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A MIXED APPROACH FOR DETERMINATION OF INITIAL CABLE FORCES IN CABLE-STAYED BRIDGES AND THE PARAMETERS VARIABILITY

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Abstract. The determination of initial cable forces in cable-stayed bridges is an important first step in design and analysis of the structure under external loads. Adjustments of stay forces are often required during construction in order to assure the requested behaviour of the bridge in terms of final geometrical configuration and internal force distribution. An accurate assessment of the stay tensioning system allows designers to obtain a good result at the end of construction, by considering the parameters involved as deterministic quantities, assuring the observance of the execution tolerances during works. Actual loads and their variations need instead a stochastic approach which can give useful indications about the effects of parameter variations. Particularly, the measurement on site of the actual values of stay elongations contains errors and the actual values of pre-stressing forces and working site loads are subjected to unknown variations. In this paper a procedure is implemented which takes into account the stochastic variation of stay elongations and the related uncertainties. The presented method does not require large computational efforts or computer memory. By approximating the probability density distribution of the variables involved by the Gaussian curve, a closed form expression of the imposed elongations given to stays and their variations during erection is provided. The main aim is to obtain an accurate prediction of the differences between the deterministic quantities found by the design analyses and the actual values of deck deformations and stresses in the erection of steel cable-stayed bridges.

Keywords: cable-stayed, bridge, stay, probability, staged construction, parameter variability.

1. Introduction

In many countries, cable-stayed bridges found wide use during the last decades (Gimsing 1997). The success of this typology is due to the introduction of innovative materials and technologies as high strength steel and orthotropic steel decks and to the development of computational analysis.

In the design of these bridges the determination of initial cable forces and the procedure of stay stress adjustments during erection are of fundamental importance, but constitutes a hard task to achieve, in order to respect the requested geometric profiles of deck and towers at the end of construction (Martinez y Cabrera, Malerba 1999).

It is not simple to state a convenient methodology of initial stay force determination for the following reasons:

– at the end of erection, the girder longitudinal profile must satisfy aesthetic and functional requirements, possibly presenting a convenient pre-camber;

– the towers must keep the vertical profile, in order to avoid second order effects and to satisfy architectural demands;

– even though geometrical requirements are satisfied in the so-called dead load configuration, after erection end, the system of stresses has to be checked to avoid high stress levels in the deck and tower members.

In order to achieve these goals, two main categories of methods were proposed in literature: the "zero-displacement" (Wang *et al.* 1993) procedure and the "static optimization" (Negrão, Simões 1997). Some researches suggest to countervail the warped shape of the bridge girder by stay prestressing, reducing or nullifying vertical dead load deflections. This procedure does not take into account the towers and the horizontal displacements induced by dead load on them. Other researchers propose to follow the "force equilibrium method" (Chen *et al.* 2000), which consists of searching a set of stay forces that give the required values of bending moments in selected points of the deck and the towers. Displacements are not checked by this methodology, disregarding the achievement of the correct geometric profile in the dead load configuration.

Other procedures based on optimization techniques, which account for cable-force and camber control, were

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also proposed (Fujisawa, Tomo 1985; Martins *et al.* 2015) and further studies (Kasuga *et al.* 1995) present some optimization criteria. Unfortunately it is very difficult to agree with all the static and kinematic demands of these bridges, especially for steel ones, which has hard problems of deformability and show a marked non linear behaviour. The number of unknown variables (initial cable forces) is less than the number of control set (static bending forces, vertical and horizontal displacements); so it is impossible to find a solution that satisfy all requirements by a mathematical procedure with a high number of constraints and a few unknowns.

The stay stressing sequence is sensitive to the actual loads applied during the bridge erection. An accurate assessment of the stay tensioning system allows for obtaining a good result at the end of construction, by considering the parameters involved as deterministic quantities, assuring the observance of the execution tolerances during works. Actual loads and their variations need instead a stochastic approach which can give useful indications about the effects of parameter variations. Particularly, the measurement on site of the actual values of stay elongations contains errors and the actual values of pre-stressing forces and working site loads are subjected to unknown variations. In this paper a procedure is implemented which takes into account the stochastic variation of stay elongations and the related uncertainties. The presented method does not require large computational efforts or computer memory. This method allows application of the Gaussian curve for stochastic modelling of the parameter (imposed stay elongation), under the hypothesis that the variability depends mainly on the error between the actual value of prestressing given to stays during erection and the predicted value found by the theoretical procedure in the design phase. No uncertainties related to geometric characteristics of the bridge are considered. An optimization of these characteristics can be found in (Simões, Negrão 2000) through a fuzzy logic approach.

The main aims of this study are:

– to find a reliable procedure for the determination of initial cable forces in the design phase which contemporarily satisfies the demands in terms of deck and tower stresses and deformations;

– to obtain an accurate prediction of the differences between the deterministic quantities found by the previous procedure applied in the design analyses and the actual values of deck deformations and stresses in the erection of steel cable-stayed bridges.

About the first target an acceptable procedure for bridge erection has to be established, because the sequence of stay stressing is of fundamental importance to achieve the desired final state of stress and deformation (Granata *et al.* 2013c; Negrão, Simões 1997). Moreover, in concrete bridges, in which creep effects can be significant, the final state can change with time (Arici *et al.* 2011; Granata *et al.* 2013b). The construction stages of a cable-stayed bridge consist of a sequence of partial schemes in which the cantilever construction is usually adopted. In order to compensate for the dead load of the cantilever segments assembled during the erection, a provisional or definitive sequence of stay stress adjustments has to be implemented. If the definitive value of stay pretension is adopted with a single-phase stressing of each stay, then a more economic solution is found, because the stressing equipment is not shifted from one stay to another. Nevertheless it implies a precise determination of displacements at each stage and the introduction of a complex staged construction analysis on a finite element software. Arici *et al.* (2011) solved the problem by stating the so-called "partial elastic scheme (PES) method", applied mainly to concrete cable-stayed bridges and to concrete arches built by the suspended cantilever method (Granata *et al.* 2013c) and by lattice cantilevers (Granata *et al.* 2013a). In some cases a single-phase stressing of each stay is not sufficient to achieve the desired result because the partial schemes are too flexible or the construction sequence is more complex. For steel-concrete composite structures or when the time-dependent phenomena as shrinkage and creep have to be considered in concrete bridges, a single stressing operation cannot be enough. In these cases one or more adjusting operations need and a transitory state of each stay is given by the initial value of cable force, which reaches the final value after the adjustments established. Generally, it is convenient to avoid too many adjustments because the steel wires of the stay can be damaged by gripping the strands in areas with a previous wedge seating, increasing the danger of fatigue collapse of stays when moving loads are applied in service life. It happens when the adjustments imply not only re-stressing but also releasing operations. During the last phase of construction, a final small regulation is always performed in order to reach the required pre-camber and to correct construction errors. Usually, in large-span cable-stayed bridges cable adjustments cannot be avoided and a complex staged construction analysis performed by finite element software need in order to establish the actual state of stress and strain (Straupe, Paeglitis 2012) by analyzing every erection stage and the mutual influence of all the parameters involved (geometry, restraints, cable system, pretension, etc…).

In this paper the problem of the initial cable force determination is faced by pointing out a mixed approach which satisfies the static demands related to the stress distribution on the deck and the pylon, establishing a convenient geometric profile with an assigned pre-camber.

About the second aim of this paper, being not the stay elongations a deterministic quantity, they undergo the peculiarities of prestressing technologies, the instrumentation used to check the displacements during the construction stages and the used equipment. Moreover the temperature variation during the different phases of construction as well as the presence of additional loads on the working site, especially during the stay stressing operation, can modify the imposed strain without a preliminary control. So, the actual values of stay elongations can be considered as random variables. The variability of the stay elongation value, respect to what predicted by the design analyses, modifies the strain field and consequently the state of stress in the structure. Every characteristic (internal force or displacement) becomes a stochastic variable and the range of variation can be very important for the structural control and monitoring during construction. There are known only a few studies which were considered the stochastic nature of the problem. In the following sections a procedure for the determination of the possible variability of deflections and stress resultants is provided.

2. The deterministic model of the stay stressing sequence

Cable-stayed bridges show a moderate geometric non linear behaviour, which is mainly due to the sag effect of cables an which can affect the deformed shape of the bridge. For the current stress levels a good approximation of the non linear solution for stays is given by the equivalent elastic modulus of a prismatic bar (Gimsing 1997; Troitsky 1977). The Ernst relation can be used in order to establish the equivalent modulus when stays of a medium span bridge have to be handled; it is possible thank to the lightness and the high yielding stress of modern stay steel (new-PWS cables) for which a quasi-linear behaviour is shown. In this way it is possible to take advantage of the effectiveness of the superposition principle and of the influence lines procedure for moving loads acting on the bridge, even though the linearization of the non-linear problem through the Ernst modulus implies an iterative solution, which is rapidly convergent.

With the specific imposed strains of stays fixed as the direct unknown variables during the stay stressing sequence, cable elongations and axial forces will be the indirect unknowns. In Fig. 1 an example of a bridge in a stressing stage is shown. The maximum number of unknowns is the same as the stay number. Design constraints instead could be expressed in terms of displacements or static requirements. The first case occurs when the geometric configuration has to be reached after the construction sequence, for which the deck and the pylon achieve the correct shape. The second case occurs when the target is to define a convenient bending moment diagram or a convenient distribution of stay forces into the deck at the end of construction. In both cases it is important to choose the smallest number of constraints for a good mathematical conditioning of the problem. These conditions are imposed in the control points that is generally convenient to establish at the anchorages of the stays along the deck and the pylon. By considering the t cables and the h-dimensional vector $\boldsymbol{d}$ of control point displacements, the following equation can be written:

$$\boldsymbol{d} = [\boldsymbol{D}] \cdot \boldsymbol{e} + \overline{\boldsymbol{d}}, \tag{1}$$

where $\overline{\boldsymbol{d}}$ – the array of displacements due to dead loads in the control points, mm; $\boldsymbol{e}$ – the array containing the specific imposed strains associated to the pretension of each stay, dimensionless; $[\boldsymbol{D}]$ – the matrix of the influence coefficients δ_{ij} which supplies the displacements in the i-th control point due to an imposed strain ej given to the j-th stay, mm.

In the same way, by applying the procedure on the static point of view, array $\boldsymbol{c}$ of the target internal forces in the control points can be defined and the following expression can be written:

$$\boldsymbol{c} = [\boldsymbol{C}] \cdot \boldsymbol{e} + \overline{\boldsymbol{c}}, \tag{2}$$

in which $\overline{\boldsymbol{c}}$ – the array of internal forces due to dead loads in the control points (e.g. kN or kNm); $[\boldsymbol{C}]$ – the matrix of the influence coefficients χ_{ij} which supplies the internal force in the i-th control point due to the imposed strain given to the j-th stay (e.g. kN or kNm).

Eq (2) is equivalent to Eq (1) but solves the problem by fixing a static target instead of a geometric one.

In the design phase generally two pathways can be followed. The first one is to impose a target array of control point displacements $\boldsymbol{d}^*$ which corresponds to a given shape or camber to the deck and to assure the verticality of pylons. Generally the choice of the camber is generated by aesthetic and functional conditions; it is common practice to give a parabolic or a polynomial camber to the deck. In this case the solution is:

$$\boldsymbol{d}^* - \overline{\boldsymbol{d}} = [\boldsymbol{D}] \cdot \boldsymbol{e}, \tag{3}$$

in which $\boldsymbol{d} = \boldsymbol{d}^*$ and $\boldsymbol{e}$ represents the array of imposed strains given to stays in order to achieve the target displacements.

The second pathway is that of fixing a convenient distribution of bending moments by defining a target array $\boldsymbol{c}^*$ of internal forces in the control points. In this case the solution is given by the following equation:

$$\boldsymbol{c}^* - \overline{\boldsymbol{c}} = [\boldsymbol{C}] \cdot \boldsymbol{e}. \tag{4}$$

If $[\boldsymbol{D}]$ and $[\boldsymbol{C}]$ are square matrices (number of control points equal to the number of stays) then the direct solution of Eqs (3) and (4) is possible and it represents an exact solution of imposed strain values $\boldsymbol{e}$ to be given to the stays. Unfortunately in most cases $[\boldsymbol{D}]$ and $[\boldsymbol{C}]$ are rectangular matrices, so the exact solution cannot be found and an approximation of it has to be attempted. In this paper three possibilities are investigated for achieving respectively:

1) a geometric shape of the structure near to the required one, by finding approximate values of target displacements established in Eq (3), disregarding the static requirements (displacement approach);

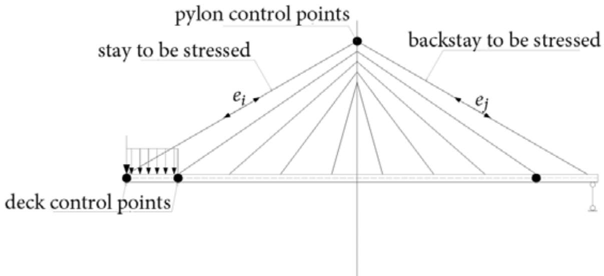

Fig. 1. Stay stressing and control points in a generic cantilever construction stage

2) an approximate distribution of internal forces, disregarding the displacement demand (static approach);

3) a static system which approximates the internal force requirements, by satisfying the displacement demand and the related dead load configuration (mixed approach).

If the first approach is followed (displacement method), the array $\Delta \boldsymbol{d}$ of displacement deviation respect to the required configuration can be defined:

$$\Delta \boldsymbol{d} = \boldsymbol{d} - \boldsymbol{d}^* = [\boldsymbol{D}] \cdot \boldsymbol{e} + \hat{\boldsymbol{d}}, \quad (5)$$

where $\hat{\boldsymbol{d}} = \overline{\boldsymbol{d}} - \boldsymbol{d}$ – the array of displacements (mm) found as the difference between those due to dead loads in the control points and those fixed by the designer, in order to achieve the desired camber. The value of deviation $\Delta \boldsymbol{d}$ has to be minimized in order to find the optimal value of stay forces through the imposed strain array $\boldsymbol{e}$. When $[\boldsymbol{D}]$ is a square matrix, the solution of Eq (5) is simply found, with a null value of deviation $\Delta \boldsymbol{d}$; it corresponds to an exact closed-form solution of the problem and to values of imposed strains $\boldsymbol{e}$ given to each stay in order to reach the exact camber shape defined by the array $\boldsymbol{d}^*$. In the other cases, when $[\boldsymbol{D}]$ is a rectangular matrix, only an approximate solution of Eq (5) can be found, by searching the minimum value of the array norm, as a function of strains $\boldsymbol{e}$:

$$\min\left(\Delta \boldsymbol{d}^{\mathrm{T}} \cdot \Delta \boldsymbol{d}\right) = \min\left[\left(\hat{\boldsymbol{d}} + [\boldsymbol{D}] \cdot \boldsymbol{e}\right)^{\mathrm{T}} \cdot \left(\hat{\boldsymbol{d}} + [\boldsymbol{D}] \cdot \boldsymbol{e}\right)\right] = \min\left[\hat{\boldsymbol{d}}^{\mathrm{T}} \cdot \hat{\boldsymbol{d}} + \left(\hat{\boldsymbol{d}}^{\mathrm{T}} \cdot [\boldsymbol{D}] \cdot \boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}} \cdot [\boldsymbol{D}]^{\mathrm{T}} \cdot \hat{\boldsymbol{d}}\right) + \boldsymbol{e}^{\mathrm{T}} \cdot [\boldsymbol{D}]^{\mathrm{T}} \cdot [\boldsymbol{D}] \cdot \boldsymbol{e}\right], \quad (6)$$

whose solution is given by the relation:

$$[\boldsymbol{D}]^{\mathrm{T}} \cdot [\boldsymbol{D}] \cdot \boldsymbol{e} + [\boldsymbol{D}]^{\mathrm{T}} \cdot \hat{\boldsymbol{d}} = 0. \quad (7)$$

In Eq (7), when the matrix $[\boldsymbol{K_D}] = [\boldsymbol{D}]^{\mathrm{T}} \cdot [\boldsymbol{D}]$ can be inverted, the problem has a single solution. This procedure is a generalization of the *zero-displacement method* (Wang *et al.* 1993), for which an approximate solution is found.

In the second approach an analogous procedure is established, in which the static control is considered. In this case an array of internal force deviation is defined

$$\Delta \boldsymbol{c} = \boldsymbol{c} - \boldsymbol{c}^* = [\boldsymbol{C}] \cdot \boldsymbol{e} + \hat{\boldsymbol{c}}, \quad (8)$$

where $\hat{\boldsymbol{c}} = \overline{\boldsymbol{c}} - \boldsymbol{c}$ – the array of internal forces found as the difference between those due to dead loads in the control points and those fixed by the designer as the static behaviour target. By following the procedure of Eqs (6) and (7), in this case the optimization problem is given by the following relation:

$$\min\left(\Delta \boldsymbol{c}^{\mathrm{T}} \cdot \Delta \boldsymbol{c}\right) = \min\left[\left(\hat{\boldsymbol{c}} + [\boldsymbol{C}] \cdot \boldsymbol{e}\right)^{\mathrm{T}} \cdot \left(\hat{\boldsymbol{c}} + [\boldsymbol{C}] \cdot \boldsymbol{e}\right)\right], \quad (9)$$

whose solution can be found by the following equation:

$$[\boldsymbol{C}]^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \boldsymbol{e} + [\boldsymbol{C}]^{\mathrm{T}} \cdot \hat{\boldsymbol{c}} = 0. \quad (10)$$

The mixed approach tries to satisfy at the same time the displacement and static requirements. It implies that a few parameters have to be fixed. In this paper the choice is that of fixing a shape profile of the bridge (as a parabolic or polynomial camber of the deck) by introducing an array $\tilde{\boldsymbol{d}}$ which represents the chosen shape.

By defining the target array of displacements as $\boldsymbol{d}^* = a\,\tilde{\boldsymbol{d}}$, the parameter a is an amplification coefficient of the "modal shape" chosen for the camber by the designer. So by recalling the Eqs (5) and (7), the following equation is established

$$\hat{\boldsymbol{d}} = \overline{\boldsymbol{d}} - a\,\tilde{\boldsymbol{d}}, \quad (11)$$

and the value of the imposed strains $\boldsymbol{e}_{alw}$ is found:

$$\boldsymbol{e}_{alw} = -[\boldsymbol{K_D}]^{-1} \cdot [\boldsymbol{D}]^{\mathrm{T}} \cdot \left(\overline{\boldsymbol{d}} - a\,\tilde{\boldsymbol{d}}\right). \quad (12)$$

Eq (11) can be re-written in the following way:

$$\boldsymbol{e}_{alw} = -[\boldsymbol{K_D}]^{-1} \cdot [\boldsymbol{D}]^{\mathrm{T}} \cdot \overline{\boldsymbol{d}} + a\left([\boldsymbol{K_D}]^{-1} \cdot [\boldsymbol{D}]^{\mathrm{T}} \cdot \tilde{\boldsymbol{d}}\right) = -\overline{\boldsymbol{e}} + a\,\tilde{\boldsymbol{e}}, \quad (13)$$

in whih $\overline{\boldsymbol{e}}$ – the array of imposed strains (pretension) applied to stays that recover the dead load displacements; while $a\,\tilde{\boldsymbol{e}}$ – the array of imposed strains that need for obtaining the required camber. The coefficient a modulates the deck internal forces, so the minimum absolute value of forces ($\boldsymbol{c}^* = \boldsymbol{0}$) can be imposed, in order to find the required camber with the minimum value of stresses. By recalling Eqs (8) and (12), the internal force deviation array is $\Delta \boldsymbol{c} = [\boldsymbol{C}] \cdot \boldsymbol{e}_{alw} + \overline{\boldsymbol{c}}$ and substituting it into Eq (9), the optimization problem becomes:

$$\min\left(\Delta \boldsymbol{c}^{\mathrm{T}} \cdot \Delta \boldsymbol{c}\right) = \min\left(\boldsymbol{e}_{alw}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \boldsymbol{e}_{alw} + \boldsymbol{e}_{alw}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot \overline{\boldsymbol{c}} + \boldsymbol{c}^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \boldsymbol{e}_{alw} + \overline{\boldsymbol{c}}^{\mathrm{T}} \cdot \overline{\boldsymbol{c}}\right). \quad (14)$$

The expression into parentheses is a function of the parameter a, through Eq (13), so the solution of the optimization problem is given by the first derivative of that function with respect to a:

$$\frac{d}{da}\left(\Delta \boldsymbol{c}^{\mathrm{T}} \cdot \Delta \boldsymbol{c}\right) = 2a\,\tilde{\boldsymbol{e}}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \tilde{\boldsymbol{e}} + \left(\tilde{\boldsymbol{e}}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot \overline{\boldsymbol{c}} + \overline{\boldsymbol{c}}^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \tilde{\boldsymbol{e}}\right) = 0, \quad (15)$$

and consequently the parameter a which minimizes the internal forces is:

$$a = -\frac{\tilde{\boldsymbol{e}}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot \overline{\boldsymbol{c}} + \overline{\boldsymbol{c}}^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \tilde{\boldsymbol{e}}}{2\,\tilde{\boldsymbol{e}}^{\mathrm{T}} \cdot [\boldsymbol{C}]^{\mathrm{T}} \cdot [\boldsymbol{C}] \cdot \tilde{\boldsymbol{e}}}. \quad (16)$$

This parameter provides a bridge deck profile which has the chosen "modal shape" giving contemporarily the configuration with the minimum values of internal forces. Naturally the satisfaction of these two geometric and static requirements cannot be exact. The procedure has to be iterated a few times, because matrices $[\boldsymbol{D}]$ and $[\boldsymbol{C}]$ as well as arrays $\overline{\boldsymbol{d}}$ and $\overline{\boldsymbol{c}}$ change when a new geometric profile is chosen. Generally the convergence is very fast.

In this way it is possible to sort out an unloaded initial configuration, in order to obtain the required profile after bridge erection. By selecting the geometric shape and by calculating the related parameter a and the specific imposed strains $\boldsymbol{e}_{alw}$, the configuration found after the stay stressing procedure is the required one. From the imposed strains in fact, the value of prestress force and that of elongation $\Delta l_i = e_{alw,i}\, L_i$ of the i-th stay, can be obtained. The previous approaches solve the problem of the initial cable force determination.

3. Variability of stay elongations and the probabilistic model

The elongation Δl_i to be given to each stay depends directly on the imposed strain value e_i found by applying the previous procedure (with any of the three approaches discussed). Nevertheless the value of Δl_i is not a deterministic quantity because it undergoes the peculiarities of prestressing technologies, the instrumentation used to check the displacements during the construction stages and the used equipment. Moreover the temperature variation during the different phases of construction as well as the presence of additional loads on the working site, especially during the stay stressing operation can modify the imposed strain without a preliminary control. So the resultant values of stay elongations can be considered as random variables. It implies that there is a difference between the theoretical value of prestressing forces and stay elongations found by the designer and the actual value during the bridge erection. The variability of prestressing parameters modifies the strain field and consequently the state of stress in the structure. Every characteristic (internal force or displacement) becomes a stochastic variable, related to the main variable that is the stay elongation, and the range of variation can be very important for the structural control and monitoring during construction.

If an elastic model is admitted for the behaviour of stays, with a linear relationship between forces and deformations, then the generic characteristic (internal force or displacement) can be calculated by a linear relationship too. It does not lose any generalization also for the geometric nonlinear behaviour if the linearization via the Ernst modulus is applied. The generic characteristic can be expressed as:

$$s = s_d + k_1 \Delta l_1 + \ldots + k_n \Delta l_n = s_d + \boldsymbol{k}^{\mathrm{T}} \cdot \boldsymbol{y} = s_d + \boldsymbol{y}^{\mathrm{T}} \cdot \boldsymbol{k}, \quad (17)$$

where s – the generic characteristic (internal force or displacement); s_d – the contribution of dead load to that characteristic; $\boldsymbol{y}$ – the array containing the values of all stay elongations Δl_i; $\boldsymbol{k}$ – the array containing the influence coefficients k_i of each stay related to the characteristic s.

A probability density function in the form of a Gaussian curve is chosen in this paper, by considering that the variation of stay elongations is of the same kind of a classical measurement error, being this variation related to the errors induced by the prestressing operations (instrumentation and equipment, operators,…), by the actual temperature on working site and by the actual loads applied during the erection procedures. So by expressing the stochastic variable s through the relation (17), the mean value of the characteristic is given by the first order statistical moment $E[s]$:

$$\mu_s = E[s] = s_d + \boldsymbol{k}^{\mathrm{T}} \cdot E[\boldsymbol{y}] = s_d + \boldsymbol{k}^{\mathrm{T}} \cdot \mu_y, \quad (18)$$

where μ_y indicates the array of the mean values of the imposed elongations to stays, which may be considered coincident with the design value of the parameter. The variance of the characteristic can be written through the second order statistical moment:

$$\sigma_s^2 = E\left[\left(s-\mu_s\right)^2\right] = s_d + \boldsymbol{k}^{\mathrm{T}} \cdot E\left[\left(\boldsymbol{y}-\mu_y\right)\cdot\left(\boldsymbol{y}-\mu_y\right)^{\mathrm{T}}\right]\cdot \boldsymbol{k} = \boldsymbol{k}^{\mathrm{T}} \cdot \left[\Sigma_y\right] \cdot \boldsymbol{k}, \quad (19)$$

where the covariance matrix $[\Sigma_y]$ is defined as follows

$$\left[\Sigma_y\right] = \begin{bmatrix} \mathrm{cov}\left(\Delta l_1, \Delta l_1\right) & \mathrm{cov}\left(\Delta l_1, \Delta l_2\right) & \cdots & \mathrm{cov}\left(\Delta l_1, \Delta l_n\right) \\ \mathrm{cov}\left(\Delta l_2, \Delta l_1\right) & \mathrm{cov}\left(\Delta l_2, \Delta l_2\right) & \cdots & \mathrm{cov}\left(\Delta l_2, \Delta l_n\right) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}\left(\Delta l_n, \Delta l_1\right) & \mathrm{cov}\left(\Delta l_n, \Delta l_2\right) & \cdots & \mathrm{cov}\left(\Delta l_n, \Delta l_n\right) \end{bmatrix}. \quad (20)$$

In some cases, it is more convenient on a mathematical point of view, to work with a single term of the matrix, expressed by the quantity:

$$\mathrm{cov}\left(\Delta l_j, \Delta l_i\right) = \rho_{ji}\, \sigma_{y,j}\, \sigma_{y,i}, \quad (21)$$

and with this notation, the covariance matrix can be expressed in the form:

$$\left[\Sigma_y\right] = \begin{bmatrix} \sigma_{y,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{y,n} \end{bmatrix} \cdot \begin{bmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{y,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{y,n} \end{bmatrix} = \left[\boldsymbol{cs}_y\right] \cdot \left[\rho\right] \cdot \left[\boldsymbol{cs}_y\right], \quad (22)$$

where ρ_{ij} represents the coefficient of correlation between stay internal forces, through the values of stay elongations. By substituting Eq (22) into Eq (19) the variance of the generic characteristic (internal force or deformation) can be simplified as:

$$\sigma_s^2 = \boldsymbol{k}^{\mathrm{T}} \cdot \left[\boldsymbol{cs}_y\right] \cdot \left[\rho\right] \cdot \left[\boldsymbol{cs}_y\right] \cdot \boldsymbol{k}. \quad (23)$$

If the array $\boldsymbol{y}$ represents the uncorrelated random variables, then the coefficients of correlation are obtained:

$$\rho_{ij} = 1 \text{ if } i = j;\ \rho_{ij} = 0 \text{ if } i \neq j. \quad (24)$$

By contrast, if the array $\boldsymbol{y}$ represents the correlated random variables, then a decay random model can be used in which the covariance matrix of the imposed strains is influenced by a negative exponential law. This decay model expresses the circumstance for which each stay elongation is strongly influenced by the mutual effect due to the stressing operation of the nearest stays but it is less influenced by the stressing operation of the far stays. So, the negative exponential law can be modulated by the coefficient z_{ij}, λ_{ij} and by the mutual distance among the stays Δx_{ij}:

$$\rho_{ij} = 1 \text{ if } i = j; \rho_{ij} = z_{ij} e^{-\frac{\Delta x_{ij}}{\lambda_{ij}}} \text{ if } i \neq j. \tag{25}$$

The model can be simplified if only two coefficients are used $z_{ij} = z$ and $\lambda_{ij} = \lambda$. Coefficient z represents the influence of the out-of-diagonal terms in the covariance matrix, while coefficient λ is related to the weight of distance among the cable-stays. From the physical point of view, the parameter z represents an inter-relation intensity between the stays. Its value is fallen within the interval [–1; 1], because out-of-diagonal terms cannot be greater than those of the diagonal. The coefficient λ represents a pseudo-distance of influence, because it rules the weight of the distance among cables in the cross terms of the covariance matrix.

In order to introduce a fully stochastic component in the model, a series of simulations can be done to verify the proposed approach. For this purpose different sets of stay elongations can be randomly generated assuming as mean values those calculated in the design analysis. In this way a number of fictitious sets of elongations could represent a stochastic sample of the possible real elongations on the working site. The random sets have to be generated on the base of a Gaussian probability distribution with variation equal to the possible error introduced during the prestressing operations (in the following examples it is assumed as an error of elongations of about 2 cm). Through these sets of elongations, an equivalent number of possible solutions in terms of internal forces (stay forces and deck moments) or displacements may be found on the bridge model, each one different from another. All randomly generated solutions are close to the theoretical one but not exactly coincident with it. The deviation from the theoretical value of moments and deflections can be simply found and the stochastic study of these generated solutions gives the expected variation respect to what determined in the design phase.

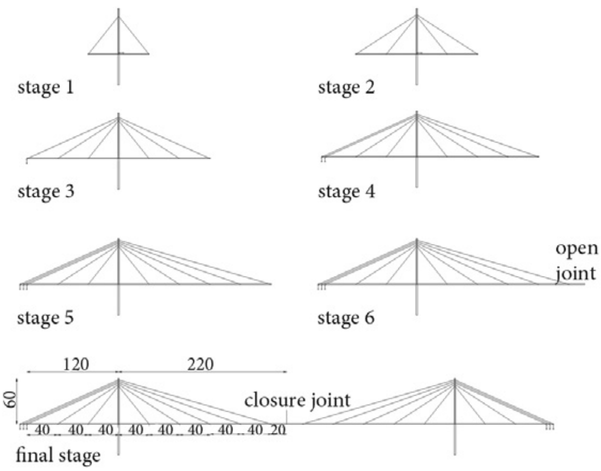

Fig. 2. Sequence of construction stages for the bridge

4. Numerical examples

Numerical examples are presented to show the effectiveness of the procedures previously explained.

4.1. First example

In the first example a bridge with the main girder composed of steel, towers made of concrete and lightweight stays (new-PWS) is considered. The structure is symmetric with respect to the midspan, both in the longitudinal and in the transverse directions. The bridge has a self-anchored deck, with the last three backstays anchored to the ground through a flexible pier that permits longitudinal movements. The deck is supported to the pylon by two vertical stays at the intersection deck-pylon, without any vertical rigid constraints. The bridge main span is 440 m long, while the side spans are 120 m long. The tower is 80 m high and the cable system is composed of 44 stays, 12 being the backstays. In the erection stages an expansion joint is arranged at the midspan. Fig. 2 shows the sequence of construction and the main characteristics of the 2D model of the bridge. The steel girder has area $A = 0.8$ m^2, moment of inertia $I = 1.88$ m^4 and the dead load is $w = 200$ kN/m. The tower is divided into two segments, the one at the base having area $A = 50$ m^2, moment of inertia $I = 350$ m^4 and dead load $w = 1650$ kN/m, while the segment of stay anchorages has area $A = 30$ m^2, moment of inertia $I = 110$ m^4 and dead load $w = 800$ kN/m. Stays have area $A_s = 2{\cdot}0.02061$ m^2, while anchored backstays have area $A_{bs} = 2{\cdot}0.02748$ m^2.

The design requirement for the dead load configuration of the deck is a parabolic pre-camber with 2.00 m of rise at the midspan, very small horizontal displacements of the stiffening girder and the vertical profile of pylons. A finite element model has been implemented and the third approach shown in section 2 has been followed (mixed approach). The geometric profile is not selected a priori but the parabolic "modal shape" is chosen. The parameter a of Eq (16) represents the amplification factor of the parabolic shape (rise at the midspan). For the half structure the vertical displacements of the stay anchorage points are selected as the 8 control points in the central and side spans, while the horizontal displacement of the girder at the intersection with the pylon is selected as 1 horizontal parameter. Moreover 5 horizontal parameters are considered in the anchorage points along the pylon. Earth-anchored stays are considered in a unique group of stays to simplify the procedure, in order to build the influence matrix and to give the pretension value to all backstays contemporarily; so by taking into account the symmetry of the structure only 11 stays are considered, gathered into 9 groups. Nine unknown variables (stay elongations) with 17 control points are considered, because the control points for the static requirements are the 8 points of the stay anchorages and the 9 midpoints between two consecutive stays, where the value of bending moment in the deck is checked.

Starting with a try value of the maximum rise $\Delta f_1 = 2.00$ m of the parabolic pre-camber, a first value of the multiplication factor a is found by Eq (16) and the imposed strain array ***ealw*** is calculated by Eq (13). This first attempt gives a result far from the expected one and a new iteration need. After only three iterations the difference between the achieved configuration and the required one is negligible and the procedure has been stopped (Fig. 3). The deck profile obtained by this process is very close to the reference dead load configuration and the maximum absolute error is about 1.5 mm. Horizontal displacements of the deck are close to zero and the bending moment diagram is balanced between positive and negative values, showing the same negative maximum value as requested by the mathematical constraint for the static requirement. The maximum displacement of the tower is about 9 mm and consequently bending moments of the pylon are negligible, being the axial force the governing internal force.

In Table 1 a comparison between the elastic modulus of cable steel (E_s = 190 GPa) and the equivalent modulus E^*, calculated by the Ernst method (Gimsing 1997) is shown. Table data are also the horizontal stay projection length L_0, the cable force and the axial stress of each stay. It can be observed how the sag effect produces in this case very small errors, if a pseudo-elastic analysis is performed instead of a geometric non-linear approach, being the length of stays limited enough, due to the geometric characteristics of the bridge.

Simulation of the construction sequence

For economical reasons during erection the choice was to provide a definitive adjustment of stays without further elongation variations. The sequence is shown in Fig. 1. Table 2 reports the values of elongation for each stay while Table 3 gives the values of initial cable forces for the six stages of the construction sequence.

As it can be seen, during the different stages of erection the stay elongations remain unchanged while the stay forces change. It means that the initial imposed strain is constant while the mutual influence of stay tensioning affects the actual stress of each stay. In this way the initial operation of stressing does not change and the pretension is applied at once. Before the midspan closure a final regulation with small adjustments always need to achieve the requested camber that is close to the calculated one. Moreover, the final adjustment is convenient in order to recover the successive positioning of the superimposed dead loads on the deck after the stiffening girder is completely assembled.

Fig. 4a shows the geometric profile of the stiffening girder at the different stages of erection, till the final adjustment. It is evident how the stay prestressing is generally preponderant, giving higher cantilever tips with respect to the final position of each segment assembled. It is due to the choice of a single stressing phase of each stay and to the constraints given to the mathematical problem both on displacements and internal force in the mixed procedure implemented.

Fig. 4b shows the bending moment diagrams in all stages, till the final distribution of internal forces is achieved as requested by the procedure. It can be seen how the

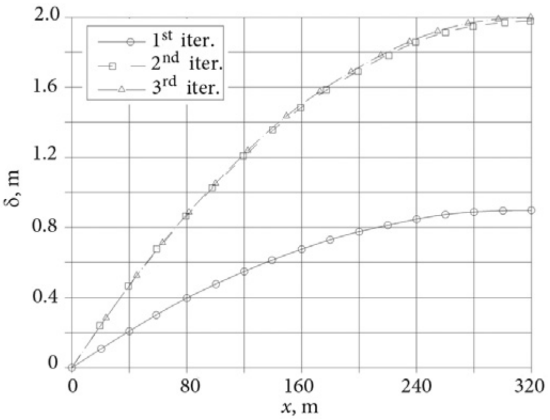

Fig. 3. Sequence of iterations required in the proposed procedure (mixed approach)

Table 1. Comparison between the elastic and the Ernst modulus of stays

Stay	Cable forces, kN	Stay section, m^2	σ_{cable}, MPa	L_0, m	E^*, GPa	Error, %
1	26229	0.05496	477.20	120.00	187.50	–1.30
2	26100	0.05496	474.90	120.00	187.40	–1.30
3	25964	0.05496	472.40	120.00	187.40	–1.40
4	16132	0.04122	391.40	80.00	188.00	–1.10
5	10175	0.04122	246.80	40.00	188.00	–1.10
6	7955	0.04122	193.00	0.00	190.00	0.00
7	10213	0.04122	247.80	40.00	188.00	–1.10
8	14842	0.04122	360.10	80.00	187.40	–1.40
9	19625	0.04122	476.10	120.00	187.50	–1.30
10	25834	0.04122	626.70	160.00	188.00	–1.00
11	30700	0.04122	744.80	200.00	188.20	–1.00

Table 2. Stay elongations (mm) in the erection stages

Stay	Stage						Final
	1	2	3	4	5	6	
1	–	–	–	–	–403	–403	–403
2	–	–	–	–389	–389	–389	–389
3	–	–	–376	–376	–376	–376	–376
4	–	–92	–92	–92	–92	–92	–92
5	266	266	266	266	266	266	266
6	592	592	592	592	592	592	592
7	520	520	520	520	520	520	520
8	–	276	276	276	276	276	276
9	–	–	17	17	17	17	17
10	–	–	–	–150	–150	–150	–150
11	–	–	–	–	–402	–402	–402

final diagram is perfectly balanced in its maximum and minimum values along the entire girder. During the erection instead maximum bending moments can be higher than the final value. It is important to follow the entire sequence in order to assure that stresses in the steel girder are always acceptable, even though the girder is generally dimensioned to face the high stress state induced by moving loads.

The followed procedure is based on a mathematical model, which attempts at obtaining the double result of a good geometric profile and an acceptable bending moment diagram along the deck. However during erection the actual values of stay elongations and stay forces can be different from the theoretical ones previously found. It is due to the effects of temperature, work site, additional loads and construction errors so that the final value of this characteristics can differ respect to the target design value. The imposed strains to stays and consequently the initial cable forces and the elongations are random variables characterized by a determined probability density function (PDF). In this example an assessment of the stochastic variability of the stress and strain field of the bridge is proposed, by applying the relations shown in section 3. The PDF assumed is a normal (Gaussian) random distribution and it is assumed that the target value of imposed stay elongations are those found in the previous analysis. They represent the mean values of the random variables that are all related to the imposed elongations to stays. Moreover, in this example a constant value of variance $\sigma_{yi} = \sigma = 2$ cm is assumed for all the n elongations calculated; this value is in agreement with the data found in the construction practice. For this evaluation the assumption of a constant variance can be acceptable, avoiding complication in the calculations, even though a more sophisticated calculus can be performed with the relations given in the previous paragraph, when the kind of sockets and the actual characteristics of cables and anchorages are known for each stay. By considering the influence of the mutual effect between stay prestressing operations with the distance between stays, the analysis was performed with different values of the coefficient λ and for three different values of coefficient z of Eq (25). Fig. 5a shows the variance of the deck vertical displacement at the midspan; the variance increases with λ but its behaviour is asymptotic,

Table 3. Stay forces (kN) in the erection stages

Stay	Stage						Final
	1	2	3	4	5	6	
1	–	–	–	–	21 741	26 230	26 288
2	–	–	–	18 131	21 490	26 101	26 167
3	–	–	13 512	17 745	21 223	25 965	26 040
4	–	9007	11 529	13 099	14 387	16 133	16 161
5	5889	8540	10 392	10 285	10 217	10 175	10 176
6	7383	9727	9396	8709	8239	7955	7950
7	5819	8857	10 935	11 227	10 920	10 214	10 211
8	–	8694	11 896	14 647	15 675	14 842	14 842
9	–	–	12 423	16 170	18 843	19 625	19 625
10	–	–	–	16 455	20 523	25 834	25 833
11	–	–	–	–	20 734	30 700	30 695

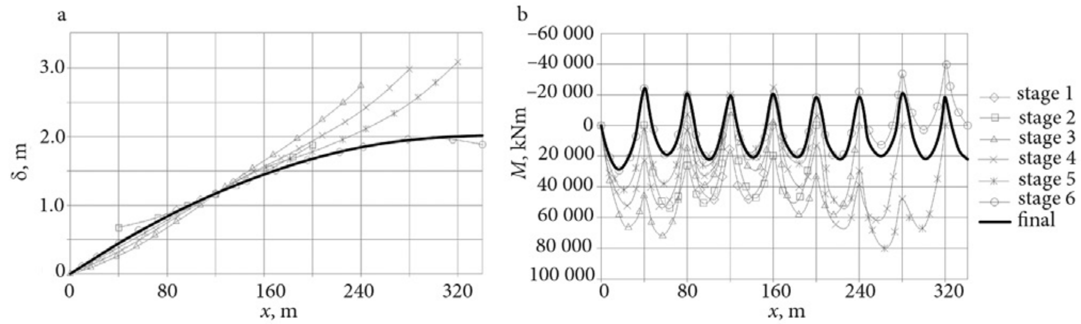

Fig. 4. Results of erection stages: a – profile of the half deck during the erection stages, m; b – Bending moment diagrams for the half deck, kNm

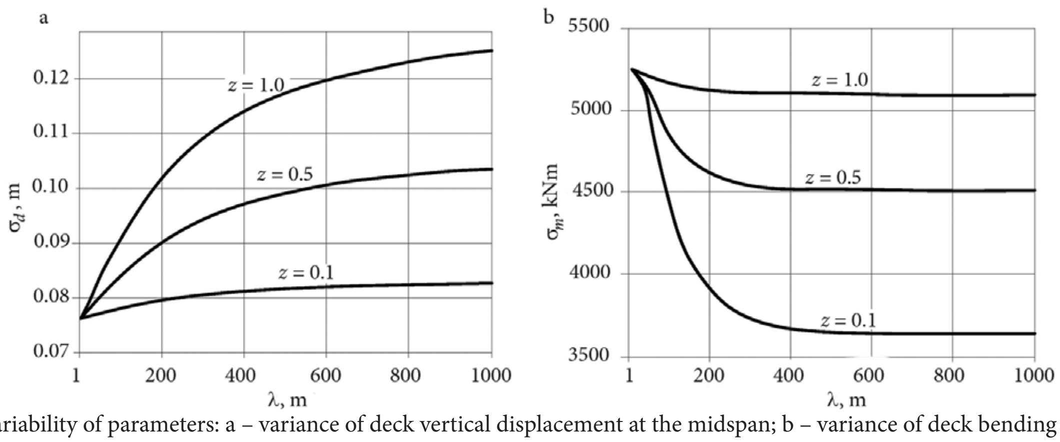

Fig. 5. Variability of parameters: a – variance of deck vertical displacement at the midspan; b – variance of deck bending moment at the midspan

remaining constant the value of variance for high values of λ and small values of *z*. For high values of λ, the curves are more sensitive to the coefficient *z*. The absolute value of displacement variance respect to the required precamber is not negligible, being it higher for higher values of *z*.

Fig. 5b shows the variance of bending moment at the midspan section of the deck. In this case a decreasing value of the variance with λ is found, being the absolute value of the variance smaller when higher is the value of λ. In this case the behaviour is asymptotic too and in practice, for $\lambda > 400$, the variance can be considered constant for any value of *z*. The minimum value of variance is not negligible respect to the mean value of bending moment.

The actual values of λ and *z* are unknown a priori, so it is necessary to check the entire field of variation of these parameters in order to find the maximum possible values of the variance both for displacements and bending moments. Reliable values of them can be found by an accurate control and by monitoring the operations on site. In this way the predictions of the design phase about the possible variations of displacements and forces can be checked in the actual erection sequence.

In this example the fully stochastic component of the model has been applied by randomly generating 100 sets of elongations. The mean values of elongations were assumed from Table 2 and the Gaussian probability distribution has variation equal to 2 cm. With these random values, 100 samples of "real" elongations have been applied to the bridge model, as possible values assumed during stressing operations on site. As a consequence 100 different solutions in terms of deflections and bending moments in the control points have been found, through the influence matrices of the proposed procedure, each one depending on the related random set of elongations. The result of the stochastic study on these fictitious solutions gave the possible deviation of midspan deflection and bending moment, as values to be compared with the previous determination of parameter variability. These variations have to be considered in relation to the mean values of deformed shape and bending moment diagrams given by Figs 4a and 4b that are the diagrams resulting from the design stage. In this case the value of 8.61 cm has been found as the deviation of vertical displacement at the midspan section, while 3860 kNm was the value of bending moment deviation in the same section. These values correspond to curves in Figs 5a and 5b that have an asymptote for $z \approx 0.20 \div 0.25$. It means that the stochastic approach supplies in this case, the prediction of a coefficient *z* of about 0.25, as the realistic value of the parameter to be expected on site.

4.2. Second example

The second example is shown in order to clarify the proposed approach, applying it on the bridge analysed in (Granata *et al.* 2012).

Fig. 6 shows the geometric characteristics of the bridge. The deck is a composite steel-concrete composite box girder. Only the probabilistic approach explained in Section 3 is applied in this case, in order to foresee the field of variation of the horizontal displacement at the top of the pylon and the axial force in the longest stay.

The same Gaussian curve of the previous example is adopted as well as the value of elongation variance $\sigma = 2$ cm, related to the jacking operation. Fig. 7 shows the variation of variance for these two characteristics. The behaviour is very similar to the one obtained for characteristics calculated in the previous example. Particularly, displacement variance remains constant for $\lambda > 100$ m while the stay axial force remains almost constant for $\lambda > 250$ m. Moreover the maximum range of variation of the horizontal displacement with respect the design value is about 2.5 cm, while the maximum range of variation of stay force is about 320 kN. Diagrams of Fig. 7 explore the entire field of variations for the characteristics investigated giving how much the actual values of displacements and forces may differ from those found in the design analysis, due to the errors in the evaluation of the actual stay elongations.

5. Conclusions

1. A methodology for the determination of the initial cable forces in steel cable-stayed bridges has been proposed. The procedure is based on a mixed approach in which the

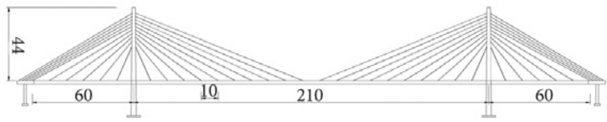

Fig. 6. Geometric layout of the bridge for the second numerical example

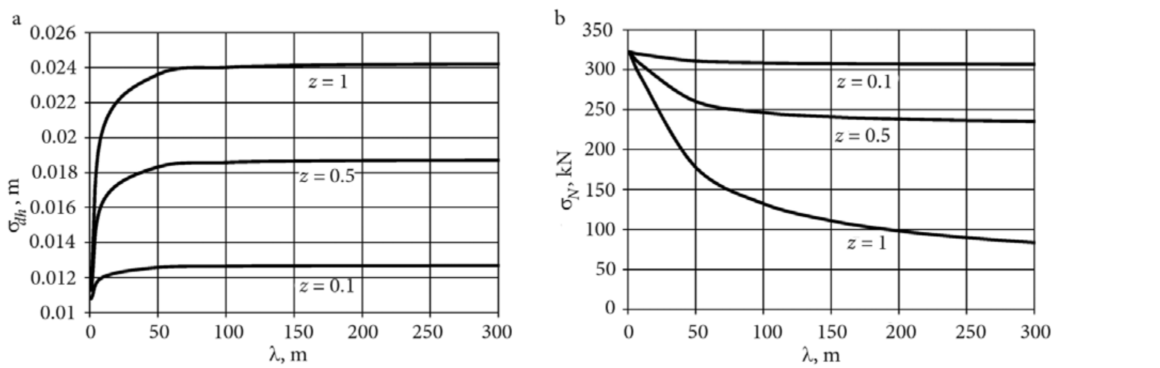

Fig. 7. Parameters variability: a – variance of pylon top horizontal displacement; b – variance of stay axial force for the longest stay

geometric profile of the deck and the required camber are satisfied together with a convenient fixed distribution of bending moments along the stiffening girder. The technique is iterative but it converges in a few steps; moreover it does not require a large computational burden. A numerical example was performed in order to make clear the proposed procedure in the actual sequence of a bridge erection.

2. Nevertheless, in the construction stages it needs to check displacements and internal forces as well as the stay forces. Deviations from the nominal values of loads and from the expected theoretical results, found in the design phase, are usually present in every stage of erection. This fact depends on the temperature, additional loads in the working sites and assembling errors. The stay stressing sequence is very sensitive to the variability of the main parameters involved that are the values of imposed elongations to stays. By considering them as random variables, a probabilistic study of these variations was proposed, by calculating the variance of the main parameters involved. The variability of the prestressing parameters modifies the stress and strain field of the bridge. They can be considered by a stochastic model which has to be not too complex, in order to use it for reliable predictions to be checked by monitoring the bridge on site during the erection stages. The results, found on the two numerical examples presented, show the importance of displacement and bending moment variations with respect to the theoretical configuration.

References

Arici, M.; Granata, M. F.; Recupero, A. 2011. The Influence of Time-Dependent Phenomena in Segmental Construction of Concrete Cable-Stayed Bridges, *Bridge Structures* 7(1): 125–137. http://dx.doi.org/10.3233/BRS-2011-030

Chen, D. W.; Au, F. T. K.; Tham, L.G.; Lee, P. K. K. 2000. Determination of Initial Cable Forces in Prestressed Concrete Cable-Stayed Bridges for Given Deck Profiles Using the Force Equilibrium Method, *Computer and Structures* 74(1): 1–9. http://dx.doi.org/10.1016/S0045-7949(98)00315-0

Fujisawa, N.; Tomo, H. 1985. Computer-Aided Cable Adjustment of Stayed Bridges, *IABSE Proc.* 9(P92): 181–190

Gimsing, N. J. 1997. *Cable Supported Bridges: Concept and Design*. 2nd edition. Chichester: John Wiley & Sons, 480 p.

Granata, M. F.; Margiotta, P.; Recupero, A.; Arici, M. 2013a. Concrete arch Bridges Built by Lattice Cantilevers, *Structural Engineering and Mechanics* 45(5): 703–722. http://dx.doi.org/10.12989/sem.2013.45.5.703

Granata, M. F.; Margiotta, P.; Arici, M. 2013b. Simplified Procedure for Evaluating the Effects of Creep and Shrinkage on Prestressed Concrete Girder Bridges and the Application of European and North American Prediction Models, *Journal of Bridge Engineering ASCE* 18(12): 1281–1297. http://dx.doi.org/10.1061/(ASCE)BE.1943-5592.0000483

Granata, M. F.; Margiotta, P.; Recupero, A.; Arici M. 2013c. Partial Elastic Scheme Method in Cantilever Construction of Concrete Arch Bridges, *Journal of Bridge Engineering ASCE* 18(7): 663–672. http://dx.doi.org/10.1061/(ASCE)BE.1943-5592.0000396

Granata, M. F.; Margiotta, P.; Arici, M.; Recupero, A. 2012. Construction Stages of Cable-Stayed Bridges with Composite Deck, *Bridge Structures* 8(4): 93–106. http://dx.doi.org/10.3233/BRS-120044

Kasuga, A.; Arai, H.; Breen, J. E.; Furukawa, K. 1995. Optimum Cable-Force Adjustments in Concrete Cable-Stayed Bridges, *Journal of Structural Engineering ASCE* 121(4): 685–694. http://dx.doi.org/10.1061/(ASCE)0733-9445(1995)121:4(685)

Martinez y Cabrera, F.; Malerba, P.G. 1999. Aspects of the Structural Behaviour of the Cable Stayed Bridges Approaching the Airport of Malpensa 2000 (in Italian), *Atti Giornate AICAP'99*: Torino, Italy. 185–194.

Martins, A. M. B.; Simões, L. M. C.; Negrão, J. H. O. 2015. Cable Stretching Force Optimization of Concrete Cable-Stayed Bridges Including Construction Stages and Time-Dependent Effects, *Structural and Multidisciplinary Optimization* 51(3): 752–772. http://dx.doi.org/10.1007/s00158-014-1153-4

Negrão, J. H. O.; Simões, L. M. C. 1997. Optimization of Cable-Stayed Bridges with Three Dimensional Modelling, *Computer and Structures* 64(14): 741–758. http://dx.doi.org/10.1016/S0045-7949(96)00166-6

Simões, L. M. C.; Negrão, J. H. O. 2000. Fuzzy Optimal Design of Cable-Stayed Bridges by the Two-Phase Method, in *2nd International Conference on Decision Making in Urban and Civil Engineering*: 20–22 November 2000, Lyon, France. 1189–1200

Straupe, V.; Paeglitis, A. 2012. Analysis of Interaction Between the Elements in Cable-Stayed Bridge, *The Baltic Journal of Road and Bridge Engineering* 7(2): 84–91. http://dx.doi.org/10.3846/bjrbe.2012.12

Troitsky, M. S. 1977 *Cable-Stayed Bridges.* Crosby Lockwood Staples: London, UK. 385 p.

Wang, P. H.; Tseng, T. C.; Yang, C. G. 1993. Initial Shape of Cable-Stayed Bridges, *Computer and Structures* 46(6): 1095–1106. http://dx.doi.org/10.1016/0045-7949(93)90095-U

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