vehicles into the following groups: SB – cars with petrol engines, SD – cars with diesel engines, CN – trucks: 2.8–3.5 t, CS – trucks >3.5 t, special purpose vehicles and agricultural vehicles, CZ – trucks with semi-trailers/trailers, BR – touring buses, BL – regular service buses.

For each of these vehicle groups it is possible to estimate the total fuel consumption by vehicles moving along a specific road section with reference to the chosen reference year. The reference year shall be understood in such a way that appropriate numerical values relating to it are determined using output data specified for the base year 1990 and attributed reduction coefficients (*WR*). Total fuel consumption in a selected vehicle group (*GP*) in a chosen i^{th} road section and chosen direction *E* (omitting deviations from normal consumption in the case of not warmed up engines) is defined as:

$$\overline{ZP}_{i,E} = \sum_{GP} \sum_{j} ZP_{GP}\left\{v_{GP,TD}(N_{i,E,j,GP}), m_{i,K}, R\right\} \cdot N_{i,E,j,GP} TD_i, \quad (11)$$

where *j* – number of period with relatively equal run characteristics; *T* – duration of period with relatively equal run characteristics; *GP* – examined vehicle group; D_i – length, m of i^{th} road section; *TD* – road type; $m_{i,E}$ – longitudinal gradient of i^{th} road section towards *E*; *J* – reference year; $N_{i,E,j,GP}$ – traffic volume of vehicles in a given *GP* group in i^{th} road section towards *E* and in the period *t* with relatively equal run characteristics; $v_{GP,TD}(N_{i,E,j,GP})$ – speed, km/h (operating speed) in the *GP* vehicle group in i^{th} road section towards *E* depending on traffic volume $N_{i,E,j,GP}$ and road type; $ZP_{GP}(v,m,J)$ – fuel consumption index for warmed up engines in the *GP* vehicle group.

Fuel consumption indexes $ZP_{GP}(v,m,J)$ consist of three components: $WZ_{GP}(v)$ – fuel consumption index referring to warmed up engine depending on driving speed *v*; $WZ_{GP}(v,m)$ – correction coefficient on account of road longitudinal gradient *m*; $WZ_{GP}(J)$ – reduction coefficient on account of technological progress in successive reference years.

Assuming a run with a warmed up engine, the fuel consumption index depends on driving speed *v*, and for $v > 20$ km/h it is defined by the following regression function:

$$WZ_{GP}(v) = c_0 + c_1 v^2 + \frac{c_2}{v}, \quad (12)$$

where parameters c_0, c_1 and c_2 refer to year 1990 and are specified in Table 4.

The impact of the longitudinal gradient *m* on the fuel consumption index for cars (groups SB and SD) within a normal driving speed range is determined using the correction coefficient $WK_S(v,m)$, which guarantees obtaining an index value equal to unity for longitudinal gradient $m = 0$, that is condition $WK_{GP}(v, m = 0) = 1$ is met, and has the following form:

$$WK_S(v,m) = \exp\left[\left(c_0 - \frac{(v - v_0)^2}{c_1}\right)\left(c_2 m + c_3 m^2\right)10^{-6}\right]. \quad (13)$$

Whereas, the following formula is applied for all other vehicle groups:

$$WK_P(m) = \exp\left[m c_1 \left(1 + c_2 m - c_3 m^2\right)\right]. \quad (14)$$

Values of coefficients c_0, c_1, c_2 and c_3 are specified in Table 5.

If future potential is taken into account as regards reduction in fuel consumption due to technological progress, which is determined by reduction coefficients *WR* (defining reduction in fuel consumption compared to base year 1990 and allowing the estimation of appropriate values with reference to years 1995, 2000, 2005 and 2010), then resultant fuel consumption indexes are obtained referring to these years. In the case of cars, resultant values of fuel consumption indexes are determined using the following formula:

Table 4. Parameters of speed-dependent fuel consumption indexes (*WZ*) for different vehicle groups

Vehicle group	c_0	c_1	c_2
SB	17.7766	0.0023606	1461.87
SD	18.9647	0.0020625	1139.17
CN	37.7695	0.0054534	1582.97
CS	31.7385	0.0151013	4048.32
CZ	139.5313	0.0111448	3984.71
BR	95.2499	0.0083955	4927.30
BL	147.4672	0	4220.44

Table 5. Correction coefficients (*WK*) on account of longitudinal gradient for different vehicle groups

Vehicle group	v_0	c_0	c_1	c_2	c_3
SB	110.0	2262.0	6.52	69.89	2.485
SD	110.0	2262.0	6.52	69.89	2.485
CN			0.1055	0.0720	0.00571
CS			0.1328	0.0650	0.00954
CZ			0.2047	0.0456	0.00997
BR			0.1409	0.1027	0.01177
BL			0.1211	0.0012	0.00567

$$ZP_S(v,m,J) = WZ_S(v)WK_S(v,m)WR(J). \quad (15)$$

In the case of all other vehicle groups, values of fuel consumption indexes are determined as:

$$ZP_{(C,B)}(v,m,J) = WZ_{(C,B)}(v)WK_P(m)WR(J). \quad (16)$$

Studies on total fuel consumption in specific road sections defined using (11) and in a certain situation dependent on consumption indexes given using (15) and (16) require separate consideration, taking into account additional route parameters, occurring in (11). They constitute the correct subject of economic analyses intended to select the optimal variant of a designed route.

However, total fuel consumption is a secondary element in relation to e.g. the geometry of route longitudinal profile. Therefore, in their studies the researchers restricted themselves to seeking those vertical arcs in the form of transition curves which would allow the fuel consumption of vehicles to be reduced. The completed analyses focused on comparing the average fuel consumption indexes in vertical arcs corresponding to each other and formed by transition curves (for indexes relating to individual vehicle groups). Fuel consumption indexes $ZP_{GP}(v,m,J)$ determined according to (15) and (16) were used for that purpose. It was assumed that arcs of transition curves corresponding to each other mean arcs characterised by the same values of minimum curvature radius and changes in the inclination of curves between the starting and end point.

5. Fuel consumption in vertical arcs formed by selected curves

Fuel consumption indexes in individual vehicle groups were determined taking into account the appropriate reduction coefficients due to technological progress. Transition curves forming vertical arcs in variant B require the use of longer tangents than conventional elements. Therefore, for the purposes of computing suitable fuel consumption indexes in variant A – according to Fig. 1 – the researchers assumed the same position of the beginning and the end of the compared grade line section, as in variant B.

All computations, the results of which are shown in Figs 7–13, have been carried out numerically. Indexes

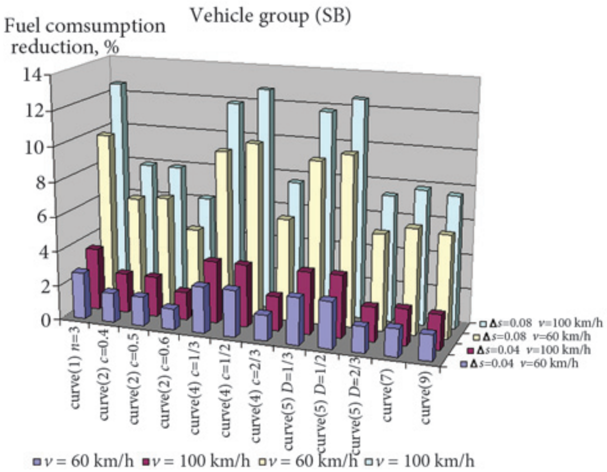

Fig. 7. Fuel consumption reduction in variant B, %, in vehicle group (SB)

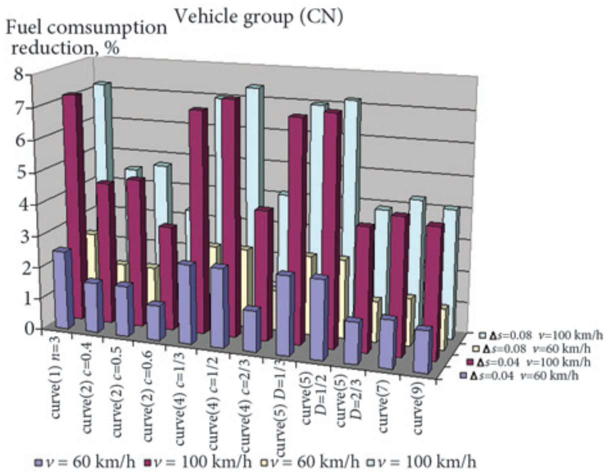

Fig. 9. Fuel consumption reduction in variant B, %, in vehicle group (CN)

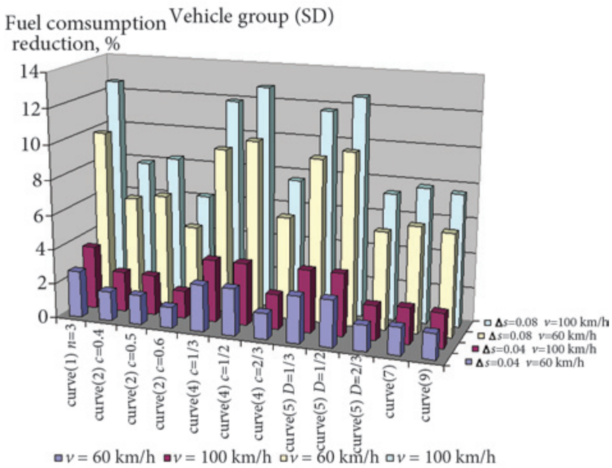

Fig. 8. Fuel consumption reduction in variant B, %, in vehicle group (SD)

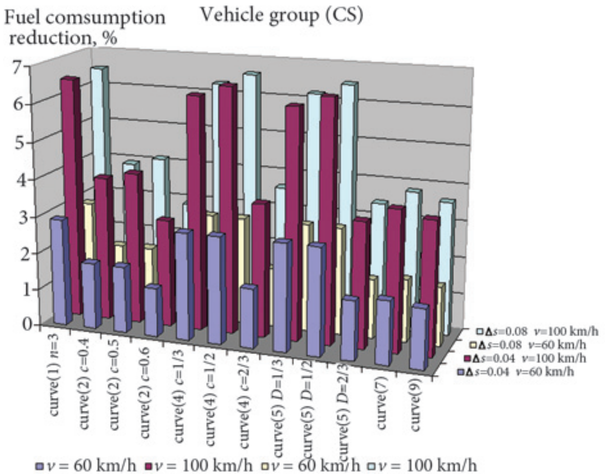

Fig. 10. Fuel consumption reduction in variant B, %, in vehicle group (CS)

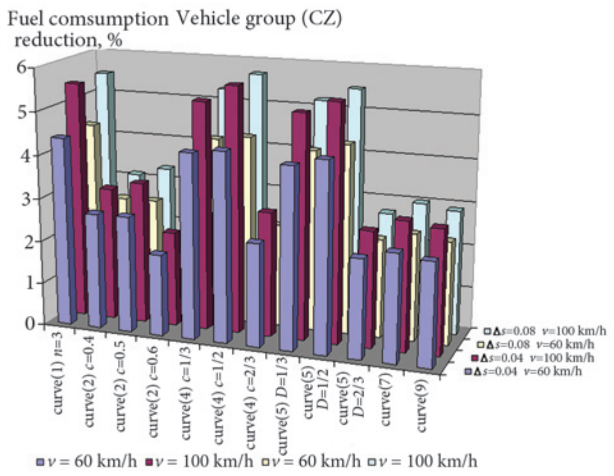

Fig. 11. Fuel consumption reduction in variant B, % in vehicle group (CZ)

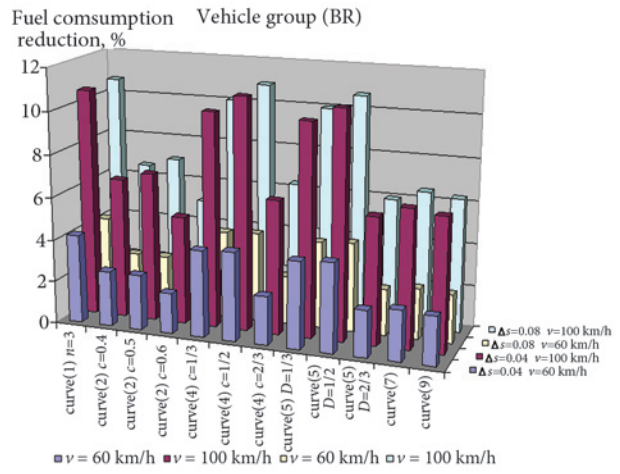

Fig. 12. Fuel consumption reduction in variant B, % in vehicle group (BR)

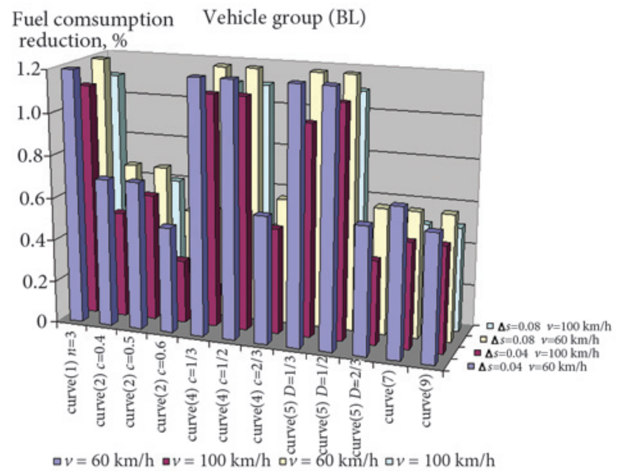

Fig. 13. Fuel consumption reduction in variant B, %, in vehicle group (BL)

specified in these tables are average values for the symmetrical vertical curve, derived from partial values obtained along the whole vertical arc with a step equal to 0.001 of the length of the arc.

Preliminary verification of the chosen computation method have proven that the values of fuel consumption indexes are not dependent on the minimum curvature radius in the top of the arc (this radius is to be equal to the radius of an appropriate circular arc). Average fuel consumption indexes in an ascending route proved to be significantly higher than along a grade line descent. Moreover, it has turned out that in a grade line ascent within rest curves and sag curves the values of average fuel consumption indexes are equal. The same is also applicable to grade line descents in a crest and sag curve. Therefore, these two arc types have not been distinguished in further analyses.

In order to be able to obtain reliable results for specific road sections, the detailed computations taking into account traffic stream and type structure of vehicles in a given route are to be carried out. Of course, this traffic characteristic would be different for each road, and thus the results obtained would characterise only a given route. It would also be difficult to relate them to any other route. Therefore, as the purpose of computations described in this paper the researchers have chosen to determine indexes of general character, which would give a view about the possible reductions in fuel consumption indexes for individual representative vehicles in the case of the chosen geometry of vertical arcs.

Figs 7–13 show possible reductions of the fuel consumption (given in percent) in variant B compared to corresponding indexes in variant A. They refer to individual automotive vehicle groups distinguished by German guidelines. The computations concern different values of inclination change between the beginning and the top of the arc and various traffic speeds.

The comparisons which have been quoted, allow to state that, depending on the speed and gradient change, a possible reduction in fuel consumption reaches different values and generally increases with growing speed and gradient change in all vehicle groups (apart from regular service buses).

As seen, the reduction of fuel consumption indexes given as a percent is highest in the group of touring buses (BR) and in both car groups (SB and SD). For example, for an inclination change of 0.08 it reaches even 7–9%. The absolute value of the possible reduction in appropriate fuel consumption indexes is significantly higher in BR group.

This is because, depending on the driving speed, the reduction scale proves to be 4 to 6 times higher compared to cars, which are a direct outcome of the fact that the values of fuel consumption indexes in buses group (BR) are 5–6 times higher than in SB and SD groups. As a result of this, if for example data corresponding to the inclination change of 0.08 is taken into account. The above consideration indicates that the absolute reduction value for the fuel consumption index in group BR is no less than one third of the fuel consumption index value for cars.

Table 6. Average values of fuel consumption indexes in variant A and average values of reduction obtainable in variant B

Vehicle group	Straight-line gradient, %	v = 60 km/h			v = 100 km/h		
		A	$B-A$	$\frac{\lvert(B-A)\rvert}{A}$, %	A	$B-A$	$\frac{\lvert(B-A)\rvert}{A}$, %
SB	4	50.7	−1.0	2.0	58.9	−1.6	2.7
	8	77.1	−5.7	7.4	102.5	−9.6	9.4
SD	4	44.9	−0.9	2.0	53.0	−1.4	2.7
	8	68.3	−5.0	7.4	91.9	−8.6	9.4
CN	4	89.0	−1.7	1.9	114.8	−2.2	1.9
	8	122.6	−7.2	5.4	158.2	−8.5	5.4
CS	4	171.9	−3.8	2.2	249.8	−5.5	2.2
	8	234.4	−11.0	4.7	340.7	−16.0	4.7
CZ	4	304.0	−10.0	3.3	359.3	−11.9	3.3
	8	425.3	−16.6	3.9	502.6	−19.6	3.9
BR	4	249.0	−7.7	3.1	274.0	−8.5	3.1
	8	409.0	−32.3	7.9	450.2	−35.6	7.9
BL	4	221.1	−1.8	0.8	192.5	−1.5	0.8
	8	239.0	−1.7	0.7	208.1	−1.4	0.7

Table 6 presents a summary of the above results. It contains the average values of fuel consumption indexes in variant A, determined on the basis of individual values applied to successive curves. They are complemented with the average reduction values given in absolute numbers and the average reduction in values of these indexes given as a percent, obtainable in variant B.

Moreover, the figures show that percentage reductions of fuel consumption indexes in individual groups of trucks are slightly lower than in the case of cars. However, due to considerably higher values of fuel consumption indexes (Fig. 14) the scale of possible benefits (especially in truck group CZ) is also significant.

Another interesting issue is: which of the considered curves guarantees the lowest values of fuel consumption indexes in variant B, and their greatest reduction compared to variant A. An analysis of Figs 7–13 allows it to be stated that, in this regard, the most advantageous are:

– polynomial transition curves (5) with a horizontal main tangent and non-smooth curvature diagram for $D=\frac{2}{3}$;

– polynomial transition curves (4) with a non-smooth curvature diagram for $C=\frac{2}{3}$;

– general polynomial transition curves (7) and (9).

The following curves are slightly worse:

– parabolic transition curves (1) for $n = 3$;

– polynomial transition curves (4) and (5) with a non-smooth curvature diagram for $C=\frac{1}{2}$ and $D=\frac{1}{2}$;

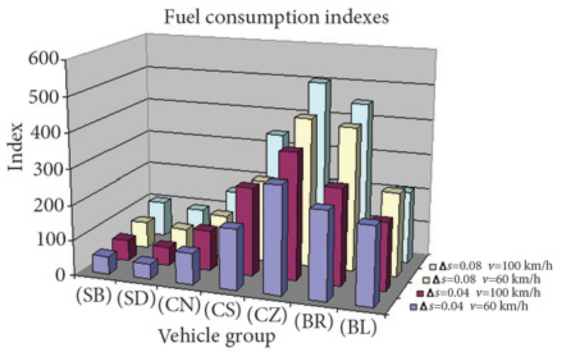

Fig. 14. Fuel consumption indexes calculated on the basis of (11)

– polynomial transition curves (2) with a smooth curvature diagram for $C = 0.6$.

Whereas, the least advantageous are polynomial transition curves (2), (4) and (5) (both with a smooth and non-smooth curvature diagram) for the minimum values of parameters C or D taken from acceptable intervals.

6. Conclusions

1. Due to the importance attributed to the issues concerning fuel reduction consumption, the results of the completed studies indicate the legitimacy of a change in the current approach to the geometrical formation of a route grade line. A reduction of fuel consumption for automotive vehicles is possible by the rounding of grade line bends with the help of the so-called vertex transition curves (that is curves with a shared tangent and identical curvature radius at contact point), instead of the conventional use of 2^{nd} degree parabolic arcs.

2. It also turns out that the type of transition curves forming vertical rounding arcs in the proposed approach is very important. The most advantageous are polynomial transition curves with a smooth and a non-smooth curvature diagram for the maximum possible values of parameters *C* or *D*. Cubical parabola give slightly worse results. Whereas, it has been proven that the least advantageous are polynomial transition curves for the lowest permissible values of parameters *C* or *D*.

3. The determined fuel consumption indexes apply to single vehicles within individual groups of automotive vehicles. The benefits resulting from this are multiply more depending on the higher vehicular traffic volume in a given road section. Even if it is assumed that the proposed approach to grade line design will sometimes require higher costs involved in the implementation of a road investment project, it is highly probable that the benefits resulting from the reduced fuel consumption and related to it, the lower environmental pollution caused by engine exhaust gases ultimately have a greater importance and justify grade line laying out with vertex transition curves.

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