



CRACKS ANALYSIS IN THE REINFORCED CONCRETE PIPES

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Abstract. In this paper, the analysis of cracking due to tensile stress in a reinforced concrete pipe is presented. The tensile strength of concrete in case of the member of a high relative curvature is significantly different from the strength of uncurved elements. The current state of research indicates that only methods of nonlinear fracture mechanics lead to a satisfactory agreement between the results of calculations and experimental studies, and allow for compiling correctly the influence of basic parameters of stretched concrete and geometrical dimensions on the load capacity of crush pipes made from concrete. Obtaining consistent results of experimental and numerical research is still a very complicated issue. This paper presents the results of the experimental investigation performed on three pipes and the numerical model of the analysed element of the pipes made using the ATENA (Advanced Tool for Engineering Nonlinear Analysis) program. The numerical analyses of cracks were compared with the results of experimental studies.

Keywords: cracks, crushing, nonlinear fracture mechanics, numerical analyses, reinforced concrete pipe, tensile strength of concrete.

1. Introduction

Small bridges and culverts constitute the majority of road structures on the existing network of roads and streets.

Each project is a bridge structure, regardless of the solution the used material is obliged to include:

- the quality and durability of materials;
- a way of protection against external conditions;
- a functionality and easy access to the individual components and their equipment (Balcerek 2011; Biliszczuk, Barcik 2011, Koch *et al.* 2015).

Particular attention shall be paid to the drainage of bridges, and thus, to the durability of the pipe elements (Chen *et alia* 2010). Durability is understood as the ability to use an object while maintaining the characteristics of strength and consumables, i.e. meeting the conditions of ultimate limit states and serviceability limit states.

Compliance of theories of concrete cracking with the results of experimental studies is much weaker than in the case of the theory of strength limit. The cracking issue is complex and depends on several poorly controlled factors, for example tensile strength of concrete (Słowik, Smarzewski 2012; Branko *et al.* 2013 Buda-Ozog, Skrzypczak 2015). The tensile strength of concrete in members of high relative curvature is significantly different from the strength of uncurved elements. The current state of research indicates

that only methods of nonlinear fracture mechanics lead to a satisfactory agreement between the results of calculations and experimental data. These methods allow for compiling correctly the influence of basic parameters of stretched concrete and geometrical dimensions on the load capacity of crush pipes made of concrete (Woliński 1991; Wallbrink *et al.* 2005; Abel 2016).

The application of the band microcracks model to describe the scale effect at the failure of concrete pipes was presented by Bazant and Cao (1985). However, obtaining consistent results of experimental and numerical research is still a very complicated issue. This article presents the analysis of cracks in the tensile concrete for reinforced concrete pipe. The relationships between the tensile strength of concrete and a relative curvature of the pipe cross-section are presented in the article.

The results of the experimental investigation performed on three pipes and the numerical model of the analysed element of the pipes made in the Advanced Tool for Engineering Nonlinear Analysis (ATENA) program are presented. The numerical analyses of the cracks were compared to the results of experimental studies. By the performed analyses and the experimental tests, the effect of the curvature of the cross-section on the tensile strength of concrete in the reinforced concrete pipes at failure was determined.

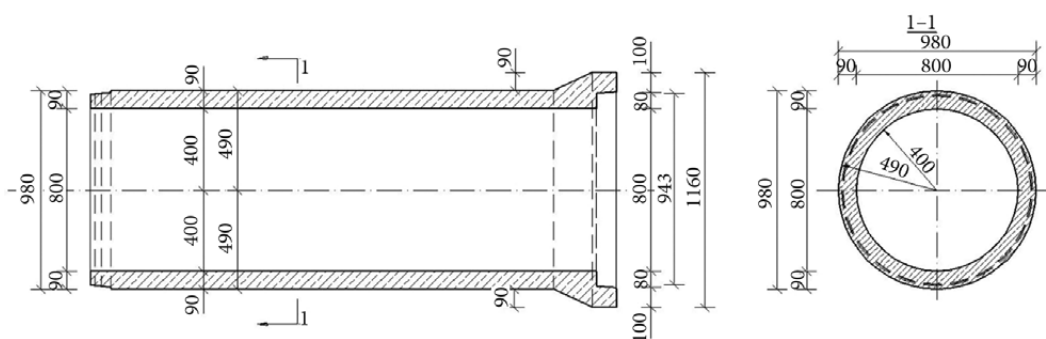


Fig. 1. The analysed reinforced concrete pipes

2. The analysed reinforced concrete pipes

The subject of the analysis was reinforced concrete pipe with an inner diameter of 800 mm and wall thickness of 90 mm. Specific dimensions of the pipes are shown in Fig. 1.

The pipes were made of concrete of the mean compressive strength $f_{cm} = 47.8 \text{ N/mm}^2$, the standard deviation $s = 4.19 \text{ N/mm}^2$. The mean strength of the concrete was obtained using a compressive test performed on four 150 mm size cubes. In the analysis of concrete structures the characteristic compressive strength f_{ck} is calculated from the formula (EN-206:2013 Concrete. Specification, Performance, Production and Conformity) – Eq (1):

$$f_{ck} = f_{cm} - 4 = 47.8 - 4 = 43.8 \text{ N/mm}^2, \quad (1)$$

where f_{ck} – the characteristic compressive strength, N/mm^2 ; f_{cm} – the mean compressive strength, N/mm^2 .

The modulus of elasticity – E_c in N/mm^2 at the concrete age of 28 days was obtained using a test for the stress-strain relation for uniaxial compression. The medium modulus of elasticity for the analysed concrete was $E_c = 28\,600 \text{ N/mm}^2$.

The pipes were reinforced with bars of 10 mm in diameter, steel RB500W of C class with reinforcement area of 872 mm^2 . The adopted cover of main reinforcement and the reinforcement spacing is shown in Fig. 2.

The analysis of the crack was performed for the diagram of test load of pipes, adopted according to recommendations of EN-1916:2005/AC Concrete Pipes and Fittings Unreinforced, Steel Fibre and Reinforced. The adopted load diagram is shown in Fig. 3.

Calculation and analysis were applied to cracks marked in Fig. 3, points A and B.

According to the recommendations in EN-1916:2005/AC and the producer’s requirements, except for the condition of strength capacity, reinforced concrete pipes are obliged to meet the cracking condition. Reinforced concrete pipe shall withstand a test load due to cracking failure force F_c equal to 0.67 without the emergence of fixed cracks of the surface greater than 0.3 mm in the continuous length of 300 mm or more in stretched zones of concrete.

3. Effect of curvature of the cross-section in the tensile strength of concrete

The energy condition of unstable crack development, i.e. failure without considering the influence of curvature on the section is as follows (Eq (2)):

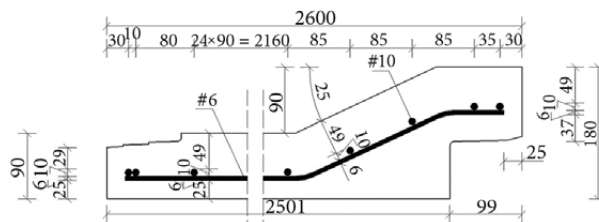


Fig. 2. The adopted cover of main reinforcement and the reinforcement spacing for the analysed the pipes

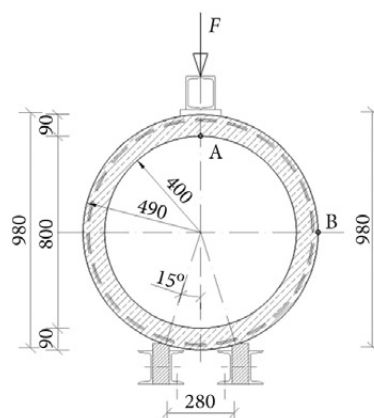


Fig. 3. Load diagram of pipes

$$\frac{\partial U}{\partial a} = G_F l, \quad (2)$$

where $U = U_1 + U_2$ – potential energy release rate of deformation in the volume of the zone of destruction U_1 and the adjacent concrete U_2 , Nm; G_F – fracture energy, N/m; l – length of the member, mm; a – length of cracks, mm.

According to the Fig. 4, it can be assumed that Eq (3):

$$\frac{\partial U}{\partial a} = l \frac{\sigma}{2E_{ct}} (2a + w_c), \quad (3)$$

where σ – mean normal stress, N/mm^2 ; E_{ct} – modulus of elasticity of concrete, N/mm^2 ; w_c – bandwidth of micro cracks, mm.

The ratio of the maximum stress was calculated taking into account the curvature of the axis σ^* to the stresses set for the element with a straight axis σ by the Eq (4):

$$\frac{\sigma^*}{\sigma} = \frac{t}{6r'} + \frac{2r'z}{t(r'-z)} \frac{J}{J^*} = \beta \left(\frac{t}{r} \right), \quad (4)$$

where σ^* – curvature stress, N/mm²; t – thickness, mm; β – constant factor, –; $z = r' - \frac{d}{2}$, mm; d – inner diameter of the pipe, mm; r – radius of curvature, mm, are shown in the Fig. 4; J – moment of inertia of the element with straight axis, mm⁴; J^* – moment of inertia of the element with flexural curved, expressed by Eq (5), mm⁴:

$$J^* = \int_A \frac{r}{r+z} z^2 dA. \quad (5)$$

Tensile strength of concrete taking into account the impact of the relative curvature of the pipe cross-section $\frac{t}{r} - f_{ctm}^*$, can be represented as follows (Eq (6)):

$$f_{ctm}^* = f_{ctm} \sqrt{\frac{2}{\beta} \left(\frac{l_{ch}}{2t + w_c} \right)^2}, \quad (6)$$

where l_{ch} – characteristic length of concrete (Eq (9)), m: $l_{ch} = \frac{G_F E_C}{f_{ct}^2}$, f_{ctm} – tensile strength of concrete (Eq (7)), N/mm²:

$$f_{ctm} = 0.3(f_{ck})^{\frac{2}{3}} = 0.3(43.8)^{\frac{2}{3}} = 3.73 \text{ N/mm}^2. \quad (7)$$

In the absence of experimental value of the fracture energy – G_F , it can be determined in accordance with the Model Code 2010 by the formulas:

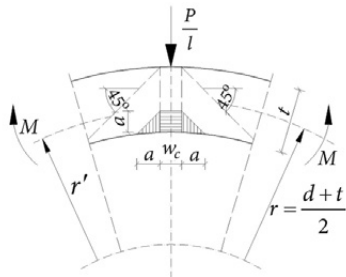


Fig. 4. Diagram of crack propagation on the thickness of the pipe wall

$$G_F = 73(f_{cm})^{0.18} = 73(47.8)^{0.18} = 146.43 \text{ N/mm}^2, \quad (8)$$

$$l_{ch} = \frac{G_F E_C}{f_{ct}^2} = \frac{146.43 \cdot 28.6}{3.73^2} = 301 \text{ mm}. \quad (9)$$

For $r \approx 5 \cdot t$, the moment of inertia of the element with the curved axis, amounts to:

$$J^* = 1.009 \frac{lt^3}{12} = 61.3 \cdot 10^6 \text{ mm}^4. \quad (10)$$

Substituting the formula (4) we received $\beta = 1.27$.

For a pipe with an internal diameter $d = 800$ mm and a wall thickness $t = 90$ mm, made of concrete C35/45, the bandwidth of micro cracks $w_c = 26$ mm, the tensile strength of concrete was obtained by crushing, taking into account the impact of the relative curvature of the cross section:

$$f_{ctm}^* = 3.73 \sqrt{\frac{2}{1.27} \left(\frac{301}{2 \cdot 90 + 26} \right)^2} = 5.66 \text{ N/mm}^2. \quad (11)$$

Development of cracking at point B depends on the tensile strength of concrete. After crossing the stress $\sigma_c > f_{ctm}^*$, the uncontrolled development of cracks follows. For the analysed pipes, the stress $\sigma_c = 5.66$ N/mm² is obtained for the load $F = 47$ kN/m.

For the load $F = 47$ kN/m the internal forces and stress in the analysed pipe are shown in Fig. 5.

4. Results of experimental test

The test was carried out on three pipes made of a single batch of concrete and reinforced in the same way. The test was done at Civil Engineering, Environmental Engineering and Architecture Lab, Rzeszow University of Technology.

A series of step loaded static tests were aimed at producing successive damage to the pipes. The development of cracks under different loading conditions was analyzed during the test. The setup of static testing is shown in Fig. 6.

Test results of experiments for three tested pipes are summarized in Table 1. The table shows the values of strength for that the first scratch and crack of width 0.3 mm over a length greater than 300 mm were observed.

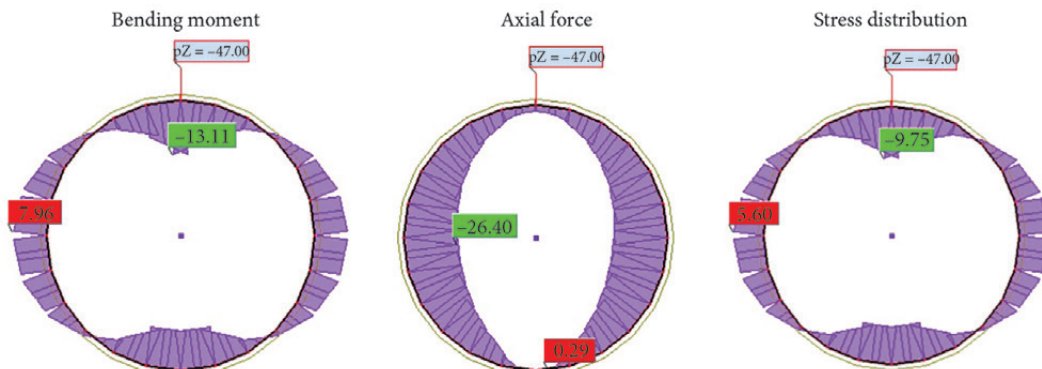


Fig. 5. The internal forces and stress values in the analysed pipe for the load $F = 47$ kN/m



Fig. 6. The setup of static testing

The failure was caused by the rapid rise of cracks at the point B (Fig. 3) and compression failure at the point A. Figure 7 presents the crack width of 0.1 mm at the point B and the failure of tested pipes.

5. Numerical model of the analysed pipe

Finite element method was applied to the construction of the numerical model of the analysed pipes.

FEM-based programming of ATENA was used (Červenka *et al.* 2014). A numerical model was considered in three-dimensional stress state. To solve static problems of reinforced concrete pipes, calculation procedure based on Newton – Raphson iterative method was applied (Crisfield 1997). Newton-Raphson method keeps the load increment unchanged and iterates displacements until equilibrium is satisfied within the given tolerance.

To model the concrete the material model SBETA (CCSbetaMaterial), proposed by ATENA (Advanced Tool for Engineering Nonlinear Analysis) was used. The material model SBETA includes the following effects of concrete behavior:

- non-linear behavior in compression including hardening and softening;
- fracture of concrete in tension based on the nonlinear fracture mechanics;
- biaxial strength failure criterion;
- reduction of compressive strength after cracking;
- tension stiffening effect;
- reduction of the shear stiffness after cracking;
- two crack models: fixed crack direction and rotated crack direction.

The perfect bond between concrete and reinforcement was assumed within the smeared concept.

However, on a macro-level, a relative slip displacement of reinforcement on concrete over a certain distance can arise if concrete is cracked or crushed. This behavior corresponds to a real mechanism of bond failure in case of the bars with ribs.

The model of material was adopted by constitutive characteristics obtained from experimental tests. The basic constitutive characteristics of concrete are shown in Table 2.

For modeling the main reinforcement, the material model “reinforcement”, proposed by ATENA was used. The model of elastic-plastic material, with characteristics corresponding to steel RB500W, was used. The characteristics of steel reinforcement are summarized in Table 3.

Table 1. Results of the experimental test of three pipes

Element	Crack loading	Loading for crack	Failure loading
		0.3 mm	
		kN/m	
P 1	63.6	76.0	148.8
P 2	60.0	71.6	132.0
P 3	62.8	74.8	129.6
mean	62.1	74.1	136.8

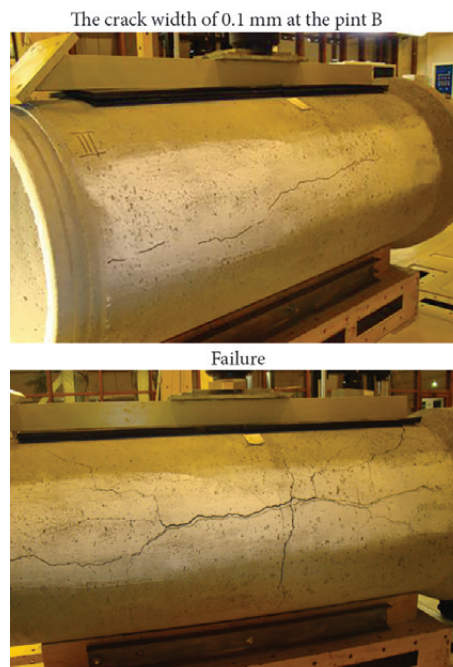


Fig. 7. Pipe during the experimental test

Table 2. The basic material characteristics of material model “concrete”

Tangent Modulus, N/mm ²	28 600
Tensile Strength, N/mm ²	5.66
Fracture energy, N/m	146.43
Uniaxial Maximum Compressive Stress, N/mm ²	48.6
Uniaxial Maximum Compressive Strain at Above Stress	0.00124
Uniaxial Ultimate Compressive Stress, N/mm ²	29.8
Uniaxial Ultimate Compressive Strain	0.0035
Poisson Ratio	0.20

Table 3. The basic material characteristics of material model “steel reinforcement”

Young's Modulus, N/mm ²	200 000
Initial Yield Stress, N/mm ²	500
Poisson Ratio	0.3

The analysed pipes were loaded by forces of values corresponding to the experimental tests. This numerical model of the analysed pipe is shown in Fig. 8.

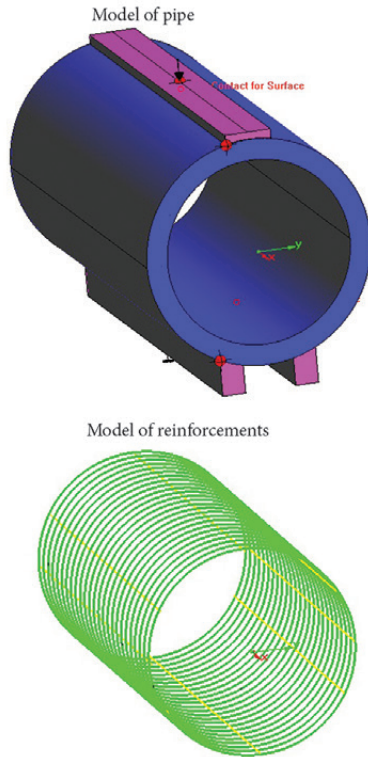


Fig. 8. Numerical model of the analysed pipe

Development of cracks depending on the applied load is shown in the Fig. 9.

Stress distribution depending on the applied load is shown in the Fig. 10.

6. Comparison of the experimental and numerical results

The applied load depending on the cracks observed at point B from the experimental and numerical test is summarized in Table 4.

In all the analysed cases, the cracking load from the numerical model was greater by about 9% in comparison to the cracking load from the experimental test. The development of scratches obtained through the experimental test is analogous to that obtained using the numerical model. The mean load for cracks of the width of 0.2 mm obtained from the numerical analysis is higher by about 10% in comparison to the experimental test.

7. Conclusions

1. Summing up the performed numerical analysis and experimental test results, the relationship between the geometric dimensions of the pipes and their load capacity can be noticed.
2. Analytically determined tensile strength of concrete taking into account the impact of the relative curvature of the pipe cross-section is almost 65% higher than it is proposed in general rules on design of concrete structures.
3. The conducted experimental test indicates that the concrete tensile strength for the tested elements was even

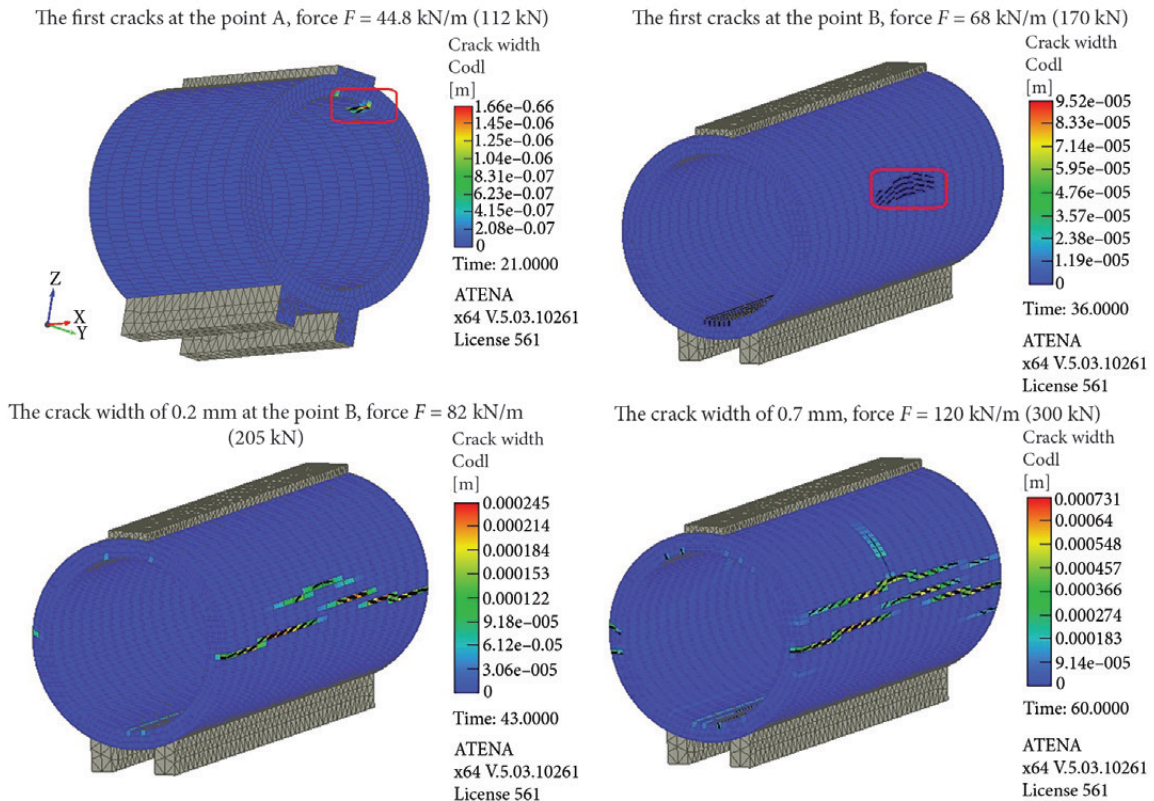


Fig. 9. Development of cracks and deformation depending on the applied load

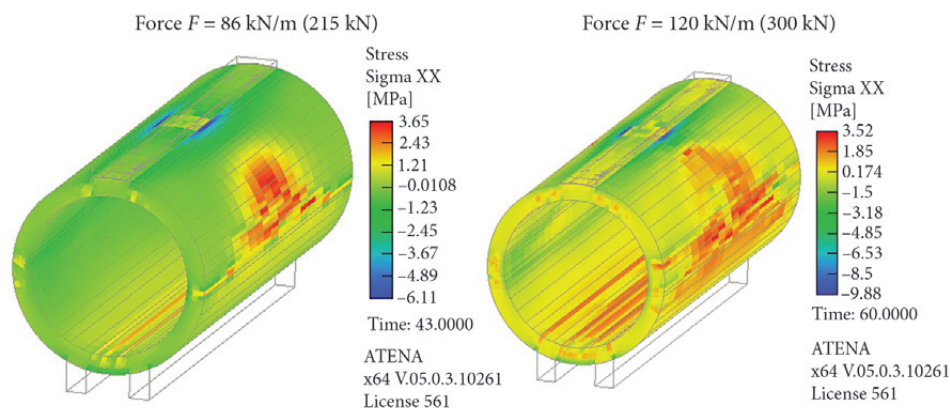


Fig. 10. Stress distribution depending on the applied load

higher than adopted. This resulted in a greater resistance to the elements to the cracking test.

4. The analysis demonstrated that the correct specification of the concrete tensile strength at failure taking into account the effect of the curvature of the cross-section is important for the optimal design of pipe elements.

5. The experimental study and the analysis are preliminary studies designed to determine the effect of geometrical parameters of pipes on their tensile strength and the optimal reinforcement.

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Table 4. The applied load depending on the cracks at point B

Cracks at point B	Applied load, kN/m	
	Experimental test	Numerical model
First crack	62.1	68.0
Crack width of 0.2 mm	74.1	82.0

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