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COMPARISON OF HYBRID METHODS WITH DIFFERENT META MODEL USED IN BRIDGE MODEL-UPDATING

Zhiyuan Xia[1], Aiqun Li[2]✉, Jianhui Li[3], Maojun Duan[4]

[1, 2]*School of Civil Engineering, Southeast University, Sipailou 2 Xuanwu District, 210096 Nanjing, China*
[2]*Beijing Advanced Innovation Center for Future Urban Design, Beijing University of Civil Engineering and Architecture, Zhanlanguanlu 1 Xicheng District, 100044 Beijing, China*
[3, 4]*School of Civil Engineering, Nanjing Forestry University, Longpanzhonglu 159 Xuanwu District, 210037 Nanjing, China*
E-mails: [1]*zhiyuanxia@seu.edu.cn;* [2]*aiqunli@seu.edu.cn;* [3]*lijianhui@njfu.edu.cn;* [4]*dmj@njfu.edu.cn*

Abstract. Two hybrid model updating methods by integration of Gaussian mutation particle swarm optimization method, Latin Hypercube Sampling technique and meta models of Kriging and Back-Propagation Neural Network respectively were proposed, and the methods make the convergence speed of the model updating process faster and the Finite Element Model more adequate. Through the application of the hybrid methods to model updating process of a self-anchored suspension bridge in-service with extra-width, which showed great necessity considering the ambient vibration test results, the comparison of the two proposed methods was made. The results indicate that frequency differences between test and modified model were narrowed compared to results between test and original model after model updating using both methods as all the values are less than 6%, which is 25%–40% initially. Furthermore, the Model Assurance Criteria increase a little illustrating that more agreeable mode shapes are obtained as all of the Model Assurance Criteria are over 0.86. The particular advancements indicate that a relatively more adequate Finite Element Model is yielded with high efficiency without losing accuracy by both methods. However, the comparison among the two hybrid methods shows that the one with Back-Propagation Neural Network meta model is better than the one with Kriging meta model as the frequency differences of the former are mostly under 5%, but the latter ones are not. Furthermore, the former has higher efficiency than the other as the convergence speed of the former is faster. Thus, the hybrid method, within Gaussian mutation particle swarm optimization method and Back-Propagation Neural Network meta model, is more suitable for model updating of engineering applications with large-scale, multi-dimensional parameter structures involving implicit performance functions.

Keywords: Back-Propagation Neural Network meta model, Gaussian mutation, Kriging meta model, Latin Hypercube Sampling, model updating, particle swarm optimization, self-anchored suspension bridge.

1. Introduction

An accuracy Finite Element Model (FEM) of real structures is used to reflect the true structural performance, but it is hard to be established due to the uncertainties in material and geometric properties as well as the stochastic factors in boundary and load conditions (Bayraktar *et al.* 2009; Deng, Cai 2010; Grigorjeva *et al.* 2008; Mottershead, Friswell 1993; Ou, Li 2010). Particularly for those large-scale and complex engineering structures, such as bridges with extra-width to meet the high increase of the requirement for vehicle throughput of bridge system today, it is harder to implement analyses like health monitoring and control, damage detection and reliability assessment without accurate FEM. By identifying and modifying the uncertain parameters, model updating technique provides an efficient way of yielding accurate FEM to predict the performance of physical structures, to which the engineers have drawn significant attention in recent years (Bayraktar *et al.* 2009).

There are two main types of methods to realize model updating. One is the non-iterative method by modifying the element matrices of stiffness and mass (Berman 1979) with perturbation, and it is based on orthogonal conditions or vibration equation to make the modified matrices meet the real ones (Ren *et al.* 2011). The other is the iterative method by changing the selected parameters like material or geometric properties with analysis of sensitivity. The drawback of the former technique is that the modified matrices lose

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their sparse character and for large and complex structures it requires the higher ability of numerical analysis with huge calculations. Furthermore, during the modifying procedure, negative stiffness occurs, this obviously has no physical meaning. Compared to previous updating technique, the latter one is transferred into an optimization method by changing modified parameters to minimize the objective function, which is always set by errors like deflections or mode frequencies between experiment and FEM (Cismasiu *et al.* 2015; Cui *et al.* 2012; Wan, Ren 2015).

For an optimization problem, the method or algorithm of optimization is the most important tool. Compared to traditional methods, easy to fall into optimal local solution under large design space, methods based on artificial intelligence (AI) have higher reliability and efficiency (Deng, Cai 2010; Perera, Torres 2006). Particle swarm optimization (PSO), one of AI methods, proposed by Kennedy and Eberhart based on Boid model, is certificated that it needs less number of iteration to get the same or better result compared to other algorithms (Do *et al.* 2014). However, due to the lack of swarm diversity at the later stage of this algorithm, its global search ability is weak as the algorithm ends prematurely. Hence, adjustments have been introduced to improve the property of PSO with modifying velocity or (and) position of particles. Gaussian mutation particle swarm optimization (GMPSO) (Liu 2012; Tian *et al.* 2015) is one of improved PSO; it gives every particle a mutation of Gaussian White Noise (GWN) disturbance (Liao *et al.* 2008) to enrich the diversity of particle swarm. Moreover, it is proven to be a useful optimization algorithm with rapid and steady global convergence.

However, during GMPSO process, the reality is that each iteration of it needs to re-run the finite element package, and this obviously increases the cost of computation, especially when the research object has a complex structure. Many numerical meta models instead of FEM are introduced to reduce the cost, in which explicit expressions between parameters and interested performances instead of implicit ones are established, namely the process of recalling FEM is replaced by recalling meta models. Response Surface Methodology (RSM) as one of meta models, proposed first by Marwala, based on intelligent experimental design, is widely used in modern model updating, especially the second order polynomial one (Ren *et al.* 2011; Wan *et al.* 2015). Deng and Cai (2010) utilized a second order polynomial response surface (RS) representing the relationship between selected parameters and structural responses; then the parameters were updated by genetic algorithm. This method was applied to model updating of a two-way bridge located in Louisiana. Ren *et al.* (2011) presented an updating method based on polynomial RSM using uniform design, and a six-girder bridge verified the proposed method. Although in most engineering applications, polynomial RSM reflects the real structure very well. If the number of parameters is over than seven, the number of regression coefficients in those response surfaces (RS) increases, and the accuracy of meta model decreases. In that case, more accuracy meta models for a large number of parameters need to be put forward, two of which are Kriging meta model and Back-Propagation Neural Network (BPNN) meta model. Through implementing the stochastic process in Kriging meta model and the ability of arbitrary high-order nonlinear function fitting in BPNN meta model, the accuracy of meta models in case of multi-parameters is guaranteed.

Generally speaking, to obtain a valid meta model, the datasets selected for establishing model must be representative of the overall design space. The simplest and most used method to generate input datasets is random sampling, but the significant disadvantage of this approach is its lack of guarantee that the chosen datasets cover the entire design space uniformly. To overcome this shortcoming, Latin Hypercube Sampling (LHS), instead of methods of orthogonal design and uniform design, has more advantages. It has a simpler operation, less calculation effort, higher accuracy, and the ability of rapidly generating samples with an interested number of parameters and dimensions (Dette, Pepelyshev 2010; Hurtado, Barbat 1998; Li *et al.* 2013).

In this paper, to realize the model updating of an in-service bridge has a complex structure with high efficiency and good accuracy, two hybrid model updating methods of GMPSO with Kriging meta model and GMPSO with BPNN meta model are proposed, and comparison among the model updating results of both approaches was made. The purpose of this study is by using GMPSO and meta models to make the convergence speed of the updating process faster and using the comparison of efficiency and accuracy among different meta models through the application to a real bridge to obtain a more proper updating method. Compared to ordinary polynomial meta models, Kriging and BPNN meta models increase the accuracy, and meanwhile, the LHS technique is first used to improve the quality of input datasets. Compared to direct GMPSO method, the only finite element analysis calls are used in forming meta models. By combining advantages of both GMPSO and meta models, the proposed hybrid methods can prohibitively reduce the computation time without losing accuracy. Thus, hybrid methods are suitable for model updating of engineering applications with large-scale, multi-dimensional parameter structures involving implicit performance functions. The architecture of the hybrid methods consists of three main stages of analysis:

1. Selecting reasonable samples of random parameters as input datasets using LHS and performing finite element analyses with the input parameters to obtain output datasets.

2. Formulating the Kriging or BPNN meta models based on input and output datasets.

3. Applying GMPSO as an optimization technique with an objective function set as errors between the experimental value and meta models to yield the updated values of selected parameters.

The paper is organized as follows. Section 2 describes types of meta models mainly about the Kriging meta model and BPNN meta model. Section 3 introduces PSO and GMPSO algorithms. Section 4 presents the procedures of two hybrid methods and the LHS method; the latter is used to form samples for meta models. The application of two hybrid methods to an existing self-anchored suspension bridge is given, and comparison results are given in Section 5 and conclusions are given in Section 6.

2. Description of meta models

2.1. Kriging meta model

A Kriging meta model, instead of FEM, initially developed by Krige in 1951 (Simon *et al.* 2016), first used in the area of mining and geostatistics, is usually implemented to illustrate the explicit relationship between selected parameters and interested performances. With the input samples of parameters and output responses of FEM, a Kriging meta model is formed as follows. Supposing an unknown function of interested performance is combined with a polynomial part and a stochastic process, providing a global model and localized departures respectively. The formula is as follows:

$$y(\mathbf{X}) = f^T(\mathbf{X})\beta + z(\mathbf{X}), \quad (1)$$

where X is a variable vector of d parameters $\left[x_1 \ldots x_d\right]$, for example, the i^{th} vector of d parameters $\mathbf{X}_i = \left[x_{i1} \ldots x_{id}\right]$; $f^T(\mathbf{X})$ is a vector of m polynomial functions, namely $f^T(\mathbf{X}) = \left[f_1(\mathbf{X}) \ldots f_m(\mathbf{X})\right]$; $\boldsymbol{\beta}$ is a vector of m regression coefficients, $\boldsymbol{\beta} = \left[\beta_1 \ldots \beta_m\right]^T$; $z(\mathbf{X})$ is a Gaussian stationary stochastic process with mean 0 and variance σ^2. The covariance of arbitrary two elements in $z(\mathbf{X})$ is as follow:

$$\operatorname{cov}\left(z(\mathbf{X}_i)z(\mathbf{X}_j)\right) = \sigma^2 R\left(\mathbf{X}_i, \mathbf{X}_j\right) = \sigma^2 \exp\left(-\sum_{k=1}^{d} \theta_k \left|x_{i,k} - x_{j,k}\right|^2\right), \quad (2)$$

where $R(\mathbf{X}_i, \mathbf{X}_j)$ is the spatial correlation of two samples, and is usually implemented by a Gaussian function $\exp\left(-\sum_{k=1}^{d} \theta_k \left|x_{i,k} - x_{j,k}\right|^2\right)$ as a core function because its calculation effect is the most welcome; θ_k is coefficient with a value over zero in the computation of spatial correlation. Supposing there are N samples, the matrices of responses and polynomial values are $\mathbf{Y} = \left[y(\mathbf{X}_1) \ldots y(\mathbf{X}_N)\right]$ and $\mathbf{F} = \left[f^T(\mathbf{X}_1) \ldots f^T(\mathbf{X}_N)\right]^T$ respectively. The estimated values of $\boldsymbol{\beta}$ and σ^2 are as follows:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F}\right)^{-1} \mathbf{F}^T \mathbf{R}^{-1} \mathbf{Y}, \quad (3)$$

$$\hat{\sigma}^2 = \frac{1}{N}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right)^T \mathbf{R}^{-1}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right), \quad (4)$$

where spatial correlation matrix

$$\mathbf{R} = \begin{bmatrix} R(\mathbf{X}_1,\mathbf{X}_1) & R(\mathbf{X}_1,\mathbf{X}_2) & \ldots & R(\mathbf{X}_1,\mathbf{X}_N) \\ R(\mathbf{X}_2,\mathbf{X}_1) & R(\mathbf{X}_2,\mathbf{X}_2) & \ldots & R(\mathbf{X}_2,\mathbf{X}_N) \\ \vdots & \vdots & \ddots & \vdots \\ R(\mathbf{X}_N,\mathbf{X}_1) & R(\mathbf{X}_N,\mathbf{X}_2) & \ldots & R(\mathbf{X}_N,\mathbf{X}_N) \end{bmatrix}. \quad (5)$$

In above formulas, before obtaining values of $\hat{\boldsymbol{\beta}}$, $\hat{\sigma}^2$ and R through Equations (3)–(5), θ_1, …, θ_d are known first. By implementing maximum likelihood estimation method, θ_1, …, θ_d are selected to get minimum value of expression (6),

$$\theta_1 > 0^{\min}, \ldots, \theta_d > 0^{\left\{\frac{1}{2}\left[N\ln(\hat{\sigma}^2) + \ln|\mathbf{R}|\right]\right\}}, \quad (6)$$

where $|\mathbf{R}|$ is a determinant of a matrix $\mathbf{R}$, the optimal values of θ_1, …, θ_d are obtained by optimization methods such as genetic algorithm (GA), particle swarm optimization (PSO) and so on with an objective function of Equation (6). Once θ_1, …, θ_d are determined, the best linear unbiased predicted value of interested performance is

$$\hat{y}(\mathbf{X}) = f^T(\mathbf{X})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{X})\hat{\boldsymbol{\alpha}}, \quad (7)$$

where $\mathbf{r}^T(\mathbf{X})\hat{\boldsymbol{\alpha}} = \hat{z}(\mathbf{X})$ denotes interpolation of residuals of the regression model $f^T(\mathbf{X})\hat{\boldsymbol{\beta}}$, vector $\mathbf{r}^T(\mathbf{X}) = \left[R(\mathbf{X},\mathbf{X}_1), R(\mathbf{X},\mathbf{X}_2), \ldots, R(\mathbf{X},\mathbf{X}_N)\right]$, vector $\hat{\boldsymbol{\alpha}} = \mathbf{R}^{-1}\left(\mathbf{Y} - \mathbf{F}\hat{\boldsymbol{\beta}}\right)$.

2.2. Back-Propagation Neural Network meta model

In recent years, Artificial Neural Network (ANN) is also widely used to illustrate the relationship between selected parameters and interested performances and a multilayer feed-forward ANN structure, trained by Back-Propagation (BP), is one of the most popular methods in engineering applications. The processing units in typical multilayer BPNN meta model are arranged in layers, i.e., an input layer, an output layer, and a number of hidden layers as shown in Fig. 1 (Li *et al.* 2013).

Suppose a BPNN meta model with one hidden layer, where the input layer contains d nodes, the hidden layer contains J nodes, and the output layer contains K nodes. Then, the output y_k is expressed as

$$y_k = f_2\left(b_{ok} + \sum_{j=1}^{j} b_{kj} f_1\left(a_{oj} + \sum_{i=1}^{d} a_{ij} x_i\right)\right), \quad (8)$$

where f_1 denotes the transfer function between input and hidden layer; f_2 denotes the transfer function between hidden and output layer; x_i denotes the ith input parameter; a_{ij} and b_{jk} (i = 1, 2, …, d; j = 1, 2, ..., J; k = 1, 2, ..., K) mean weights between input and hidden layer and between hidden and output layer respectively; a_{oj} and b_{ok} mean deviations of hidden layer and output layer respectively.

In this study a three-layer BPNN meta model was built with a sigmoid transfer function, $f_1(x) = \frac{1}{1+e^{-x}}$, in the hidden layer and a linear transfer function, $f_2(x) = x$, at the output layer. It showed that the multilayer feed-forward networks with as few as a single hidden layer and an appropriately smooth hidden layer transfer function are capable of arbitrarily accurate approximation to an arbitrary function and its derivatives. A deficiency when training a BPNN meta model is that the network overfits on the training set, and the way to prevent this disadvantage is using an early stopping technique, in which the available data are divided into three subsets: the training set, validation set, and test set (Li *et al.* 2013). A random generator randomly selects all input datasets divided into those three subsets. The disadvantage is the lack of guarantee that the selected dataset covers the entire design space uniformly, especially when the number of variables is large, and the number of input dataset is relatively small. In this paper, three datasets, guaranteed to be relatively uniformly distributed over each dimension, are created by the LHS technique dependently, represented in the following section. Thus, it overcomes the referred disadvantage (Li *et al.* 2013).

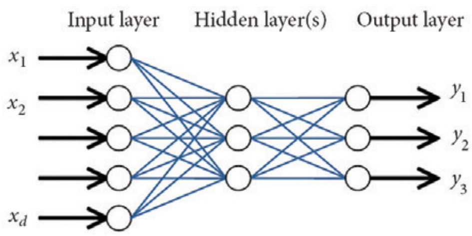

Fig. 1. Typical structure for the three layers of Back-Propagation Neural Network meta model

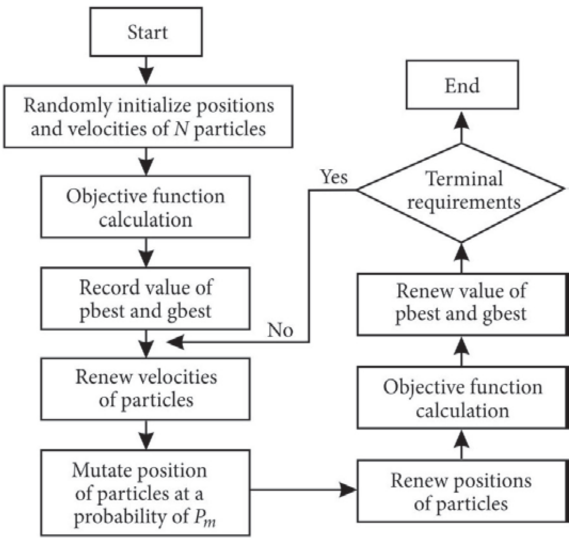

Fig. 2. Flow chat of Gaussian mutation particle swarm optimization algorithm

3. Gaussian mutation particle swarm optimization

For standard PSO, (Liu 2012; Tian *et al.* 2015), supposing there are N random initialization particles with a d – dimensional search space, in which the position and velocity of particle I are $\mathbf{X}_i = [x_{i1}, x_{i2}, \ldots, x_{id}]$ and $\mathbf{V}_i = [v_{i1}, v_{i2}, \ldots, v_{id}]$ respectively. After every iteration process, particle I obtains two best solutions of the objective function, one is the personal best (*pbest*), $\mathbf{P}_i = [p_{i1}, p_{i2}, \ldots, p_{id}]$ and the other is the best of indivisible particles called global best (*gbest*), $\mathbf{P}_g = [p_{g1}, p_{g2}, \ldots, p_{gd}]$. Then particle I adjusts its velocity and position according to the following formulas:

$$v_{ij}(t+1) = \omega v_{ij}(t) + c_1 r_1 \left[p_{ij} - x_{ij}(t) \right] + c_2 r_2 \left[p_{gj} - x_{ij}(t) \right], \quad (9)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)\Delta t, \quad (10)$$

where $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, d$; $\Delta t = 1$, t is the t^{th} iteration; ω is inertia weight coefficient; c_1 and c_2 are positive learning coefficient, namely acceleration coefficient; r_1 and r_2 are random numbers uniformly distributed between 0 and 1; x_{ij} and p_{ij} mean the component of aforementioned vector $\mathbf{X}$ and $\mathbf{P}$. In the Equation (9) of subsequent velocity, the first item represents the inheritance of current state; the second item means self-cognition namely experiences and memories of itself, enabling the ability of global search and the last item reflects group-cognition based on experiences of total particles. That is why the PSO method improves that global search ability.

The standard PSO has an aforementioned main disadvantage that due to the lack of swarm diversity at the later stage of this algorithm, its global search ability is influenced as the algorithm ends prematurely. The methods of improvements are all based on techniques of changing positions, speeds and directions of partial particles to avoid falling into local best solution. Gaussian mutation used in PSO (GMPSO) is one, giving every particle a mutation of GWN disturbance (Liao *et al.* 2008) to enrich the diversity of particle swarm. Gaussian White Noise, a noise whose probability density obeys Gaussian normal distribution and power spectrum density is constant in the frequency domain, is usually used to fit unknown random process due to its excellent statistic characteristic. In Gaussian mutation particle swarm optimization (GMPSO) algorithm, the position of particle I, $\mathbf{X}_i = [x_{i1}, x_{i2}, \ldots, x_{id}]$, is mutated by disturbance of GWN at a probability of P_m whose value is generally between 0.05 and 0.15. The mutation formula is as follows:

$$x_{ij} = p_{gj}(0.5+\sigma), \quad (11)$$

where σ is a random number of GWN, the other meanings of items are like before. The flow chat of GMPSO algorithm is shown in Fig. 2.

The detail steps are as follows:

- Step 1: Randomly initialize positions and velocities of N particles and set the values of serials of parameters such as w, c_1, c_2, d, P_m, maximum iteration number and the upper and lower bounds of search space;
- Step 2: Establish the objective function, called fitness function, and calculate its value of each particle. Record each current position of each particle and function value in *pbest*, the best position and best value of *pbest* are recorded in *gbest*;
- Step 3: Renew velocity of particles based on Equation (9);
- Step 4: Mutate position of particles at a probability of P_m based on Equation (11);
- Step 5: Renew position of particles based on Equation (10);
- Step 6: Calculate function value again and replace *pbest,* if the current value is better than the former one.
- Step 7: Replace *gbest* in the current *pbest* and *gbest* according to the best value.
- Step 8: Quit search, if the iteration number reaches the maximum one or accuracy meets the limitation, otherwise, return to Step 3.

4. Proposed methods of model updating

The proposed two methods of model updating aim to improve the efficiency of model updating with no accuracy decreasing with the integration of LHS, GMPSO and two different meta models of Kriging and BPNN.

4.1. Latin Hypercube Sampling

Before establishing meta models, computer experiments are needed to be designed for forming samples of parameters and interested performances of FEM. Latin Hypercube Sampling, first proposed by McKay (Li *et al.* 2013), instead of methods of Orthogonal Design, Uniform Design, Central Composite Design, and so on, has simpler operation, smaller calculation, higher accuracy and the ability of rapidly generating samples with wanted number of parameters in total design spaces (Dette, Pepelyshev 2010; Li *et al.* 2013).

For uncertain but bounded parameters, supposing X is a variable vector of d parameters $[x_1, x_2, \ldots, x_d]$, $([l_1, u_1], [l_2, u_2], \ldots, [l_d, u_d])$ is the lower and upper boundary value of each parameter, N is the necessary number of samples. The steps of LHS method is as follows. Firstly, for each parameter, it is divided into N equal parts according to its boundary. Namely $\left[l_i, l_i+\frac{u_i-l_i}{N}\right]=P_{1i} < P_{2i} < \ldots < P_{Ni} = \left[l_i+\frac{u_i-l_i}{N}(N-1),\ u_i\right]$, $i = 1, 2, \ldots, d$. Secondly, a $N \times d$ matrix S_1:

$$\mathbf{S}_1 = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1d} \\ P_{21} & P_{22} & \cdots & P_{2d} \\ \cdots & \cdots & \ddots & \cdots \\ P_{N1} & P_{N2} & \cdots & P_{Nd} \end{bmatrix}. \quad (12)$$

is obtained, and every element in S_1 is a small boundary. Thirdly, a matrix S_2 is transformed from matrix S_1 with elements at each column, namely $P_{1i}, P_{2i}, \ldots, P_{Ni}$, are arranged to be in random order. Then, a matrix S is transformed from matrix S_2 with elements chosen randomly in small corresponding boundaries. Lastly, every row of the $N \times d$ matrix S is one sample of parameter vector and N rows mean N parameter samples. After taking these parameter samples into FEM, N results of interested performance are obtained. The N parameter samples with its corresponding N results of interested performance comprise N samples.

4.2. Two hybrid methods

As aforementioned, a model updating problem is an optimization problem. If the optimization algorithm of GMPSO is directly used in model updating, the algorithm needs to take current parameter vector into FEM to calculate the results in every process of calculation of objective function, shown in Fig. 3. Here supposing one particle denotes

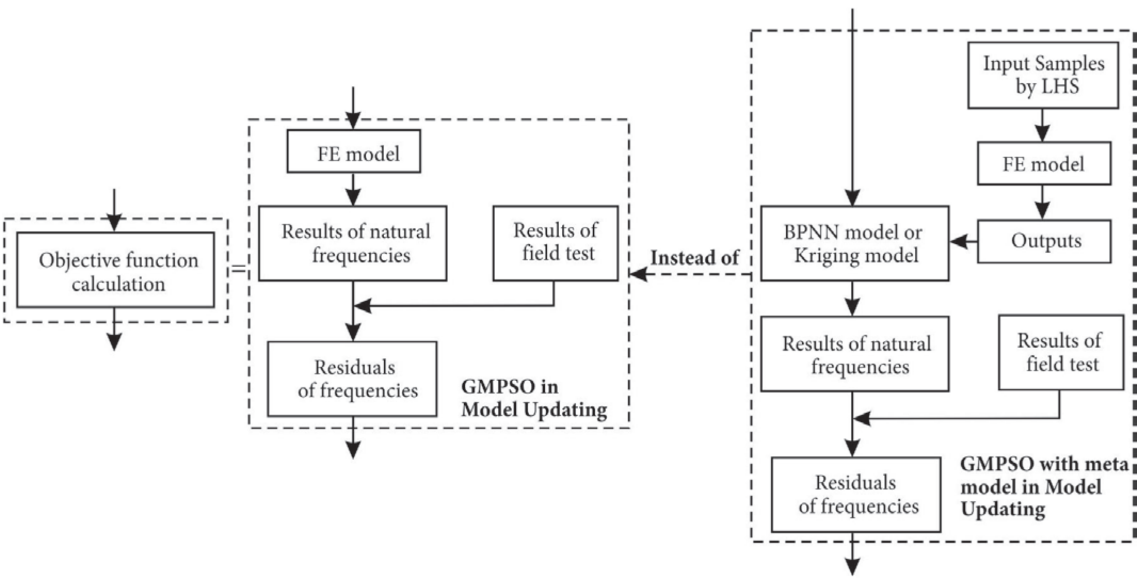

Fig. 3. Meta model integrated with Gaussian mutation particle swarm optimization

one parameter vector and objective function is residuals of frequencies between FEM and test results. The repeated calculation of each FEM is obviously a waste of time for a complex structure. However, when BPNN or Kriging meta models is implemented in the process, the calculation of meta model with the explicit expression showing the relationship between frequencies and parameters instead of FEM shorten the calculation time. Namely recalling the finite element package is replaced by recalling the meta model. The two proposed hybrid model updating methods use optimization algorithm GMPSO with Kriging meta model and GMPSO with BPNN meta model respectively. A combined program based on both MATLAB-code and ANSYS-APDL is developed to apply these hybrid methods to solve model updating problem, and the program is performed in modular form so that it is easily used in other model updating of structures. The flow chat of hybrid methods is shown in Fig. 3. Moreover, the architecture of the hybrid methods consists of three main stages of analysis:

- Step 1: Select reasonable samples of random parameters as input datasets using LHS and performing finite element analyses with the input parameters to obtain output datasets;
- Step 2: Formulate the Kriging or BPNN meta models based on input and output datasets;
- Step 3: Apply GMPSO as an optimization technique with an objective function set as errors between the experimental value and meta models to yield the updated values of selected parameters.

Fig. 4. Picture of Hunan Road Bridge

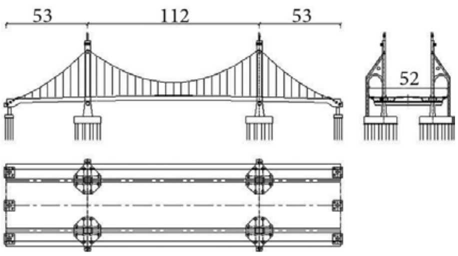

Fig. 5. General layout (unit in metres)

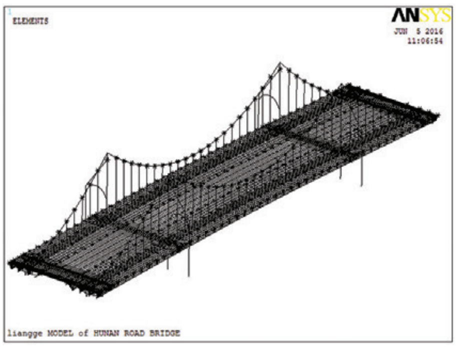

Fig. 6. Finite Element Model of Hunan Road Bridge

5. Application and comparison of hybrid methods

5.1. Description and Finite Element Model

The model updating process of an existing bridge is applied to make a comparison among the proposed hybrid methods. The prestressed concrete bridge, Hunan Road Bridge in Liaocheng City of Shandong Province in China shown in Fig. 4, a self-anchored suspension one with double-tower and double-cable plane, has a span arrangement of 53 m + 112 m + 53 m and a total width of 52 m with an arrangement of 3.5 m (pavement) + 5.5 m (non-motor vehicle lane) + 2.5 m (region of lateral separation, towers and hangers) + 29 m (8 vehicle lanes) + 2.5 m (region of lateral separation, towers and hangers) + 5.5 m (non-motor vehicle lane) + 3.5 m (pavement). The main girder with the material of concrete has a section of the double-lateral box connected by transverse beams with a height of 2.8 m of the standard section and a height from 4.3 m to 5.6 m of sections in the region of main cable anchorage. The tower with the material of concrete, consisting of the main tower and vice tower with a gate-shape, has a height (including the spire) of 45 m over the deck. The two cables are arranged symmetrically with a core-to-core transverse distance of 31.7 m, and the rise-span ration of the middle span is 1/5.276. There are 37 pairs of hangers of this bridge, 8 pairs in each two side spans and 21 pairs in middle span, with a standard distance of 5 m among hangers and a distance of 6 m between the axis of the tower and lateral hangers. The general layout of this bridge is shown Fig. 5.

Based on the software of ANSYS, the three-dimensional FEM of Hunan Road Bridge was established shown in Fig. 6. The main girder, transverse beam and tower are simulated with a BEAM-44 element in ANSYS, and due to the large width of box girder, it is divided into eight small girders with the connection of transverse beams and virtual weightless deck using SHELL-63 element. Both the main cable and hanger are simulated with LINK-10 element only considering tension, and the cable is dispersed by the position of hanging points of hangers. Its empty cable curve is decided based on segmental catenary theory considering self-weight. The longitudinal and transverse prestressed steels are all simulated with the LINK-8 element, and the weights of cable clamps, main saddles, and secondary dead load are all transferred to the MASS-21 element. The four connections of each tower and girder are simulated with the COMBINE-14 element. In those four connections, the connection conditions between left tower and girder are restrained in the vertical direction and longitudinal direction, while the conditions between right tower and girder are restrained only in vertical direction. The six connections of girder and foundation are also simulated with the COMBINE-14 element. In that six connections, the connection conditions are all restrained in the vertical direction and, especially, in the transverse direction when referred to connections on the longitudinal axis of the girder. The bottom of the tower is restrained in all directions of six freedoms. A total of 15 530 nodes and 20 875 elements are found in the model, and the gravity acceleration is 9.806 m/s^2, the material damping ratio of the girder, beam, and the tower is 0.05, and it is 0.02 of cable, hanger and prestressed steel.

5.2. Theoretical and experimental dynamic characteristic results

Besides the finite element dynamic analysis of the model above, an in-site ambient excitation test was conducted. Seventy four measured points, used to get the dynamic features of the main girder, were arranged during the experiment. Figure 7 shows the setup for the dynamic test carried out on the physical structure. Wireless accelerometers on both sides of the bridge deck under each hanger recorded the vibration responses under ambient excitation. The accelerometers operated as the wireless transmitter and the computer together with the repeater operated as a wireless receiver. Based on the application of stochastic subspace identification, the first five mode frequencies and shapes were identified. Figure 8 shows the mode shapes of the main girder from the first FEM. Table 1 gives the shapes description and detailed frequencies from initial model and vibration test. The detailed mode shape values of both model and experiment are compared in Fig. 9.

From the results of the main girder in Table 1 and Fig. 9, the first mode frequency is 0.7208 Hz from initial FEM and 0.9030 Hz from field test with the difference among them up to 25.28%. The frequency difference between theoretical and experimental torsion shape is 28.44% as each value is 1.0643 Hz and 1.3670 Hz. In second and third order of vertical bending it is up to 38.60% and 38.08% respectively, while in transverse bending, the difference is 3.08% as the smallest one. However, from the value of MACs (Model Assurance Criteria), it shows that the mode shapes calculated by FEM have an agreement with the filed ones as the lowest value is around 85%. Hence, due to the big differences in mode

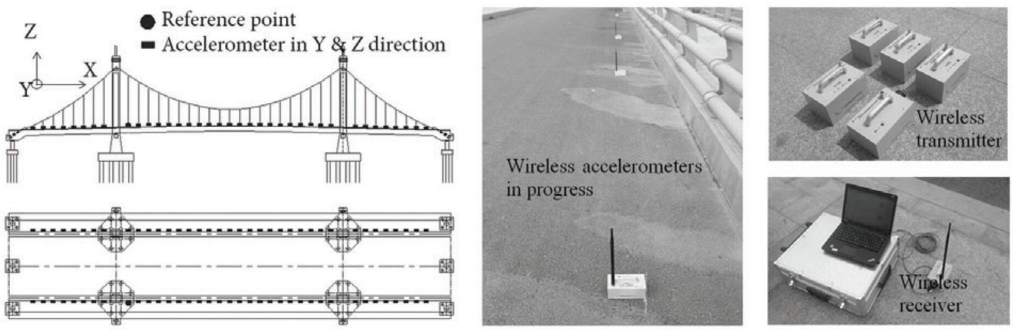

Fig. 7. Set up for ambient excitation test

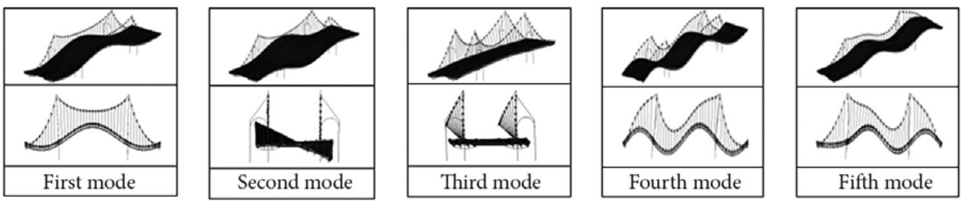

Fig. 8. The first five mode shapes of the main girder of Hunan Road Bridge from Finite Element Model

Table 1. Detail dynamic characteristics of the main girder from theoretical and experimental analysis

Mode	Theoretical frequency, Hz	Modal shape	Experimental frequency, Hz	Difference, %	Model Assurance Criteria
1	0.7208	First order of vertical bending	0.9030	25.28	0.979
2	1.0643	First order of torsion	1.3670	28.44	0.973
3	1.3378	First order of transverse bending	1.3790	3.08	0.915
4	1.3892	Second order of vertical bending	1.9290	38.60	0.894
5	1.7504	Third order of vertical bending	2.4170	38.08	0.849

Note: $\text{Difference} = \frac{|\text{Experimental frequency} - \text{Theoretical frequency}|}{\text{Theoretical frequency}} \cdot 100\%$.

frequencies, a model updating work should be implemented. Moreover, the sum of errors of frequencies was selected as the objective function. In this section, considering the complexity of this bridge structure, the model is updated by the two proposed methods of GMPSO with meta models instead of GMPSO with FEM.

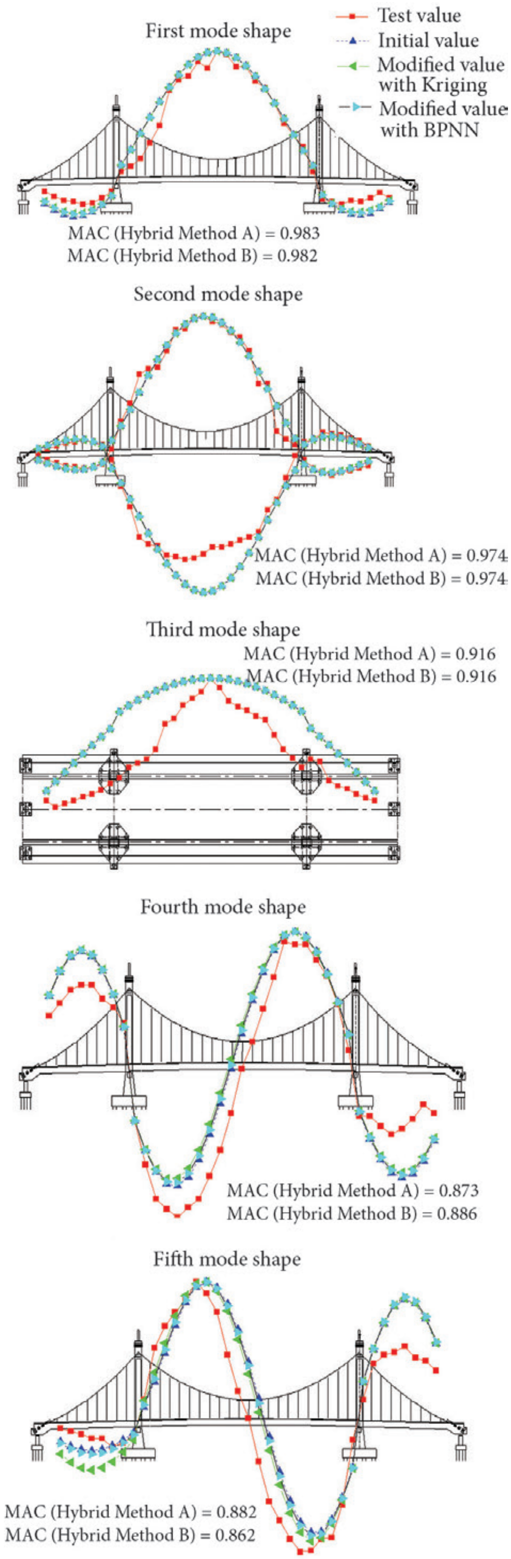

Fig. 9. Comparison of mode shape among test, initial model, and modified models

5.3. Model updating process using hybrid methods

In this section, Hybrid Method A denotes the optimization process with Kriging meta model, and Hybrid Method B denotes the process with BPNN meta model. For both of the two hybrid methods, the parameters selected as updated ones are module of main girder, main tower and main cable, density of main girder, moment of inertia of vertical bending, transverse bending and torsion, moment of inertia of main tower, area of section of main girder and secondary dead load respectively. Moreover, a particle is described as $\mathbf{X}_i = [x_{i1}, x_{i2}, \ldots, x_{i10}]$ with ten parameters, and x_{i1} to x_{i10} are the ratio coefficients (real value/original value) of selected ones (for example, the parameter vector of initial FEM is [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]). The original values of the geometric and material parameters are set by the design drawings and the Chinese bridge code respectively. If the parameter is over or below one, it means the real value is larger or smaller than the original one. Due to the existing differences between field values and design or specified values in drawing and code, and especially the value must be changed when the bridge is in-service for a few years, the real value is larger or smaller than original ones. Hence, the lower and upper boundary of each parameter is set as 0.5 and 1.5. With the constrained parameters, 400 samples of datasets for establishing Kriging or BPNN meta models are obtained using the LHS technique.

In both of the two hybrid methods, a number of particles N is 200, the inertia weight w is 0.55, the positive learning coefficients c_1 and c_2 are both 2, the probability of P_m is set as 0.1, and the maximum iteration number is 200, the search boundary is 0.6 to 1.4 and the objective function is the sum of residuals of frequencies of first five modes between modified and experimental modes like Equation (13), where N_j denotes the number of considered modes, f_j^t and f_j^c denote the test and finite element calculated frequency value of jth mode respectively.

$$Obj = \sum_{j=1}^{N_j} \left(\frac{f_j^t - f_j^c}{f_j^t} \right)^2 . \tag{13}$$

Table 2 shows the updated values of parameters by the two hybrid methods and the modified frequencies together with the differences compared to test values are given in Table 3. Meanwhile, the modified mode shapes together with the initial and test values are shown in Fig. 9.

Table 2 indicates that the modified values of material modules are higher than the original value provided by the code, values of inertia moment are greater than the initial value calculated according to the design drawings, but the cross-section and the second dead load are less than the design value. All this shows that the stiffness of the real structure is higher than designed one, but the actual mass of actual structure is less than designed one. From Table 3 the updated parameter values and the frequency values are obtained. Especially in Table 3 the frequency differences between test and modified model reduce sharply compared

Table 2. Parameter values of modified model of bridge with two proposed methods

Parameter	1	2	3	4	5	6	7	8	9	10
Hybrid Method A	1.394	1.324	1.100	1.166	1.281	1.262	1.038	1.240	0.703	0.803
Hybrid Method B	1.241	1.392	1.148	1.000	1.291	1.175	1.126	1.243	0.725	0.897

Table 3. The comparison between modified and experimental frequencies

Mode	Experimental frequency, Hz	Modified frequency, Hz		Difference, %	
		Hybrid Method A	Hybrid Method B	Hybrid Method A	Hybrid Method B
1	0.9030	0.9161	0.9250	1.45	2.43
2	1.3670	1.3427	1.3624	1.78	0.33
3	1.3790	1.4600	1.4535	5.87	5.40
4	1.9290	1.8246	1.8337	5.41	4.94
5	2.4170	2.2810	2.3023	5.63	4.75

Note: $\text{Difference}=\frac{|\text{Modified frequency}-\text{Experimental frequency}|}{\text{Experimental frequency}}\cdot 100\%$.

to results of the initial model in Table 1 as all the values are less than 6%, but they are around 25%–40% in the original model. Furthermore, from the MACs in Fig. 9, although the values increase a little compared to those in Table 1, this also illustrates that more agreeable mode shapes are obtained after model updating as all of the MACs are over 0.86. The particular advancements in modified FEM, especially the improvements in frequency values, indicate that a relatively more adequate FEM of this self-anchored suspension bridge with extra-width is achieved with relatively high efficiency without losing accuracy. However, after comparison among the two hybrid methods, the differences in Table 3 and the MACs in Fig. 9 illustrate that the frequency results of Hybrid Method B are relatively better than method one when the mode shapes of both methods show the same results. On the other hand, the efficiencies of these two approaches show the significant difference as Hybrid Method A occupies more analysis time than Hybrid Method B. These two hybrid methods show high efficiency and accuracy compared to direct GMPSO optimization process. However, through the coparison between the two Hybrid Methods, it still indicates the Hybrid Method B has higher efficiency than Hybrid Method A with the relatively same accuracy. It is concluded that the proposed hybrid model updating method two, namely the method with BPNN meta model, has more significant value in model updating of engineering applications.

6. Conclusions

Two hybrid model updating methods by integration of Gaussian mutation particle swarm optimization, Latin Hypercube Sampling method and meta models of Kriging and Back-Propagation Neural Network respectively was proposed in this article. Compared to ordinary model updating technique with polynomial meta model, Kriging or Back-Propagation Neural Network meta models with improved particle swarm optimization increase the accuracy and the efficiency. Furthermore, the Latin Hypercube Sampling technique is first used in establishing meta model process to improve the quality of input datasets. With the application of the hybrid methods to model updating of a self-anchored suspension bridge with extra-width located in China, the comparison among the two proposed methods was made, and several conclusions are drawn as follows.

1. Based on the results of the dynamic characteristic of original Finite Element Model and ambient excitation test of Hunan Road Bridge, the differences in frequencies are large enough as most of the values are around 25%–40%. However, the mode shapes of the test show high agreement with the theoretical ones as the lowest modal assurance criteria value is 0.849. Furthermore, because of the complex structure of extra-width, the torsion mode emerges earlier than in ordinary suspension bridges. Hence, it is needed to yield an adequate Finite Element Model for further performance analyses.

2. After model updating with the two proposed method, the frequency differences between test and modified model reduce sharply compared to results of the initial model as all the difference values are below 6%. The values of modal assurance criteria increase a little, and this illustrates that more agreeable mode shapes are obtained after model updating with all of the values of modal assurance criteria over 0.86. The particular advancements in modified Finite Element Model, especially in frequency values, indicate that a relatively more adequate model of this self-anchored suspension bridge has been yielded.

3. With the comparison of the two hybrid methods in the process of model updating of the bridge, it is concluded that the hybrid method with Back-Propagation Neural Network meta model is better than the one with Kriging meta model. Under the same results of the mode shapes, the frequency differences of the former method mostly under 5% but those of the latter one are not. Furthermore, the efficiencies of these two approaches show the significant difference as the latter occupies more analysis time than the former. Compared to direct Gaussian mutation particle swarm optimization, these two hybrid methods

show high efficiency and accuracy, but the hybrid method with Back-Propagation Neural Network meta model has higher efficiency than the one with Kriging meta model within the relatively same precision. Thus, this method is suitable for model updating of engineering applications with large-scale, multi-dimensional parameter structures involving implicit performance functions.

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