DESIGN PROCEDURES FOR FOOTBRIDGES SUBJECTED TO WALKING LOADS: COMPARISON AND REMARKS

Alberto Maria Avossa[1✉], Cristoforo Demartino[2], Francesco Ricciardelli[3]

[1, 3]*Dept of Civil Engineering, Design, Building and Environment, University of Campania "Luigi Vanvitelli",
Via Roma, 29, Aversa, Caserta, 81031, Italy,*
[2]*College of Civil Engineering, Nanjing University, Hankou Road, 22, Nanjing Jiangsu, 210093, P.R. of China,
E-mails:* [1] *albertomaria.avossa@unicampania.it;* [2] *cristoforo@njtech.edu.cn;* [3] *friccia@unicampania.it*

Abstract. This paper aims at pointing out some misconceptions concerning the evaluation of the walking-induced dynamic response of footbridges, and their impact on design procedures. First, a review of the existing Code provisions is briefly presented. In particular single-walker models and multiple-walker models are addressed; in doing so, models originally presented in different forms are made homogeneous for the purpose of comparison; their limits of applicability and advantages are pointed out. Then, the response of six steel box girder footbridges with different spans is evaluated following the provisions of existing Standards and Guidelines, and compared with allowable comfort levels. The comparison showed a wide scatter of the results, revealing some inconsistencies of the procedures, and underlining a clear need for their critical revision.

Keywords: footbridge response models, footbridges, structural vibration assessment, vibration serviceability limit state, walking loading models, walking-induced vibrations.

1. Introduction

Footbridges, especially if built within urban areas, are taking an ever-growing importance both from the functional aspect point of view and for the role of architectural icons they are often given.

The modest live loads considered in the design process allow sizing extremely slender structures, with low stiffness and low damping; this brings the risk of unacceptable pedestrian-induced vibrations. The necessity follows from providing the structural engineer with methods of dynamic analysis and reliable comfort criteria.

The issue of the evaluation of the dynamic response of footbridges subjected to human-induced loads was first introduced more than a century ago by Tilden (1913). This was a pioneering work, where most of the aspects of human loading of structures seem to have already been recognized, though not quantified. However, for many following decades most of the design procedures have been exclusively based on the application of static vertical loads.

Indeed, pedestrian movements produce dynamic forces in three directions, varying in time and space. These depend on the activity carried out (e.g. walking, jogging, running) and on the characteristics of the pedestrian. In addition, they are modified due to interaction with other pedestrians

and due to perception of structural vibrations. In particular, when walking is considered, the maximum level of vibration risk is associated with vertical vibration frequencies in the range of 1.0 Hz to 3.0 Hz and lateral vibration frequencies in the range of 0.5 Hz to 1.5 Hz; longitudinal vibration effects are usually neglected (Živanović *et al.* 2005).

It is known that gait is a non-periodic activity, therefore to different steps of the same walker different force time histories correspond; this aspect is referred to as intra-subject variability of gait. In addition, if the statistics of the gait parameters of different walkers are compared, they are found to differ from one another; this aspect is referred to as inter-subject variability of gait.

In spite of this, the characteristic of the forces exerted by a walker to a footbridge (Ground Reaction Forces – GRFs) was initially studied with a deterministic approach, assuming these to be periodic in time (considering one or more harmonics of their Fourier expansion), and moving with constant velocity along the bridge. Sinusoidal load models derive from a Fourier decomposition of the GRFs, which Bachmann and Ammann (1987) and Bachmann *et al.* (1995) first introduced, and Živanović *et al.* (2005) reviewed; these fail to account for intra-subject variability of gait. As an alternative, stochastic load models have been formulated in which the load is expressed in the frequency

domain through its Power Spectral Density Function (Brownjohn *et al.* 2004; Butz 2006; Ricciardelli 2005; Ricciardelli, Pizzimenti 2007), or in the time domain through the sum of Gaussian-shaped functions with random parameters (Brownjohn *et al.* 2004; Racic *et al.* 2009). Deterministic and stochastic load models all fail to account for the effects induced by the footbridge vibration on gait, and therefore on GRFs.

Furthermore, the action of a single pedestrian is not necessarily the most severe load scenario for a footbridge; it is therefore also necessary to consider the common case of more than one pedestrian crossing the bridge at the same time. This fact gives rise to other load scenarios, such as groups or streams of walkers; in these cases, walking loads differ from one subject to another, and inter-subject variability must be considered.

In 1999 and 2000, the two vibration incidents at the Paris Passerelle Solferino and the London Millennium Bridge triggered a major revision of existing knowledge concerning footbridge response to walkers. Both footbridges experienced significant lateral vibrations on their opening day, the causes of which are clearly found in the interaction between the dynamics of the structures and the walkers. As a consequence, in the last fifteen years many valuable scientific papers (Avossa *et al.* 2017; Ingolfsson 2012; Racic *et al.* 2009; Venuti, Bruno 2009; Živanović *et al.* 2005), and Standards and Guidelines have been published, aiming at the evaluation of pedestrian-induced vibrations, also accounting for inter-walker and walker-structure interactions.

Nevertheless, regarding these two last topics, it seems that no significant and definitive result has been reached to be used in the engineering practice. In fact, there is still need for clarifying what real improvements have been achieved in design procedures, as current approaches lead to results that are almost never consistent with each other (Ricciardelli, Demartino 2016). Moreover, only a few cases of comparison among results obtained through different procedures are available in the literature (Van Nimmen *et al.* 2014).

Within this framework, the paper classifies various approaches currently available for the design of footbridges against walking-induced vibrations. A critical analysis of their pros and cons is made, after comparing background hypotheses, the field of applicability, and the results provided. Only models having immediate implication in the design practice are considered, whereas more sophisticated models, especially with respect to interaction phenomena, still under discussion within the scientific community are neglected. The various procedures are finally compared through application to six steel box girder footbridges, with and without a concrete slab, designed according to Eurocodes. The results revealed a clear need for a critical revision of design procedures that, although inspired by the same principles and applying common rules, bring rather different outcomes.

2. Design approaches and walking force modelling

Current approaches to the analysis of walking-induced footbridge vertical and lateral accelerations are based on various load scenarios. These are primarily classified as static (Level 0 approach) and dynamic (Levels 1 – 3) approaches, as shown in Fig. 1. Then a further subdivision is made based on the number of walkers crossing the footbridge.

In particular, Level 1 approaches aim at the evaluation of the footbridge maximum transient acceleration due to a single walker resonant with one of the natural frequencies: the "worst pedestrian" ever crossing the footbridge. Level 2 approaches aim at the evaluation of the footbridge maximum transient acceleration due to the crossing of a group of walkers with a high level of synchronization (a "group of friends" crossing the footbridge together). Level 3 approaches aim at the evaluation of the maximum stationary acceleration induced by a stream of walkers (a "continuous flow" in which the level of synchronization increases with density).

Experimental results show that GRFs are characterized by a rather significant level of randomness, this is the result of intra-subject and inter-subject variability of gait (Eriksson, 1988). Moreover, GRFs are modified by the interaction between the walkers and the structure. Natural (i.e. unaffected) walking frequency is reasonably described by a Gaussian distribution with a mean of 2 Hz and a Standard Deviation of approximately 0.20 Hz (Matsumoto *et al.* 1978, Ricciardelli, Pansera 2010). Neglecting intra-subject variability, GRFs are modelled as periodic in time and moving through space at a constant velocity; so doing, their vertical and lateral components are written as:

$$F_V = W\left[1 + \sum\nolimits_j DLF_{V,j} \sin\left(\omega_{V,j} t + \varphi_{Vj}\right)\right];$$

$$F_L = W\left[\sum\nolimits_j DLF_{L,j} \sin\left(\omega_{L,j} t + \varphi_{Lj}\right)\right], \qquad (1)$$

where W – the weight of the walker (generally taken equal to 700 N), N; $DLF_{V,j}$ and $DLF_{L,j}$ – the jth Dynamic Load Factors (DLFs); $\omega_{V,j}$ and $\omega_{L,j}$ – the vertical and lateral walking frequencies, rad/s; and φ_{Vj} and φ_{Lj} – the vertical and lateral phase lags of the jth harmonic, rad. The values proposed for the first DLF by different Standards and Guidelines are shown in Table 1.
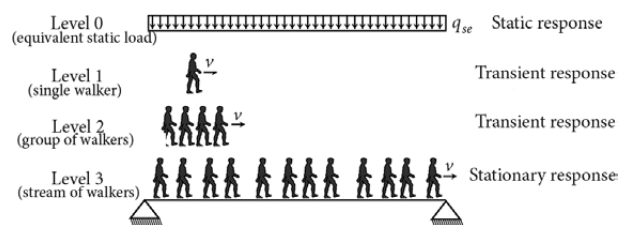

Fig. 1. Approaches for the evaluation of walking-induced response of footbridges

Table 1. *DLFs* proposed by different Standards and Guidelines

	BS5400	OHBDC	AASHTO	AISC	ENV-1995	EN-1995	FIB Bulletin	Sétra	HiVoSS
DLF_{V1}	0.257	0.257	0.5	0.5	0.23	0.286	0.5	0.4	0.4
DLF_{L1}	–	–	–	–	0.057	0.071	0.05	0.05	0.05

2.1. Level 0 approaches: equivalent static live load

Level 0 approaches are based on the definition of an equivalent static load, which gives rise to the same maximum stress level as the pedestrian dynamic load. Some Standards, such as *EN-1991 Eurocode 1:2003 Actions on Structures, BS5400:2006 Part 2: Steel, Concrete and Composite Bridges. Specification for Loads, CHBDC:2006 Canadian Highway Bridge Design Code*, and the Italian *D.M. 04.05.1990: Aggiornamento delle Norme Tecniche per la Progettazione, la Esecuzione e il Collaudo dei Ponti Stradali* allow for a reduction of the live load with increasing span length. Such reduction, however, is never applied when the footbridge is expected to experience high-density traffic. Other Standards, such as Japanese *FDC:1979 Footbridge Design Code*, Swiss *SIA 260:2004 Actions on Structures*, U.S. *UBC: 1984 Uniform Building Code*, and the Italian *D.M. 14.01.2008: Norme Tecniche per le Costruzioni* prescribe load values independent of span length. Level 0 approaches allow to perform safety checks regarding strength, and to control the maximum bridge deflection but are unable to predict accelerations. Moreover, they completely neglect lateral loads. A summary of the live load values reported in different Codes is provided in Fig. 2; these range between 5.6 kN/m² prescribed by *D.M. 04.05.1990* for very short spans, to 1.6 kN/m² prescribed by *CHBDC:2006* for spans over 100 m. Also, *D.M. 14.01.2008* and *UBC:1984* prescribe constant load values of 5 kN/m² and 4.9 kN/m², respectively, corresponding to a density of about 7 walkers/m².

2.2. Level 1 dynamic approaches: single walker

Level 1 are the simplest approaches for the analysis of walking-induced footbridge vibrations. These approaches are based on the evaluation of the transient response to a single resonant walker. Standards and Guidelines consider that such condition is likely to occur for footbridges having vertical vibration frequencies lower than approximately 5 Hz or horizontal vibration frequencies lower than approximately 2.5 Hz, being able to represent the worst case
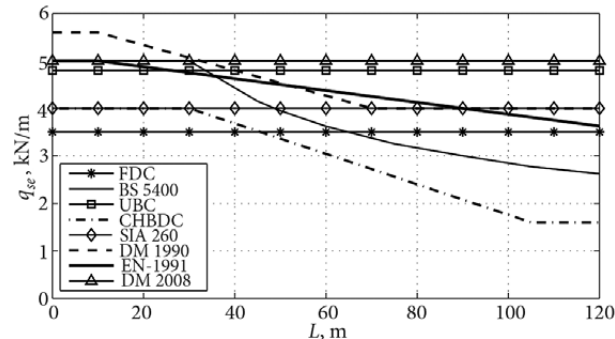

Fig. 2. Live load (nominal) prescribed by different Standards as a function of span length

scenario. In this case, the maximum acceleration $\ddot{y}_{\max,i}(1)$ in all the relevant modes must be calculated as:

$$\ddot{y}_{\max,i}(1) = \ddot{y}_{stat,i} \cdot \varphi(\varepsilon_i) = \frac{DLF_j \cdot W}{2\xi_i m_i} \varphi(\varepsilon_i), \qquad (2)$$

where m_i and ξ_i – the modal mass and damping ratio in the ith mode, kg; $\varphi(\varepsilon_i)$ – a Transient Resonant Response Coefficient (TRRC) accounts for the load motion. The TRRC is a stationary index, usually available for simply supported beams, that is found to depend on the parameter ε_i defined as follows (Ricciardelli, Briatico 2011):

$$\varepsilon_i = \frac{1}{2}\frac{2L}{l}\xi_i = \frac{1}{l}n\xi_i, \qquad (3)$$

where n is the total number of cycles the sinusoidal force applies to the beam; this is equal to twice the ratio of the beam span L to the wavelength $l = \frac{2\pi v}{\omega_j}$ of the load, expressed as a function of the walker velocity v. In particular, large values of ε_i are associated with long spans, with a short stride length, with low vibration modes, and with high damping, in that case the TRRC approaches 1. On the other hand, small values of ε_i are associated with short spans, with a long stride length, with a high vibration mode, and with low damping values. In this case, the TRRC is close to 0 and the response is highly non-stationary.

In Eq (2) DLF_j is the Dynamic Load Factor associated with the closest load harmonic to the ith natural frequency.

The first expression for the TRRC was developed by Fryba (1973), who proposed a closed-form solution of φ based on the wrong assumption that the maximum acceleration occurs when the walker is located at midspan:

$$\varphi(\varepsilon_i) = \frac{\varepsilon_i}{1+\varepsilon_i^2}\left[\exp\left(-\frac{\pi}{2}\varepsilon_i\right)+\varepsilon_i\right]. \qquad (4)$$

Later, Blanchard *et al.* (1977) proposed a numerical solution for the TRRC, evaluated the simply supported beams through numerical integration of the equation of motion; the TRRC is given in the form of a graph for four values of ξ_1 and L in the range of 10 m to 50 m. This model was incorporated into *BS5400* and into *OHA, Ontario Highway Association. Ontario Highway Bridge Design Code, 1983*.

Then, based on the measurements carried out on two footbridges, Allen and Murray (1993) suggested the use of a constant value $\varphi = 0.7$ regardless of the bridge characteristic. This solution was then acknowledged by *AISC* (Allen, Murray 1993) and by *Guide Specifications for Design of*

Pedestrian Bridges, American Association of State Highway and Transportation Officials *(AASHTO:2009)*.

Another approach is to consider the response of a Single Degree of Freedom system loaded by a transient harmonic load located at midspan, acting for a duration equal to the crossing time $T = \dfrac{L}{v}$:

$$\varphi(n, \xi_1) = \left[1 - \exp(-2\pi n \xi_1) \right], \qquad (5)$$

This formulation accounts for the transient behaviour of the load neglecting the mode shape, and the solution is largely on the safe side for all values of ε_i. Thus, it is necessary to tune the solution with a reduction coefficient. The pre-standard version of *CEN 1995. Eurocode 5. Design of Timber Structures. ENV-1995. Comitè Européen de Normalisation,* adopted this solution, with which the reduced values of *DLF* are associated (Table 1).

Also according to the last model, Grundman *et al.* (1993) suggested that the reduction coefficient of 0.6 can be incorporated in Eq (5). *FIB 2005. Guidelines for the Design of Footbridges. Bulletin n. 32. Fédération Internationale du Beton, Lausanne, Switzerland* also adopted their approach - to evaluate the maximum resonant acceleration.

The final version *CEN 2005. Eurocode 5. Design of Timber Structures. EN-1995. Comitè Européen de Normalisation,* suggests φ = 0.7 regardless of the structure characteristics and in agreement with Allen and Murray (1993), combined with values of the *DLFs* higher than those given in *ENV*-1995. It also requires that the vertical response is halved for vibration frequencies larger than 2.5 Hz and that the lateral response is neglected for vibration frequencies lower than 0.5 Hz.

Moreover, Ricciardelli and Briatico (2011) found an approximated closed-form solution for the TRRC of simply supported beams, as follows:

$$\varphi(\varepsilon_i) = \frac{\varepsilon_i}{1 + \varepsilon_i^2} \left\{ \sqrt{1 + \varepsilon_i^2} + \exp\left[\varepsilon_i \left(-\frac{\pi}{2} + \arctan\frac{1}{\varepsilon_i} \right) \right] \right\}, \quad (6)$$

results of which are in good agreement with the numerical solution of Blanchard *et al.* (1977). A more comprehensive comparison among the values of TRRC is developed by Ricciardelli and Demartino (2016). It is shown that the numerical results, provided by Blanchard *et al.* (1977), based on the correct assumption that the maximum acceleration occur when the walker is over the midspan, are well estimated by the closed-form solution provided in the paper. Moreover, Piccardo and Tubino (2012) provided another closed-form solution for the TRRC of beams with any end condition; in case of simple supports this provides the same results as those of Ricciardelli and Briatico (2011). Fig. 3 compares the values of the TRRC arising from application of different approaches, indicating a clear disagreement among one another. The light-grey area shows the typical range of ε_i for common footbridges. Thus, the

use of the above mentioned closed-form solutions allows to overcome inaccuracy in the estimation of TRRC.

2.3. Level 2 dynamic approaches: group of walkers

Level 2 approaches account for a small group of walkers crossing a footbridge, usually less than 10 to 15 pedestrians, having a high level of synchronization and with an average walking frequency close to a natural frequency of the footbridge. The high level of synchronization is justified by the possibility that a small number of pedestrians walks together in a compact manner; despite the low average density (evaluated on the entire deck area), the high level of synchronization makes, in some cases, this condition to be the dominant load case.

Usually, the group is modelled as a *macro-walker*, whose effects in terms of maximum vertical or lateral accelerations are obtained from those induced by a single walker, amplified by the equivalent number of walkers N_e. The latter is defined as the number of perfectly correlated walkers who produce the same maximum acceleration as the actual, partially correlated N walkers. The maximum acceleration response to a group of N walkers $\ddot{y}_{max,i}(N)$, either in the vertical or in the lateral direction, is given as:

$$\ddot{y}_{max,i}(N) = N_e \ddot{y}_{max,i}(1) \Psi(f_i), \qquad (7)$$

where $\Psi(f_i)$ is a reduction coefficient, accounting for the non-coincidence of the vibration frequency f_i and the load harmonic f_j. In fact, the probability that the members of a group of walkers are resonant with the vibration frequency reduces as the latter moves away from the considered load harmonic.

The values of $\Psi(f_i)$ for the vertical and lateral components proposed by various Standards and Guidelines are shown in Fig. 4. The grey and light-grey areas indicate the frequency ranges for the first and second load harmonics of a standard walker. The frequency at which $\Psi(f_i)$ takes the largest values indicates the ranges where resonance is most likely to occur; $\Psi(f_i) = 1$ means that all the equivalent walkers N_e are resonant.

This load scenario was first suggested by *ENV*-1995, that considered a group of three equivalent walkers
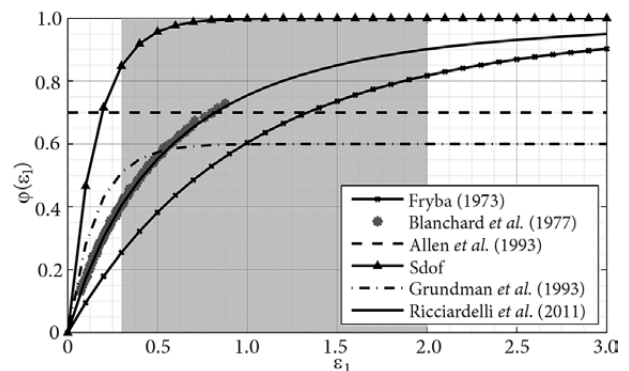

Fig. 3. TRRC values proposed by different authors for simply supported beams (Ricciardelli, Demartino 2016)

(N_e = 3) without specifying the actual number N of walkers to which this corresponds. This load scenario proves to be dominant in the Level 3 approaches only for very small footbridges, with a deck surface area smaller than 37 m^2; for larger footbridges the later approach is in order. Then, *EN*-1995 considers the same N_e = 3 for the vertical direction, and N_e = 2.34 for the lateral direction; in this case it is specified that this corresponds to a group of fewer than 13 walkers. This load scenario is found to be dominant for footbridges with a deck surface area smaller than 22 m^2.

Similarly, *FIB Bulletin n.* 32 considers the case of a group of fewer than 10 walkers, assuming N_e = 3; this gives the same result as the application of the Level 3 method of *ENV*-1995 for a density of 0.36 walkers/m^2. It must be noted that for the same values of N_e, different values of the coefficient $\varphi(\varepsilon_i)$ are given by *ENV*-1995, *EN*-1995, and *FIB Bulletin n.* 32 leading to different results regarding to the maximum acceleration.

2.4. Level 3 dynamic approaches: stream of walker

Level 3 approaches consider a continuous flow of walkers, with a level of synchronization increasing with density δ. In this case, the characteristics of the walkers must be described on a probabilistic basis, and the response needs to be evaluated either in the frequency domain through the Random Vibration Theory, or in the time domain through Monte Carlo simulations. Despite this, Standards and Guidelines erroneously refer to a transient response under deterministic walking load, according to Eq (2).

The first attempt to evaluate the load exerted by a number N of walkers is proposed by Matsumoto *et al.* (1978). He noted that when a footbridge is crossed by a stream of walkers having same frequency and phases uniformly distributed between 0 and 2π (uncorrelated walkers), the Root Mean Square (RMS) response varies with the square root of N walkers (or with the square root of the walkers density $\delta = \dfrac{N}{BL}$, walkers/m^2, B being the width of the deck, m).

According to this result, some Standards and Guidelines consider a conventional density of 0.6 walkers/m^2 and evaluate N by multiplying this density by the area of the deck surface. Then, the equivalent number of walkers N_e

is obtained by multiplying N by a coefficient taking into account the level of correlation among the walkers. Thus, the same equations of Level 2 approaches are used. The first implementation of this approach is that of *ENV*-1995, where such coefficient is set equal to 0.135. Then *EN*-1995 modified it to 0.23 and 0.18 for the vertical and lateral directions, respectively. Finally, *FIB Bulletin n.* 32 suggested the value of 0.225.

Subsequently, the *Technical Guide – Footbridge: Assessment of Vibrational Behaviour of Footbridges under Pedestrian Loading (Sétra:2006)* and *HiVoss:2008 Human Induced Vibration of Steel Structures: Design Guideline and Background Report (HiVoss:2008)* proposed approaches based on the definition of traffic classes, expressed in terms of walker density (values of 0.5 walkers/m^2, 0.8 walkers/m^2 and 1.0 walkers/m^2 for *Sétra:2006*, and of 0.2 walkers/m^2, 0.5 walkers/m^2, 1.0 walkers/m^2 and 1.5 walkers/m^2 for *HiVoSS*). The number of walkers is again calculated multiplying the deck surface area by the density, and the equivalent number of walkers is calculated as:

$$N_e(N) = \begin{cases} 10.8\sqrt{\xi N} & \text{for } \delta < 1 \text{ walker / m}^2 \\ 1.85\sqrt{N} & \text{for } \delta \geq 1 \text{ walker / m}^2 \end{cases}. \quad (8)$$

The walkers given by Eq (8) are considered to give rise to a stationary load, therefore $\varphi(\varepsilon_i)$ is set equal to 1, making the approach closer to the actual physical behaviour.

The coefficients $\Psi(f_i)$ to be used are shown in Fig. 4. Also, *Sétra:2006* gives the possibility to account also for the second harmonic of both vertical (DLF_{V2} = 0.1) and lateral (DLF_{L2} = 0.01) loads, whereas *HiVoSS* accounts for it only for the vertical direction (DLF_{V2} = 0.1).

Only *HiVoSS Guidelines* give method in which the footbridge acceleration is evaluated from the spectral characteristics of the load. The model was calibrated through Monte Carlo simulations, based on numerical integration of the equations of motion of a variety of footbridges crossed by pedestrian streams of different characteristics (Butz 2006). In particular, the 95th fractile of the peak acceleration $\ddot{y}_{\max}(N)$ due to a stream of N walkers is evaluated applying the empirical peak factor k_p (ranging between
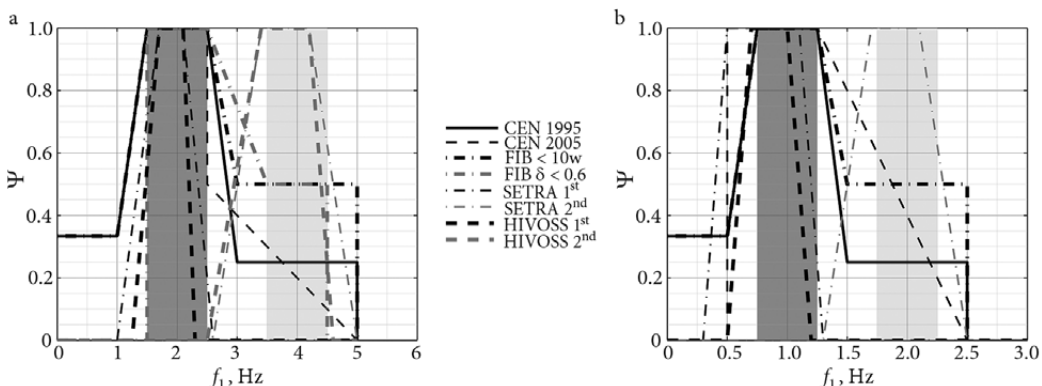

Fig. 4. Comparison of the $\Psi(f_i)$ coefficients suggested by different Standards and Guidelines for (a) vertical and (b) lateral components; the grey and light grey areas indicate the frequency range for the first and second harmonics of GRFs (Ricciardelli, Demartino 2016)

3.63 and 3.92, depending on density) to the RMS response due to N uncorrelated walkers:

$$\ddot{y}_{max}(N) = k_p \sqrt{N} \sigma_a = k_p \sqrt{N} \sqrt{\int_0^\infty (2\pi f)^4 |H_i(f)|^2 S_F(f)df} =$$

$$k_p \sqrt{N} \sqrt{\frac{C}{m_i^2} \frac{k_1(f_i)}{\xi_i^{k_2(f_i)}} k_f} , \qquad (9)$$

where σ_a is the Standard Deviation (STD) of the acceleration due to a single stationary walker, whose action is considered as uniformly distributed on the deck surface; this is obtained by integration of the Power Spectral Density Function of the modal load $S_F(f)$ associated with one walker, multiplied by the square of the modulus of the Frequency Response Function $|H_i(f)|^2$. $S_F(f)$ is assumed to have a Gaussian shape, and the scale parameter C is evaluated using 5000 Monte Carlo simulations carried out for each value of density and of span length considered. In particular, walkers with mass, step frequency, and *DLF* extracted according to probabilistic distributions, and crossing the bridge with Poisson-distributed arrival times are taken into account. Moreover, the walking speed, common to all walkers, depends only on density. The variance of the acceleration response σ_a^2 is expressed in closed form from interpolation of numerical analyses, assuming that vibration frequency coincides with the mean walking frequency. Finally, in the Eq (9), k_f is the variance of the load induced by a single walker (varying in the range of 17 034 N^2 to 35 400 N^2 for the vertical direction and of 1938 N^2 to 3591 N^2 for the lateral direction). Moreover, k_1 is a correction factor (ranging between 0 and 1.4) depending on natural frequency and crowd density, and k_2 is a correction factor depending on the modal force bandwidth, the value of which (ranging between 1 and 1.25) increases with increasing bandwidth. The correction factors $k_1(f_j)$ and $k_2(f_j)$ are given in the form of second-order polynomial functions, whose coefficients are available for crowd densities of $\delta < 0.5$ walkers/m², $\delta = 1.0$ walker/m² and $\delta = 1.5$ walkers/ m², for the vertical and lateral directions, respectively. The role of $\Psi(f_i)$ is embedded in the product of $k_1(f_j)$ and $k_2(f_j)$.

3. Application to case studies of footbridges with steel box girder

With the purpose of comparing the results obtained through the application of the approaches described in Section 2 – *Design Approach and Walking Force Modeling*, the design of prototype steel footbridges has been developed. In particular, bridges with spans of 10 m, 20 m, 40 m, 60 m and 80 m have been considered. To allow a better comparison among the behaviour of the various footbridges, only one deck width equal to 3 m was considered.

The design was carried out according to Eurocodes, that classify footbridges as bridges of the third category, for which an equivalent static vertical uniform live load of 5 kN/m² is prescribed. The structures consisted of simply supported, single- cell steel box girders, with a constant trapezoidal section, without or with a concrete slab 10 cm thick. The cross-sectional geometry was selected by setting the maximum deflection under live loads to 1/400 of the span length (Fig. 5). Grade S355 steel was used in the design. Assuming Class 4 sections, safety checks are carried out using effective geometric properties, according to the
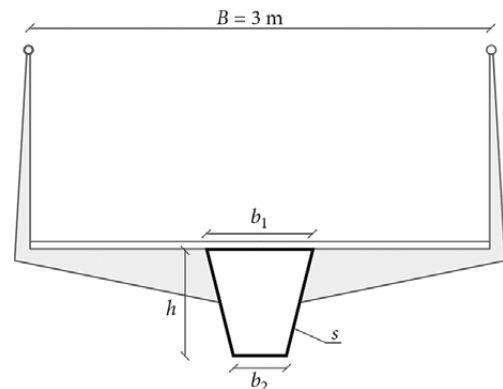

Fig. 5. Cross section of the prototype footbridges

Table 2. Geometric and dynamic properties of the prototype footbridges

Footbridges prototypes without concrete slab							Footbridges prototypes with concrete slab						
	Span length, m							Span length, m					
	10	20	30	40	60	80		10	20	30	40	60	80
h, mm	600	800	1050	1200	1600	2000	h, mm	600	800	1050	1200	1600	2000
b_1, mm	400	600	900	900	1400	2000	b_1, mm	400	600	900	900	1400	2000
b_2, mm	200	300	450	450	700	1000	b_2, mm	200	300	450	450	700	1000
s, mm	5	6	8	10	12	15	s, mm	5	6	8	10	12	15
m, kg/m	865	943	1026	1104	1322	1672	m, kg/m	1465	1543	1626	1704	1922	2272
$f_{1,v}$, Hz	1.72	1.24	0.78	0.58	0.45	0.41	$f_{1,v}$, Hz	1.32	0.97	0.62	0.47	0.38	0.35
$f_{2,v}$, Hz	6.87	4.94	3.14	2.32	1.82	1.64	$f_{2,v}$, Hz	5.28	3.86	2.49	1.87	1.51	1.41
$f_{1,l}$, Hz	0.93	0.57	0.40	0.25	0.21	0.19	$f_{1,l}$, Hz	0.71	0.44	0.32	0.20	0.17	0.16
$f_{2,l}$, Hz	3.71	2.26	1.59	1.01	0.83	0.75	$f_{2,l}$, Hz	2.85	1.77	1.26	0.81	0.69	0.65
ξ, %	1.2	0.8	0.6	0.6	0.4	0.4	ξ, %	1.5	1.2	0.9	0.6	0.4	0.4

procedure suggested by *EN-1993 Eurocode3:2006 Design of Steel Structures.*

Finally, natural frequencies were calculated assuming simple support end conditions, in both vertical and lateral directions. Damping ratios were chosen in agreement with general values available in literature, decreasing with increasing span length. The geometric and dynamic properties of the prototype footbridges, without and with concrete slab are shown in Tab. 2.

3.1. Comparison of the accelerations evaluated for the footbridges without concrete slab

The maximum acceleration induced in the first mode by a single resonant walker (Level 1 approaches) or by a group of walkers (Level 2 approaches) in the vertical and lateral directions are shown in Figs 6–7, respectively. In particular, in both cases, the accelerations decrease with increasing span, as the effect of the reduction of the vibration frequency. The scatter of the results obtained applying different methods is mainly related to the variation of the TRRC coefficient. Furthermore, the Level 1 approach proposed by *EN*-1995 (Fig. 6b) leads to a nil value of lateral acceleration for footbridges with span length larger than 20 m, because their first lateral frequency is lower than 0.5 Hz, therefore outside the amplification range of Fig. 4.

For Level 2 approaches (Fig. 7), on the other hand, the scatter in the results is mainly due to the different values of the coefficient $\Psi(f_i)$. Comparison between the results obtained with Level 1 and Level 2 approaches points out significant differences in both vertical and lateral directions, for the footbridge with a span length of 10 m, and only for the lateral direction, for the footbridge with a span length of 20 m when *EN*-1995 is applied. This outcome is

primarily due to the values, close to 1, taken by $\Psi(f_i)$. On the other hand, the two approaches lead to almost coincident results for footbridges of the larger span, because $\Psi(f_i)$ takes values close to 0.33, making it ineffective the amplification of the single walker acceleration due to some equivalent walkers $N_e = 3$.

The results of the analysis of the response induced by a stream of walkers (Level 3 approaches), considering a fixed density value of 0.6 walkers/m² are shown in Fig. 8; the calculations have also been repeated considering the additional mass brought to the system by the walkers, having the effect of reducing the vibration frequency. In particular, it is noticed that neglecting the contribution of the mass of the walkers brings a visiable increase in the accelerations, especially for shorter footbridges. The overall results obtained through the application of Level 3 approaches, assuming the fixed walker density, is that these bring accelerations are always larger than those obtained with the application of Level 2 approaches. The only exception is the 20 m footbridge analysed according to *ENV-1995* provisions that suggest the implementation of a Level 2 approach for very small footbridges (in particular when the deck surface area is less than 37 m² as already pointed out in Section 2.3). Moreover, for vertical vibrations the results obtained applying Level 3 approaches proposed by *EN-1995* and by *FIB Bulletin n. 32* are similar to each other and larger than those obtained through the application of *ENV-1995*, due to the different values of N_e. Differently, in the case of lateral vibrations, the results obtained by *ENV-1995* and by *FIB Bulletin n. 32* are very close to each other, while *EN-1995* gives larger accelerations for footbridges with span length up to 20 m, and nil values for
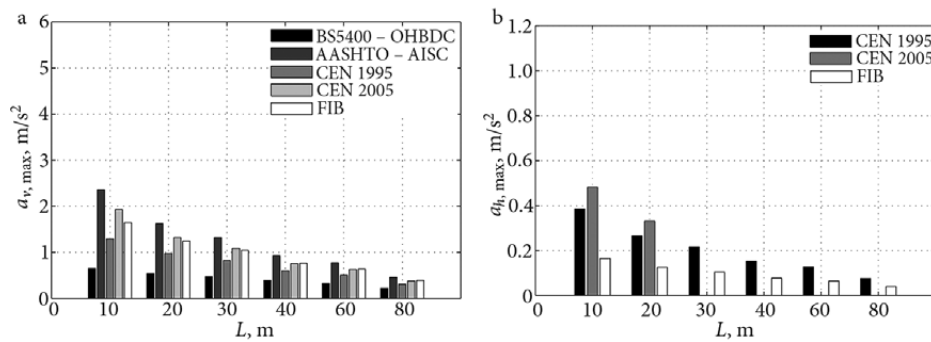

Fig. 6. Vertical and lateral accelerations induced by a single resonant walker
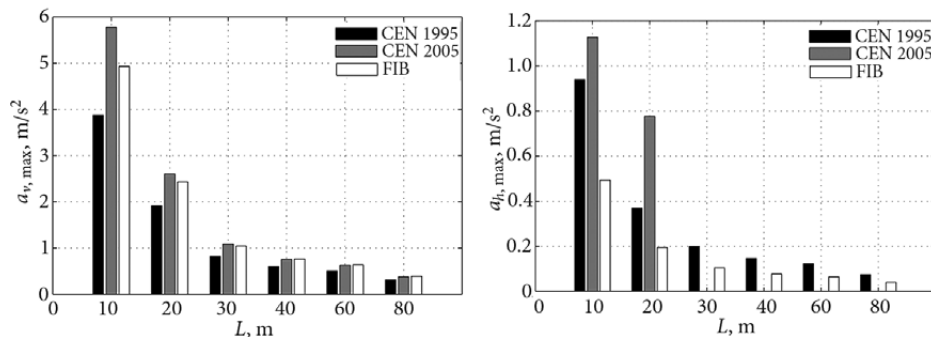

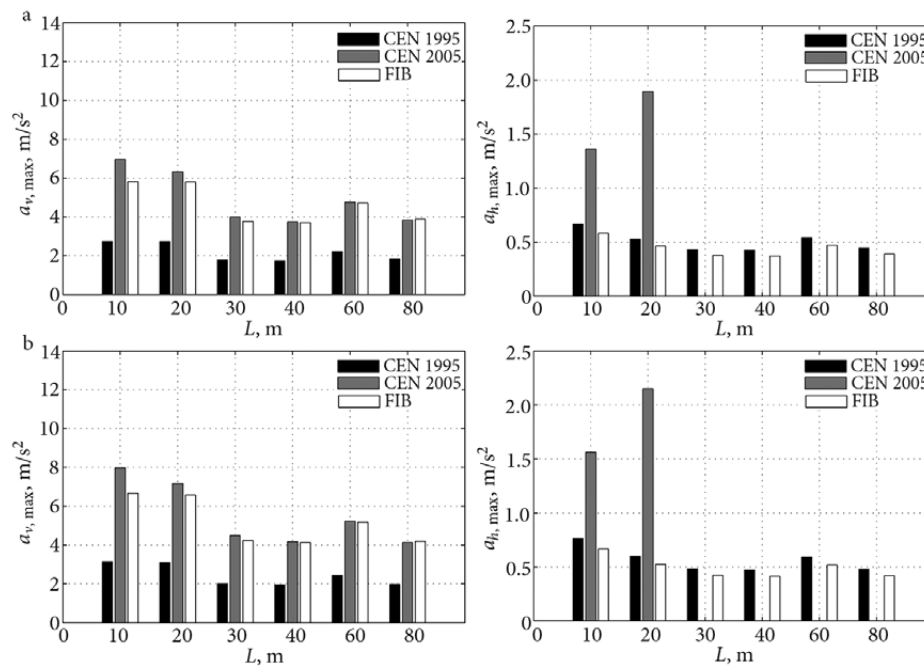**Fig. 7.** Vertical and lateral accelerations induced by a group of walkers

Fig. 8. Vertical and lateral accelerations induced by a flow of 0.6 walkers/m^2: comparison between the response with (a) and without (b) mass of the walkers

Table 3. Vibration frequency variation due to the mass of the walkers (footbridges without concrete slab)

Density δ, walkers/m^2	Mass variation, kg/m	Span length, m											
		10		20		30		40		60		80	
		f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l
		Hz											
Single walker or group of walkers	–	–	1.72^1	0.93^1	1.24^1	0.57^1	0.78^1	0.40^1	0.58^1	0.25^1	0.45^1	0.21^1	0.41^1	0.19^1
Stream of walkers with fixed density	0.6	126	1.60^1	0.87^1	1.16^1	0.53^1	0.74^1	0.38^1	0.55^1	0.24^1	0.43^1	0.20^1	0.40^1	0.18^1
Stream of walkers with variable density	0.2	42	1.68^1	0.91^1	1.21^1	0.55^1	3.07^2	1.56^2	2.28^2	0.99^2	1.79^2	0.81^2	1.62^2	0.75^2
	0.5	105	1.62^1	0.88^1	1.17^1	0.54^1	2.99^2	1.51^2	2.22^2	0.96^2	1.75^2	0.79^2	1.59^2	0.73^2
	0.8	168	1.57^1	0.85^1	1.14^1	0.52^1	2.91^2	1.47^2	2.16^2	0.94^2	1.71^2	0.78^2	1.57^2	0.72^2
	1.0	210	1.54^1	0.83^1	1.12^1	0.50^1	2.86^2	1.45^2	2.13^2	0.92^2	1.69^2	0.77^2	1.55^2	0.71^2
	1.5	315	1.47^1	0.79^1	1.07^1	0.48^1	2.74^2	1.40^2	2.05^2	0.89^2	1.63^2	0.74^2	1.51^2	0.69^2

Note: [1] first mode; [2] second mode.

footbridges with larger span lengths. This outcome is mainly due to the values of $\Psi(f_i)$, discussed above.

The maximum accelerations, obtained applying Level 3 approaches of *Sétra:2006* and *HiVoSS* Guidelines, are shown in Fig. 9, for the different density values. Also, in this case, the variation of vibration frequency due to the mass of the walkers is taken into account. Table 3 provides an overall framework of the vertical and lateral frequencies, depending on walker density. The effects of the mass of the walkers is neglected for the single walker and the group of walkers. Therefore, the first line in the table contains the frequency values corresponding to the bare footbridge, as given in Table 2.

Sétra:2006 and *HiVoSS* Guidelines consider three acceleration thresholds (0.5 m/s^2, 1.0 m/s^2 and 2.5 m/s^2 for vertical vibrations and 0.1 m/s^2, 0.3 m/s^2 and 0.8 m/s^2 for lateral vibrations) that define four comfort levels (maximum, average, minimum, unacceptable); these are also shown in Fig. 9 with lighter to darker grey hatches.

The response is calculated for the frequency maximizing $\Psi(f_i)$. It must be noted that when the second harmonic is considered, then the acceleration is lower because the modal load is halved.

Comparison between the results obtained with Level 3 approaches highlights that fixed density models bring larger accelerations when similar values of density

are considered. Moreover, when density increases there is a growth of the accelerations; in particular, for density values equal or more than 1 walkers/m^2 the accelerations are considerably larger due to the discontinuity in the definition of the equivalent number of walkers (Eq (8)). For longer spans, the accelerations are globally comparable to those obtained with constant density models, even when larger densities are considered. In these in, the response is calculated in the second mode. Conversely, much larger accelerations are computed for the span length of 10 m in the vertical direction and for the span length of 20 m in the lateral direction, given that $\Psi(f_i)$ is almost equal to 1. The differences between the results obtained for the span length of 20 m in the lateral direction are mainly due to the different value taken by $\Psi(f_i)$ (0.45 and 1 for *HiVoSS* and *Sètrà:2006*, respectively). Furthermore, vertical accelerations are almost zero for footbridges with a span length of 20 m, and lateral accelerations are almost zero for the footbridge with a span length of 30 m, because $\Psi(f_i)$ takes a value close to zero.

Finally, the results obtained through the application of the spectral approach proposed by *HiVoSS* are shown in Fig. 10. In particular, the vertical accelerations for the three density δ values of 0.5 walkers/m^2, 1.0 walkers/m^2 and 1.5 walkers/m^2 are surprisingly similar to each other, and lower than those obtained through the application of the other variable density pedestrian flow model proposed by *HiVoSS* Guidelines. The results achieved for the lateral direction for density values of 1.0 walkers/m^2 and 1.5 walkers/m^2 are comparable to those obtained with variable density model, whereas they are larger for a density of 0.5 walkers/m^2.

3.2. Comparison of the accelerations evaluated for the footbridges with concrete slab

The calculations presented in Section 3.1 *Comparison of the Acceleration Evaluated for the Footbridge without Concrete Slab,* have been repeated for the case of footbridges provided with a reinforced concrete slab. The accelerations evaluated are in general smaller than those of the footbridges without a concrete slab, due to the increase of mass. The maximum vertical and lateral accelerations induced by a single resonant walker or by a group of walkers confirm the conclusions presented for the footbridges without concrete slab (Figs 11–12). In particular, the maximum accelerations decrease with increasing footbridge span, due to the variation of the vibration frequency. Again, the main differences between the results obtained through the application of the different methods lay in the different values taken by $\Psi(f_i)$.

Then the results of the analysis of the footbridge response to a stream of walkers (Level 3 approaches), considering fixed density and variable density models are shown in Figs 13–15. The change of vibration frequencies due to the mass of the walkers is again taken into account, and these are listed in Table 4.

However, in this case, only a slight variation of the vibration frequencies is found when the mass of the walkers is taken into account. The results obtained applying all the Level 3 approaches confirm the results obtained for footbridges without a concrete slab. Quite different results are found only when variable density models are applied. In these cases, larger accelerations are found for longer footbridges. Finally, the results obtained through the application of the spectral approach for all densities are again similar to each other.
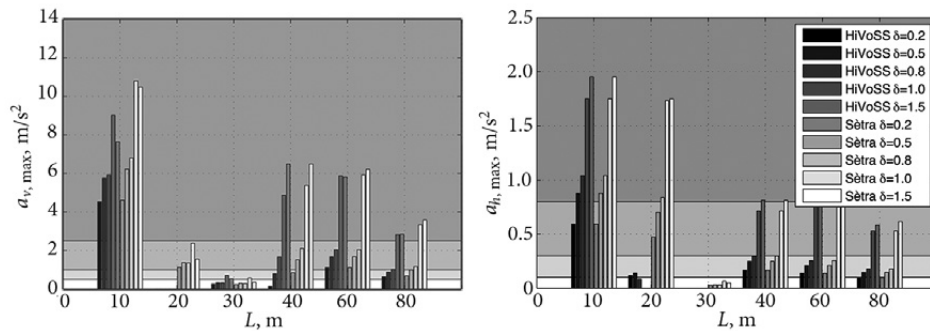

Fig. 9. Vertical and lateral accelerations induced by a flow of walkers: variable density
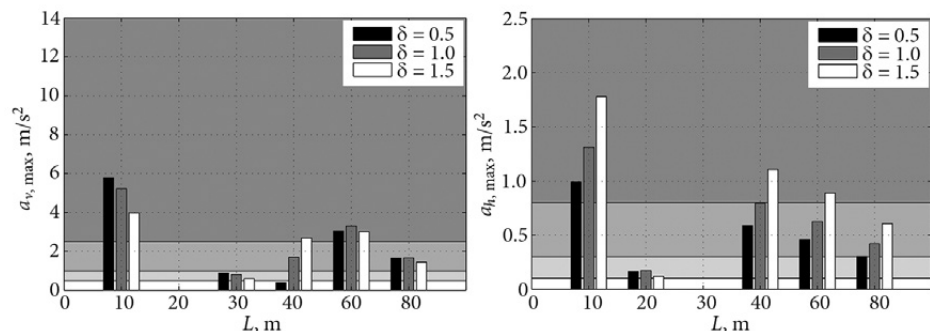

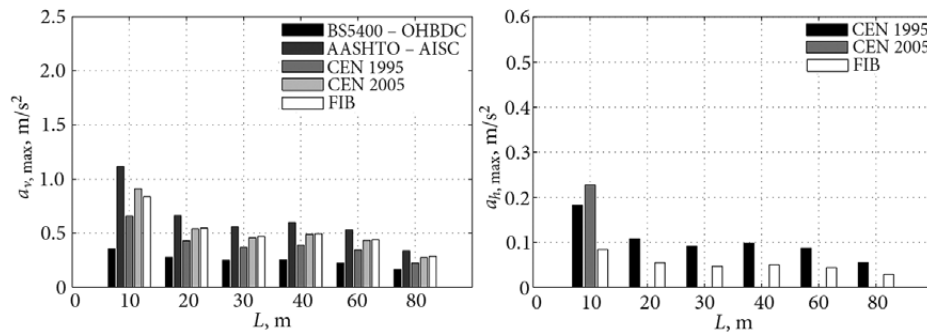**Fig. 10.** Vertical and lateral accelerations induced by a flow of walkers: spectral approach of *HiVoSS*

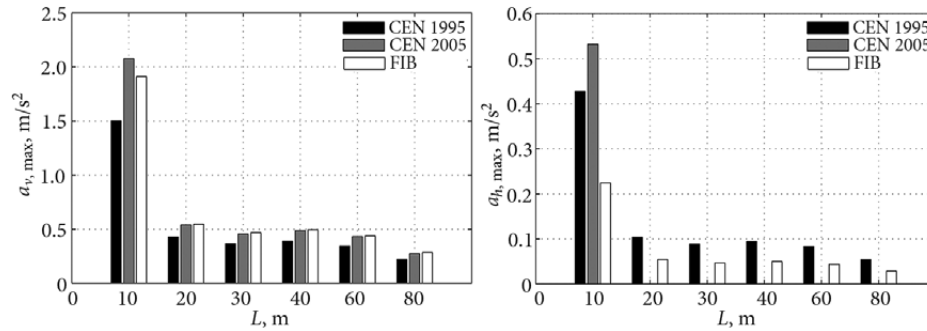**Fig. 11.** Vertical and lateral accelerations induced by a single resonant walker

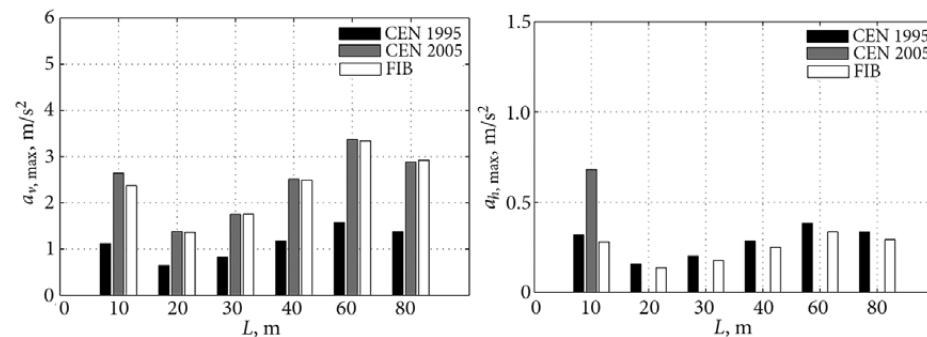**Fig. 12.** Vertical and lateral accelerations induced by a group of walkers

Fig. 13. Vertical and lateral accelerations induced by a flow of 0.6 walkers/m^2

Table 4. Vibration frequency variation due to the mass of the walkers (footbridges with concrete slab)

|	Density δ, walkers/m^2	Mass Variation, kg/m	Span length, m											
			10		20		30		40		60		80	
			f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l	f_v	f_l
			Hz											
Single walker or group of walkers	–	–	1.32[1]	0.71[1]	0.97[1]	0.44[1]	0.62[1]	0.32[1]	0.47[1]	0.20[1]	0.38[1]	0.17[1]	0.35[1]	0.16[1]
Stream of walkers with fixed density	0.6	126	1.27[1]	0.69[1]	0.93[1]	0.43[1]	0.60[1]	0.30[1]	0.45[1]	0.20[1]	0.37[1]	0.17[1]	0.34[1]	0.16[1]
Stream of walkers with variable density	0.2	42	1.30[1]	0.70[1]	0.95[1]	0.44[1]	2.46[2]	1.25[2]	1.85[2]	0.80[2]	1.49[2]	0.68[2]	1.40[2]	0.64[2]
	0.5	105	1.27[1]	0.69[1]	0.94[1]	0.43[1]	2.42[2]	1.22[2]	1.81[2]	0.79[2]	1.47[2]	0.67[2]	1.38[2]	0.63[2]
	0.8	168	1.25[1]	0.68[1]	0.92[1]	0.42[1]	2.37[2]	1.20[2]	1.78[2]	0.78[2]	1.45[2]	0.66[2]	1.36[2]	0.63[2]
	1.0	210	1.23[1]	0.67[1]	0.91[1]	0.41[1]	2.35[2]	1.19[2]	1.76[2]	0.77[2]	1.43[2]	0.65[2]	1.35[2]	0.62[2]
	1.5	315	1.20[1]	0.65[1]	0.88[1]	0.40[1]	2.28[2]	1.16[2]	1.72[2]	0.75[2]	1.40[2]	0.64[2]	1.32[2]	0.61[2]

Note: [1] first mode; [2] second mode.
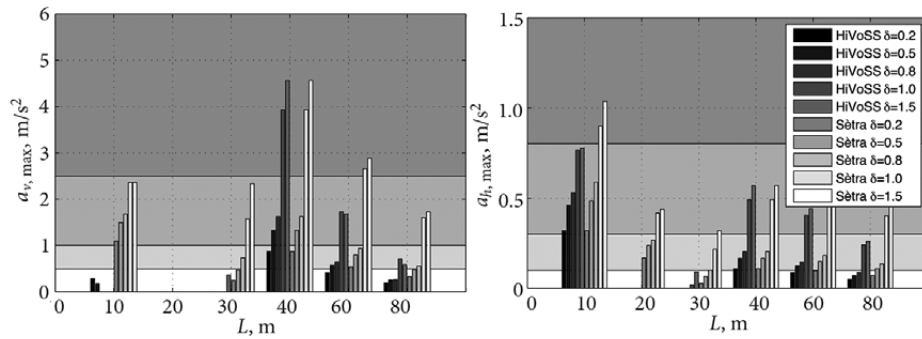
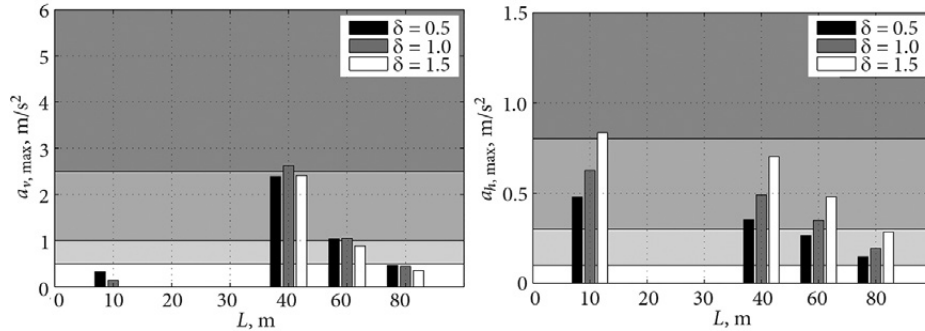**Fig. 14.** Vertical and lateral accelerations induced by a flow of walkers: variable density

Fig. 15. Vertical and lateral accelerations induced by a flow of walkers: spectral approach of *HiVoSS*

4. Conclusions

1. In this paper, a classification of the different approaches proposed by Standards and Guidelines for the assessment of the dynamic response of footbridges to walkers has been given. A critical analysis of their limitations and advantages after comparison of background hypotheses, the field of applicability, the results obtained, have been developed. In particular, the comparison among the models brings to the following conclusions:

- the values of Transient Resonant Response Coefficient, provided by different Standard and Guidelines, show a clear disagreement among one and another; moreover, the expressions proposed by Ricciardelli and Briatico and by Piccardo and Tubino, well reproduce the numerical results proposed by Blanchard, providing an alternative to overcome this inaccuracy;
- comparison of the maximum transient acceleration due to one resonant walker, $\ddot{y}_{\max}(1)$ shows a considerable scatter;
- definition of the equivalent number of walkers, N_e appears rather arbitrary, and never clearly justified; also, a quite large scatter for N_e was found;
- the reduction coefficient accounting for non-resonant conditions, $\Psi(f_1)$ shows only minor differences from one procedure to another;
- the use of the transient response model under deterministic walking load, for the evaluation of the maximum stationary acceleration induced by a stream of pedestrians, appears as a clear misconception.

2. Then the dynamic response of prototype footbridges with steel box girder of different spans, designed according

to the Eurocodes, is evaluated, considering the various loading scenarios. The following conclusions are drawn:

- the design of footbridges for vertical static loads results in vertical and lateral vibration frequencies that may fall in the ranges of high susceptibility to walking-induced vibrations;
- the results obtained applying similar procedures for the assessment of the maximum vertical and lateral accelerations often show a considerable and unjustified scatter;
- the accelerations evaluated with Level 1 (single walker) and Level 2 (group of walkers) methods are lower than those obtained with Level 3 (the flow of walkers) methods;
- the accelerations achieved with fixed density Level 3 methods are larger than those of other models when similar values of density are considered;
- the accelerations achieved with variable density Level 3 methods grow with increasing density; they also contain inconsistency due to the variation of the equivalent number of the walkers as a function of density;
- the accelerations obtained with the spectral approach proposed by *HiVoSS* are lower than those obtained with the other pedestrian flow models.

Concluding, in spite of the impressive amount of research developed in the last fifteen years in the given field, the changes, which appeared in design procedures seem to be still insufficient for an appropriate design of footbridges against walking-induced vibrations. Moreover, it seems there is not full agreement on the most appropriate methodologies to be used. It is evident, that there is the

need that the huge amount of experimental and numerical results, achieved by the scientific community, are synthesized in a way to provide designers with straightforward and reliable procedures.

References

Allen, D. E.; Murray, T. M. 1993. Design Criterion for Vibrations due to Walking, *Engineering Journal American Institute of Steel Construction* 30(4): 117–129.

Avossa, A. M.; Demartino, C.; Ricciardelli F. 2017. Design Criterion for Vibrations due to Walking, in *Proc. of 10th International Conference on Structural Dynamics EURODYN2017, 10–13 September, 2017, Rome, Italy.*

Bachmann, H.; Ammann, W. 1987. *Vibrations in Structures Induced by Man and Machines, Structural Engineering Documents* vol. 3e, International Association of Bridge and Structural Engineering, Zürich, Switzerland.

Bachmann, H.; Pretlove, A. J.; Rainer, H. 1995. *Dynamic Forces from Rhythmical Human Body Motions*. Vibration Problems in Structures: Practical Guidelines, Birkhäuser, Basel, Switzerland.

Blanchard, J.; Davies, B. L.; Smith, J. W. 1977. Design Criteria and Analysis for Dynamic Loading of Footbridges, in *Proc. of a Symposium on Dynamic Behaviour of Bridges at the Transport and Road Research Laboratory*, 19 May, 1977, Crowthorne, UK, 90–106.

Brownjohn, J. M. W.; Pavic, A.; Omenzetter, P. 2004. A Spectral Density Approach for Modelling Continuous Vertical Forces on Pedestrian Structures due to Walking, *Canadian Journal of Civil Engineering* 31(1): 65–77. https://doi.org/10.1139/l03-072

Butz, C. 2006. Beitrag zur Berechnung fußgängerinduzierter Brückenschwingungen (On the Calculation of Pedestrian-Induced Vibration of Bridges). Ph.D. thesis, RWTH, Aachen, Germany (in German).

Eriksson, P.E.; Ohlsson S.V. 1988. Dynamic Footfall Loading from Groups of Walking People, in *Proc. of Symposium Workshop on Serviceability of Buildings (Movements, Deformations, Vibrations)*, Vol.1, National Research Council, 16–18 May, 1988, Ottawa, Canada, 497.

Fryba, L. 1973. Vibration of Solids and Structures under Moving Loads. Noordhoff, Groningen, Netherlands.

Grundmann, H.; Kreuzinger, H.; Schneider, M. 1993. Dynamic Calculations of Footbridges, *Bauingenier* 68(5): 215–225. (in German)

Ingólfsson, E. T.; Georgakis, C. T.; Jönsson, J. 2012. Pedestrian-Induced Lateral Vibrations of Footbridges: a Literature Review, *Engineering Structures*, 45: 21–52. http://dx.doi.org/10.1016/j.engstruct.2012.05.

Matsumoto, Y.; Nishioka, T.; Shiojiri, H.; Matsuzaki, K. 1978. Dynamic Design of Footbridges, in *Proc. of International Association for Bridge and Structural Engineering*, Zurich, Switzerland, 1–15.

Piccardo, G.; Tubino, F. 2012. Dynamic Response of Euler-Bernoulli Beams to Resonant Harmonic Moving Loads, *Structural Engineering and Mechanics* 44(5): 681–704. https://doi.org/10.12989/sem.2012.44.5.681

Racic, V.; Pavic, A.; Brownjohn, J. M. W. 2009. Experimental Identification and Analytical Modelling of Human Walking Forces: Literature Review, *Journal of Sound Vibrations* 326(1–2): 1–49. http://dx.doi.org/10.1016/j.jsv.2009.04.02

Ricciardelli, F. 2005. Lateral Loading of Footbridges by Walkers, in *Proc. of Footbridge 2005*, *2nd International Conference*, 6–8 December, 2005, Venice, Italy.

Ricciardelli, F.; Pizzimenti, A. D. 2007. Lateral Walking-Induced Forces on Footbridges, *Journal of Bridge Engineering* 12(6): 677–688. https://doi.org/10.1061/(ASCE)1084-0702(2007)12:6(677)

Ricciardelli, F; Pansera, A. 2010. An Experimental Investigation into the Interation among Walkers in Groups and Crowd, in *Proc. of 10th International Conference Recent Advances in Structural Dynamics*, 12–14 July, 2010, Southampton, UK.

Ricciardelli, F.; Briatico, C. 2011. Transient Response of Supported Beams to Moving Forces with Sinusoidal Time Variation, *Journal of Engineering Mechanics* 137(6): 422–430. https://doi.org/10.1061/(ASCE)EM.1943-7889.0000241

Ricciardelli, F.; Demartino, C. 2016. Design of Footbridge against Pedestrian-Induced Vibrations, *Journal of Bridge Engineering* 21(8), C4015003. http://dx.doi.org/10.1061/(ASCE)BE.1943-5592.0000825#sthash.WZYa5TPy.dpuf

Van Nimmen, K.; Lombaert, G.; De Roeck, P.; Van den Broeck, P. 2014. Vibration Serviceability of Footbridges: Evaluation of the Current Codes of Practice, *Engineering Structures* 59(2014): 448–461. https://doi.org/10.1016/j.engstruct.2013.11.006

Venuti, F.; Bruno, L. 2009. Crowd-Structure Interaction in Lively Footbridges under Synchronous Lateral Excitation: a Literature Review, *Physics Life Review* 6(3): 176–206. https://doi.org/10.1016/j.plrev.2009.07.001

Tilden, C. J. 1913. Kinetic Effects of Crowds, in *Proc. of American Society of Civil Engineers* 39(3): 325–340. https://doi.org/10.1038/scientificamerican09131913-172supp

Zivanovic, S.; Pavic, A.; Reynolds, P. 2005. Vibration Serviceability of Footbridges under Human-Induced Excitation: a Literature Review, *Journal of Sound and Vibrations* 279(1–2): 1–74. https://doi.org/10.1016/j.jsv.2004.01.019