

# A NETWORK-BASED IMPORTANCE MEASUREMENT INDEX FOR BRIDGE SECURITY RISK ASSESSMENT AND PRIORITISATION

MEHDI DEZFULI NEZHAD, REZA RAOUFI\*,  
AHMAD DALVAND

*Department of Civil Engineering, Ahvaz Branch, Islamic Azad University,  
Ahvaz, Iran*

Received 28 November 2020; accepted 22 October 2021

**Abstract.** In the related literature, conventional approaches to assessing security risk and prioritising bridges have focused on unique characteristics. Although the unique characteristics appropriately reflect the economic and social consequences of failure, they neglect the consequences of a bridge failure at the network level. If network owners and operators prioritise bridges solely based on their unique characteristics, bridges with low object-level importance and high network-level importance have very low chances to get priority. In this paper, a bridge importance measurement index  $\alpha(e)$  has been presented, prioritising bridges based on their unique characteristics, location and network topology. To describe how to use this index  $\alpha(e)$ , three numerical examples were provided. While the first example was related to a simple hypothetical network, the second and third examples were real networks related to the bridges of Wrocław city. Using these examples, the results of bridge prioritisation obtained in the unique-characteristics-only state were compared to the state in which  $\alpha(e)$  had been used. Results showed that considering the location of the

\* Corresponding author. E-mail: [r\\_raoufi@iauahvaz.ac.ir](mailto:r_raoufi@iauahvaz.ac.ir)

Mehdi DEZFULI NEZHAD (ORCID ID 0000-0003-1386-2155)

Reza RAOUFI (ORCID ID 0000-0002-0328-1886)

Ahmad DALVAND (ORCID ID 0000-0003-3042-451X)

Copyright © 2022 The Author(s). Published by RTU Press

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

bridge and the topological characteristics of the network change the bridges prioritisation. For instance, in the second example, it was observed that the use of the  $\alpha(e)$ , made bridge Bolesława Krzywoustego the essential bridge, while bridge Grunwaldzki was the essential bridge under the previous prioritisation made by researchers. However, the results of the third example showed that bridge Milenijny, which was considered the essential network bridge as stated in the previous prioritisation made by researchers, was again selected as the most critical bridge based on the  $\alpha(e)$ .

**Keywords:** bridge, disaster, resilience, security risk assessment, transportation network.

## Introduction

Transportation infrastructures have been considered attractive targets for terrorists due to their accessibility and potential impact on human lives and economic activities (Yi et al., 2014). Bridges are the most vulnerable components of the transportation network (Garg et al., 2020), and their failure due to destructive events leads to substantial economic and social consequences. Research works on the most important causes of bridge failure in different countries have shown that security threats such as terrorist attacks are as crucial as other threats (Diaz et al., 2009; Frangopol et al., 2010).

Furthermore, research works have proved that the economic consequences of a large-scale terrorist attack could be as much as a severe earthquake or hurricane (on the order of \$100 billion) (Al Kazimi & MacKenzie, 2016). Also, terrorist attacks of a smaller magnitude, such as the destruction of significant bridges, have cost estimates on par with less severe hurricanes (Al Kazimi & MacKenzie, 2016). For example, some researchers have estimated that an attack that destroys bridges over the Mississippi River or leads into Denver could lead to losses in the US economy of \$17.8 billion (Richardson et al., 2014).

Their reliability against any terrorist attacks is increased by allocating financial resources and implementing countermeasures such as retrofit actions to prevent economic and social consequences of bridges failure. However, retrofitting all the bridges in the network is impossible (Williamson & Winget, 2005) because the cost of building bridges capable of resisting all possible potential blasts would be very high (Deng et al., 2016). Moreover, available financial resources are often significantly insufficient to cover all the bridges in the network. Therefore, network owners need to identify critical bridges through a proper security risk assessment and prioritisation. The risk assessment model and criteria used to identify and prioritise critical bridges significantly impact decision-making (Kučas, 2015; Macek & Mestanová,

2009). Changing the criteria used for ranking the network bridges lead to selecting different bridges to receive the financial resources.

In the related literature, conventional approaches to assessing security risk and prioritising bridges focus on criteria based on unique characteristics, such as geometric dimensions, transit traffic, symbolic importance, cost, and reconstruction time.

However, some literature has turned its attention to network-level characteristics and the concept of resilience. Although the unique characteristics appropriately reflect its economic and social failure consequences, they neglect them at the network level. In addition to unique characteristics, the bridge failure consequences are also related to its network-level characteristics. The mere use of indicators based on the unique characteristics of the bridge without considering the topological characteristics of the network and branch on which the bridge is located reduces the quality of decision making. It prevents the identification of real critical bridges.

In case of resource constraints in which a limited number of network bridges are allowed to be retrofitted, if network owners prioritise bridges only based on their unique characteristics, bridges with lower object-level importance and higher network-level importance have very low chances to get priority. This article aims to present an important measurement index  $\alpha(e)$  for bridge security risk assessment and prioritisation, which prioritises bridges based on their unique characteristics and considering their location in the network and its effect on reducing the resilience network on which a bridge is located.

The remaining part of this paper is divided into five major sections and a conclusion section as follows:

1. background;
2. resilience concept and measures;
3. problem statement and objective;
4. importance measurement index;
5. numerical examples.

The first section gives a brief overview of security risk models, equations, and criteria used in other research work to identify critical bridges. The second section begins by explaining various definitions of the resilience concept. It then reviews and summarises research related to resilience and focuses on resource allocation to the bridge. Also, a brief overview of various metrics used to measure the resilience of transportation network are given at the end of this section. In the third section, a hypothetical example is provided to determine the research problem and research questions to describe the aim and objective of the study. The introduction and methodology of calculation of importance measurement index  $\alpha(e)$  are available in the fourth section. In the fifth

section, three numerical examples are presented to compare differences in bridges prioritisation based on the unique-characteristics-only used in other research works and bridges prioritisation based on  $\alpha(e)$ . Finally, conclusions are drawn in the last section.

## 1. Background

Special attention has been paid in the literature to research security risk and man-made disasters (Garcia & von Winterfeldt, 2016; Rios & Insua, 2012; Zhang & Reniers, 2016). However, much of this research has focused on other assets, such as buildings, aeroplanes, chemical, and industrial facilities, and less has focused on bridges. For example, the US Federal Emergency Management Agency (FEMA) has published many different guidelines related to security risk assessment (Chipley et al., 2003, 2012; Chipley & Lasch, 2007; Hinman et al., 2003; Kennett et al., 2005; Krimgold, 2003; Smith et al., 2004), all of which have focused on assessing and managing security risks in a variety of buildings (residential, commercial, school, shelters and, safe rooms).

Risk assessment methods for mitigation decisions related to natural hazards are sufficiently well established. The application of these methods to non-natural hazards is relatively narrow. There is a shortage of well-established comprehensive procedures to determine how to allocate limited resources among bridges (Roberts et al., 2003). Some research related to bridge security risk has applied approaches presented for natural disasters to prioritise bridges (Roberts et al., 2003). Other researchers have proposed separate models specific to man-made disasters. Table 1 lists these studies.

The research work mentioned in Table 1 presented different models for security risk assessment and prioritisation of network bridges. The general form of these models is as follows Equation (1):

$$R = O \cdot V \cdot I. \quad (1)$$

In Equation (1), the risk is defined as the multiplication of three parameters of occurrence ( $O$ ), vulnerability ( $V$ ), and the importance of the bridge ( $I$ ) (Valeo et al., 2012). The occurrence factor ( $O$ ) approximates the likelihood of an attack on the bridge and is usually expressed as a relative probability for different bridges (Roberts et al., 2003). Some researchers have considered the vulnerability ( $V$ ) and the importance of the bridge ( $I$ ) to estimate this relative probability (Duchaczek & Skorupka, 2013, 2016; Issa, 2008; Nassif et al. 2006) so that meaningful and vulnerable bridges are attractive targets. The intelligence and strategic nature of man-made threats and the existence of a correlation between these three parameters have led

conventional approaches to estimate this probability to be criticised by some researchers (Brown & Cox, 2011; Cox, 2008; Greenberg et al., 2012; National Research Council, 2010). They have suggested using Nash equilibrium to analyse this probability instead of considering a constant relative probability for different bridges. Nash equilibrium is a concept within game theory where the optimal outcome of a game is no incentive to deviate from the initial strategy (Feng et al., 2016).

Table 1. Research works related to bridges security risk assessment and attributes for identifying the critical bridge

Risk assessment equation; reference	Level	Attributes for identifying the critical bridge
$TBR = \sum \left[ I_j \sum (O_{ij} \cdot V_{ij}) \right];$ <p>Ray, 2007</p>	Among individual components of a single bridge	<ul style="list-style-type: none"> <li>- structural importance;</li> <li>- historical and symbolic importance;</li> <li>- repair cost;</li> <li>- time out of service;</li> <li>- span ratio;</li> <li>- span length.</li> </ul>
$BCI = \left[ \frac{1}{8} \left( \sum_{i=1}^8 CI_i \frac{F_{CI_i}}{CI_{i_{max}}} \right) \right] RF;$ <p>Rummel et al., 2002</p>	Among multiple bridges	<ul style="list-style-type: none"> <li>- commerce importance – truck ADT in vpd;</li> <li>- transportation needs importance – ADT in vpd, Detour length;</li> <li>- connectivity importance – ADT in vpd, on the intersecting route, interstate intersection;</li> <li>- navigational access importance – if the bridge requires a Coast Guard permit;</li> <li>- international access importance – borders bridge;</li> <li>- military movement importance – if the bridge is located on the Strategic Highway Network;</li> <li>- replacement and repair importance – structural complexity and span length.</li> </ul>
$R_{ij} = \sum_{k=1}^6 \left[ w_k^A \sqrt{\left( \sqrt{a_{ij}^{k_1} a_{ij}^{k_2}} \sum_{l=1}^3 \left[ w_l^C c_{ij}^{k_l} \right] \right)} \right];$ <p>Li et al., 2016</p>	Among individual components of a single bridge	<ul style="list-style-type: none"> <li>- economic loss;</li> <li>- time lost;</li> <li>- social impact.</li> </ul>

Risk assessment equation; reference	Level	Attributes for identifying the critical bridge
$R = p \cdot c;$ Duchaczek & Skorupka, 2016	Among multiple bridges	<ul style="list-style-type: none"><li>- traffic volume – ADT in vpd;</li><li>- length of the analysed span – bridges with spans longer than 30 m;</li><li>- construction material – wood, stone, concrete, reinforced concrete, and steel;</li><li>- spans construction – cable-stayed, suspended bridges, truss and beam bridges;</li><li>- access to the span bottom – height of the spans above water or land;</li><li>- bridge protection – distance from the city centre.</li></ul>
$R_i = \frac{p_i c_i}{\sum_{i=1}^n (p_i c_i)};$ Duchaczek & Skorupka, 2013	Among multiple bridges	<ul style="list-style-type: none"><li>- maximum span length;</li><li>- number of lanes on the bridge;</li><li>- number of indirect pillars;</li><li>- traffic volume;</li><li>- alternative passage.</li></ul>
$RS = IF \sum [OF_i \cdot VF_i];$ Roberts et al., 2003	Among individual components of a single bridge	<ul style="list-style-type: none"><li>- historic and symbolic;</li><li>- replacement value;</li><li>- evacuation route;</li><li>- regional economy;</li><li>- transportation network;</li><li>- annual revenue;</li><li>- attached utilities;</li><li>- military route;</li><li>- exposed population.</li></ul>
$R = O \cdot V \cdot I;$ Davis et al., 2017	Among multiple bridges	<ul style="list-style-type: none"><li>- the social and economic impact of bridge loss;</li><li>- the role played by the bridge in defence and security of the region, state and nation;</li><li>- average daily traffic in vpd;</li><li>- average daily traffic of heavy vehicles in vpd;</li><li>- distance to nearest detour;</li><li>- symbolic importance.</li></ul>

Risk assessment equation; reference	Level	Attributes for identifying the critical bridge
$CS(B) = \frac{x}{c_{max}} 100;$ Smith et al., 2002	Among multiple assets (bridges)	<ul style="list-style-type: none"> <li>- loss and damage consequences – casualty risk, environmental impact, replacement cost, replacement and downtime;</li> <li>- consequences to public services – emergency response function, government continuity, military importance;</li> <li>- consequences to the general public – available alternate, communication dependency, economic impact, functional importance;</li> <li>- symbolic importance.</li> </ul>
Risk severity matrix; Leung et al., 2004	Among multiple bridges	<ul style="list-style-type: none"> <li>- multidimensional at a national level;</li> <li>- military operation impact;</li> <li>- economic impact at a national level;</li> <li>- impact on an evacuation route.</li> </ul>
$R = O \cdot V \cdot I;$ Issa, 2008; Valeo et al., 2012	Among multiple bridges	<ul style="list-style-type: none"> <li>- near, or on route to a high-value target;</li> <li>- over or near a chemical, refinery, or industrial facility;</li> <li>- length of the longest span;</li> <li>- annual average daily traffic in vpd;</li> <li>- part of an evacuation route;</li> <li>- culturally or historically significant.</li> </ul>

*Note:*  $TBR$  – total bridge risk;  $j$  – individual bridge component;  $i$  – primary threat;  $I_j$  – importance of an individual component,  $j$ , to the bridge;  $O_{ij}$  – a measure of the relative probability of a primary threat,  $i$  occurring against the given component,  $j$ ;  $V_{ij}$  – a measure of the relative vulnerability of the given component,  $j$ , given the occurrence of the primary threat,  $i$ ;  $BCI$  – bridge criticality index;  $CI_j$  – importance criteria;  $F_{CLI}$  – relative importance of criterion  $i$  to the other criteria;  $CI_{imax}$  – the maximum value of criterion  $i$  among all bridges;  $RF$  – replacement factor;  $R_{ij}$  – total risk score weighted by six attack methods;  $w_k^A$  – weight of the possibility of different attack methods;  $c_{ij}^{k_1}$  – an indicator of comprehensive loss;  $a_{ij}^{k_1}$  – evaluation criteria for an indicator of attack convenience;  $a_{ij}^{k_2}$  – evaluation criteria for an indicator of attack concealment;  $w_i^C$  – weight of indicator of comprehensive loss;  $R$  – risk of damage to the bridge structure;  $p$  – the probability of a terrorist attack on a bridge;  $c$  – a consequence of the destruction (damage);  $R_i$  – risk of terrorist attack occurrence at bridge  $i$ ;  $p_i$  – probability of destructing bridge  $i$ ;  $c_i$  – a consequence of destructing bridge  $i$ ;  $RS$  – risk score;  $IF$  – importance factor;  $OF_i$  – occurrence factor;  $VF_i$  – vulnerability factor;  $RS$  – risk;  $I$  – importance;  $O$  – occurrence factor;  $V$  – vulnerability;  $CS(B)$  – bridge score coordinate;  $x$  – critical asset values;  $c_{max}$  – the maximum possible criticality value.

Some researchers have used multi-criteria approaches to determine the vulnerability of bridges ( $V$ ) (Li et al., 2016; Valeo et al., 2012). However, some others have considered this parameter based on the probability of bridge failure in the case of a successful attack (Dillon et al., 2009; Ezell et al., 2010; Keeney & Von Winterfeldt, 2011). Also, to determine vulnerability, the fragility curve of a bridge has been used (Kim & Lee, 2020). As blasting is the most common attack mode, some studies have used the scaled distance ( $z$ ) to measure the bridge fragility curve (Singh et al., 2020; Yu et al., 2018). It is worth mentioning that this paper only focuses on the criteria used to measure ( $I$ ) and the methods of measuring ( $V$ ) and ( $O$ ) are out of the scope of this work.

The parameter of bridge importance ( $I$ ) indicates the economic and social consequences of bridge failure (Issa, 2008; Roberts et al., 2003). Various criteria have been proposed to assess this parameter. Some of them are presented in the last column of Table 1. The criteria proposed in the reviewed research for determining the importance of the bridge mainly focused on the unique characteristics of the bridge, while the topological characteristics of the branch and network on which the bridge is located received less attention.

## 2. Resilience concept and measures

There are various definitions for resilience in different resources (Petersen et al., 2020). A general standard definition is a rapid return to the initial conditions after the disruptive event (Hosseini et al., 2016). Concerning the critical infrastructure, also a definition is presented by the National Infrastructure Advisory Council (2009) (NIAC), which has been accepted worldwide defines resilience as the ability to predict, absorb, adapt to and rapid exit and recover to the initial conditions against the probable disruptive events (Kameshwar et al., 2019). In the case of road infrastructures, resilience refers to the ability to deliver a particular service level even after an extreme event and recover their proper functionality as fast as possible (Giunta, 2017).

Furthermore, Bruneau et al. (2003) have introduced four different indices for the concept of resilience as follow:

1. robustness – ability to stand against significant events and crises and preserve a predefined and a specific level of service after an occurrence of the disasters;
2. redundancy – the ability of the elements and constituent components of a system in replacing and covering each other;
3. resourcefulness – the organisational power to apply disaster management including the ability to recognise and understand the



disaster and problem, the ability to prioritise the problems, the ability of planning and organising the human forces and financial resources after the disaster;

4. rapidity – returning to a specific and acceptable service level in a short time (Zhang & Wang, 2016).

Much research related to resilience has dealt mainly with resource allocation to the bridges to increase network resilience. This group of studies is classified into three categories in terms of the applied resource allocation strategy:

1. the research concentrated upon the pre-disaster period;
2. the research concentrated upon the post-disaster period;
3. the research concentrated simultaneously upon the pre and post-disaster periods.

Table 2 shows some related research works agreeing to the classification mentioned above.

**Table 2. Research works related to resilience and focuses on resource allocation to the bridges**

Phase(s)	General problem	Reference
Risk mitigation, preparedness	Selecting bridges to be retrofitted and optimising allocation of retrofit resources to bridges	Chang et al., 2012; Dong et al., 2014; Liu et al., 2009; Lu et al., 2016; Zhang & Wang, 2016.
Emergency response, recovery	Optimising post-disaster bridge restoration sequence and optimising post-disaster repair strategies to minimise total costs	Bocchini & Frangopol, 2012a, 2012b; Decò et al., 2013; Frangopol & Bocchini, 2011; Karamlou & Bocchini, 2014; Li et al., 2019; Liu et al., 2020; Merschman et al., 2020; Vugrin et al., 2014; Zhang et al., 2017.
Mitigation, preparedness, response and recovery	Optimising preparedness and recovery actions	Faturechi & Miller-Hooks, 2014; Liao et al., 2018; Zhang & Alipour, 2020; Zhang et al., 2015, 2018.

Various indices have been introduced to measure network resilience. These metrics are generally classified into three groups:

1. topological metrics;
2. property-based metrics;

3. performance-based metrics (Zhou et al., 2019).

In this paper, the topological metrics less discussed in the bridge security-related literature have been used. This group of metrics are generally based on properties of the network, such as the betweenness centrality or shortest path length. Table 3 depicts some of these metrics (Zhou et al., 2019).

Table 3. Some of the topological metrics to measure the transportation network resilience

Reference	Metrics	Reference	Metrics
Schintler et al., 2007	- network diameter - average shortest paths	Testa, 2015	- average node degree; - clustering coefficient; - betweenness centrality; - redundancy.
Berche et al., 2009	- size of a giant component - average shortest paths	Chopra et al., 2016	- degree assortativity; coefficient.
Osei-Asamoah & Lownes, 2014	- efficiency - size of a giant component	Aydin et al., 2018	- betweenness centrality; - size of a giant component; - efficiency.
Hartmann, 2014	- backup capacity	Zhang et al., 2015	- average degree; - network diameter; - cyclicity.
Liao et al., 2018	- network connectivity		

Note: by Zhou et al. (2019).

3. Problem statement and objective of the work

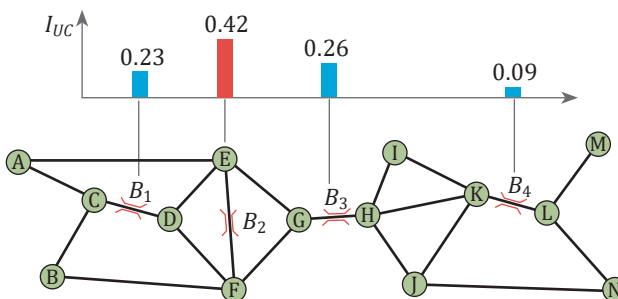
Figure 1 shows a hypothetical transportation network. As shown, this network has four bridges in different locations ( $B_1, B_2, B_3, B_4$ ). The network owner intends to increase the reliability of bridges against possible terrorist attacks and minimise the economic and social consequences of the failure of the bridge through allocating financial resources. It is assumed that ( $O$ ) and ( $V$ ) factors are the same for all bridges, so the main factor for selecting a bridge as the target is the importance ( $I$ ).

Also, it is assumed that attackers be able to damage one bridge at most. The attacked bridge remains intact if the resources are allocated; otherwise, it completely collapses. Also, it is assumed that, due to limited financial resources, the network owners could only protect one of the four bridges through risk evaluation and prioritisation (Cox, 2009; Feng et al., 2016; Zhang & Ramirez-Marquez, 2013).

The above diagram in Figure 1 shows the relative score obtained from evaluating the importance of the bridges based on their unique characteristics ( $I_{UC}$ ). It is observed that bridge  $B_2$  has gained the highest importance score due to its unique characteristics (more replacement cost and replacement and downtime, more transit traffic, larger span, its symbolic value). If the location of the bridge in the network and network topology are disregarded, this bridge  $B_2$  is selected as the essential bridge by the owners.

However, bridge  $B_3$  is more central than other bridges in the network. The centrality of this bridge allows it to play a role in more communication routes. Therefore, the failure of this bridge compared to a bridge  $B_2$  causes more communication routes to be lost. Moreover, it is observed that if a bridge  $B_3$  collapses, the network divides into two completely separate parts. Because there are no alternative routes for this bridge ( $B_3$ ). But if bridge  $B_2$  collapses, there are two alternative routes for this bridge ( $B_2$ ), which are accessible until the route reopens.

The economic and social consequences of a bridge failure and its unique characteristics are also related to some of its characteristics at the network level. In case of resource constraints in which a limited number of network bridges are allowed to be retrofitted, if network owners prioritise bridges only based on their unique characteristics, a bridge with low object-level importance and high network-level importance has very low chances to get priority.



**Figure 1.** Importance score of bridges  $B_1$ ,  $B_2$ ,  $B_3$ , and  $B_4$ , based on their unique characteristics

## 4. Importance measurement index $\alpha(e)$

In this paper, an important measurement index  $\alpha(e)$  has been presented, along with the unique characteristics of the compared bridges, it considers their location in the network and the network topology to decide on the importance of the bridge.

$$\alpha(e) = f[I_{UC}(e), I_{BCW}(e), I_{IPW}(e)]. \quad (2)$$

In the above relation (Equation (2))  $I_{UC}$  is the measure of bridge importance based on its unique characteristics,  $I_{BCW}$  indicates a bridge communication-related importance measure in normal service conditions (normal functionality aspect), and  $I_{IPW}$  indicates a bridge communication-related importance measure in emergencies (resilience aspect). This paper assumed that  $I_{UC}$  is calculated based on the criteria presented in previous research. Therefore, in the following,  $I_{BCW}$  and  $I_{IPW}$  indices and the way of their calculation have been explained.

### 4.1. Communication importance of bridge under normal conditions $I_{BCW}(e)$

The basic function of transportation systems is to transfer traffic from a source node to a destination node (Zhang & Wang, 2016). Under normal circumstances, users of this system logically choose the shortest path between the source and destination nodes (Ramazani et al., 2011). Each path between network  $O-D$  pairs consists of one or more branches connected in series. Suppose a branch is located in a large number of short paths between different nodes. In that case, its communication importance increases under normal conditions since its removal affects many short paths generally used by network users.

The number of short paths associated with a branch is measured based on its betweenness centrality. Betweenness centrality is a standard topological measure in graph theory. It is calculable for both nodes and branches (Lu & Zhang, 2013). This index for branch  $(e)$ , as shown in Equation (3) is the sum of the ratio of the number of short paths between nodes  $(i$  and  $j)$  that pass-through branch  $e(b_{ij}(e))$  to the total number of short paths between nodes  $i, j, (b_{ij})$  (Girvan & Newman, 2002).

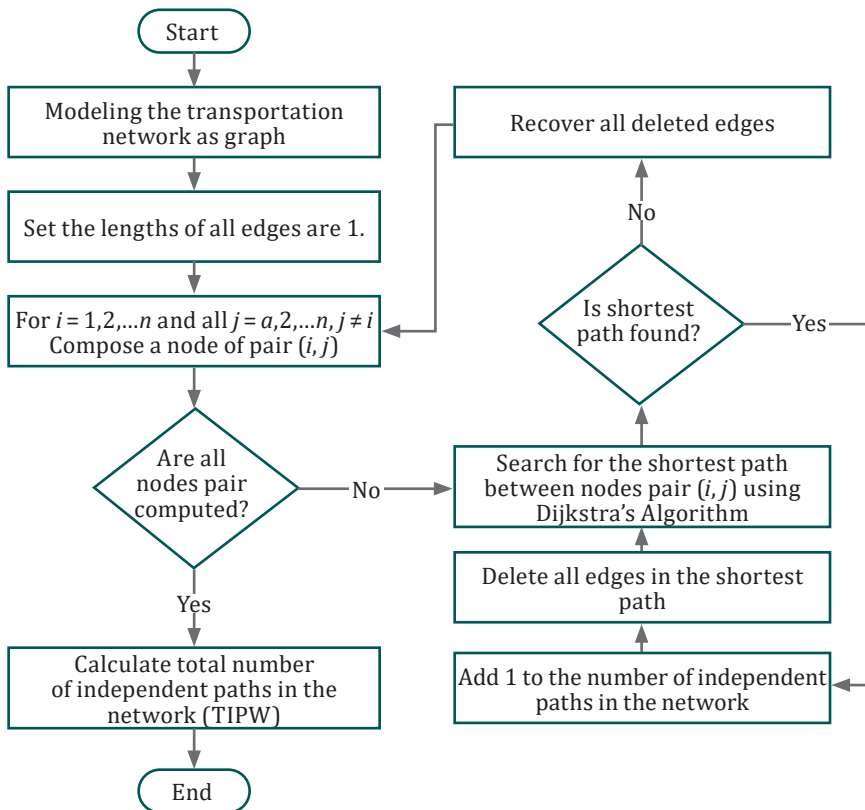
$$I_{BCW}(e) = \sum_{i,j \in V} \frac{b_{ij}(e)}{b_{ij}}. \quad (3)$$

Betweenness centrality is calculable for all network branches (with or without bridges). This paper presents a calculation of the betweenness centrality only for branches with bridges. Bridges with

a more significant betweenness centrality are critical because more shortest paths are lost if they collapse.

#### 4.2. Communication importance of bridge in emergencies $I_{IPW}(e)$

In comparing the importance of bridges, it is essential to consider the effects of their failure on both typical network communication performance and reduced network preparedness for emergency response. In this article, the preparedness of the network for emergency response was measured based on the number of independent pathways. More independent pathways in a network result in more network preparedness to deal with disruptive events. Two pathways between the same  $O-D$  pair are independent when they do not share any common road link or branch (Zhang & Wang, 2016). Figure 2 illustrates finding the number of independent paths in a network.



**Figure 2.** The process of finding the  $(T_{IPW})$

To calculate  $I_{IPW}(e)$ , the total number of network-independent paths ( $T_{IPW}^0$ ) is counted once based on the process shown in Figure 2. The studied branch ( $e$ ) is then removed from the network, and the total number of independent paths of the network ( $T_{IPW}^e$ ) is recounted. As stated in Equation (4) emergency importance of a branch ( $e$ ) is equal to the number of reduced independent paths ( $\Delta_{TIPW}$ ) due to the removal of that branch.

$$I_{IPW}(e) = \Delta_{TIPW} = T_{IPW}^0 - T_{IPW}^e. \quad (4)$$

It is worth mentioning that the number of network-independent paths is not one of the common topological indicators in graph theory. It is a novel indicator presented for the first time by Ip & Wang (2011) and then developed by Zhang & Wang (2016).

It should also be noted that  $I_{IPW}$  is directly related to the number of alternative routes of a bridge. However, it is quite different from the criteria related to the alternative route used in some studies previously mentioned in Table 1. For example, the distance to the nearest detour (Davis et al., 2017) or the criterion of the detour length (Rummel et al., 2002) reflect the indirect economic impact of bridge failure and are irrelevant to resilience aspects of the network. It is worth mentioning that, in case of bridge failure, a longer route must be taken by drivers. Therefore, the operation cost of vehicles in the detour ( $C_{op}$ ) as well as cost due to vehicle time loss ( $C_{TL}$ ) increase due to longer travel time for users (Banerjee et al., 2019). Some researchers have suggested the following relationships to estimate the indirect cost. In these relationships, the effect of increasing the alternative route length on indirect costs is well seen (Equations (5)–(6)):

$$C_{op} = P_E \left[ C_{op.car} \left( 1 - \frac{TR_D}{100} \right) + C_{op.truck} \left( \frac{TR_D}{100} \right) \right] D_I \cdot ADT. \quad (5)$$

$$C_{TL} = P_E \left[ C_{AW} O_{car} \left( 1 - \frac{TR_D}{100} \right) + (C_{ATC} O_{truck} + C_{good}) \left( \frac{TR_D}{100} \right) \right] \cdot \left[ ADT \frac{D_I}{S} + ADE \left( \frac{l}{S_D} - \frac{l}{S_0} \right) \right]. \quad (6)$$

In the above relation (Equations (5)–(6)),  $C_{op.car}$  and  $C_{op.truck}$  are the average costs of operation of car and truck per kilometre length in USD/km, respectively,  $D_I$  is the detour length in km,  $ADT$  is the average daily traffic in vpd, and  $TR_D$  is the average daily truck traffic ratio in per cent. Also,  $C_{AW}$ ,  $C_{ATC}$  and  $C_{good}$  are average wage per hour in USD/h, average total compensation per hour in USD/h, and monetary value of time taken to transport goods in cargo in USD/h, respectively.  $O_{car}$  and

$O_{truck}$  are average vehicle occupancies for car and truck, respectively,  $l$  is the length of link in km,  $S$  is the average speed on detour in km/h,  $S_D$  and  $S_0$  are average speeds on the damaged and intact bridge in km/h, respectively, and  $ADE$  is average daily traffic remaining on the bridge after the event in vpd. It should be noted that the economic analysis of bridge failure is out of the scope of this article.

Also,  $I_{IPW}$  is different from the “lack of alternative route” criterion used in some studies (Duchaczek & Skorupka, 2016; Smith et al., 2002). In these research works, the lack of alternative routes increased the importance of a bridge. However,  $I_{IPW}$  is a quantitative measure used to evaluate the level of communication performance for all bridges (with or without an alternative route). For example, in Figure 1, bridges  $B_1$ ,  $B_2$ , and  $B_4$  have alternate routes. However, the failure of a bridge  $B_4$  further reduces the number of independent routes in the network. Under this index, any bridge that loses more independent routes due to its failure is more important. Failure of bridges with fewer or no alternative routes reduces network-independent pathways and is considered essential.

#### 4.3. Calculation of bridge importance measurement index $\alpha(e)$

To calculate  $\alpha(e)$ , values of  $I_{BCW}$  and  $I_{IPW}$  were first calculated using the relationships presented in the previous sections. It was also assumed that  $I_{UC}$  had been calculated using the criteria presented in (Duchaczek & Skorupka, 2013, 2016). Then, the above indicators were combined, and a single index was considered to compare the failure consequences of each bridge.

The entropy method is a scientific and reasonable method to combine multiple subjective and objective factors (Guo et al., 2017). This method has also been used in the present research to consider the synthetic effects of different indicators. Therefore, an initial matrix was first established using Equation (7) for the whole network:

$$r = [\tilde{r}_{ij}]_{m,n}. \quad (7)$$

In Equation (7),  $\tilde{r}_{ij}$  is a standardised non-dimensional value of the  $i^{\text{th}}$  index ( $I_{UC}$ ,  $I_{BCW}$ ,  $I_{IPW}$ ) in the  $j^{\text{th}}$  bridge,  $m$  is the number of indexes affecting the importance, and  $n$  is the total number of bridges compared in the network. Because increasing all the three studied indexes promotes the importance of the bridge, so they were standardised as non-dimensional parameters using Equation (8):

$$\tilde{r}_{ij} = \frac{r_{ij} - \min(r_{ij})}{\max(r_{ij}) - \min(r_{ij})}. \quad (8)$$

Let  $P_{ij}$  be the proportion of importance index  $i$  of bridge  $j$  (Equation (9)) (Guo et al., 2017):

$$P_{ij} = \frac{\tilde{r}_{ij}}{\sum_{j=1}^n \tilde{r}_{ij}}. \quad (9)$$

Then, the entropy value of  $P_{ij}$  was calculated using Equation (10) (Hsu & Lin, 2006).

$$S_i = -\frac{1}{\ln(n)} \sum_{j=1}^n P_{ij} \ln(P_{ij}). \quad (10)$$

In particular, in Equation (10), it is assumed that,  $P_{ij} \ln(P_{ij}) = 0$  when  $P_{ij} = 0$  (Guo et al., 2017). The smaller the value of  $S_i$ , the less the importance of  $i^{\text{th}}$  index. Therefore, the weight of each index affecting the importance is calculated using Equation (11):

$$\omega_i = \frac{1 - S_i}{m - \sum_{i=1}^m S_i}. \quad (11)$$

On the other hand, the weight of each index affecting the importance based on the judgment of decision-makers must be considered  $\lambda_i$ , so the final weight of each indicator is considered using Equation (12):

$$\kappa_i = \frac{\lambda_i \omega_i}{\sum_{i=1}^m \lambda_i \omega_i}. \quad (12)$$

It should be noted that in this paper, the weight of each index affecting the importance based on the judgment of network owners was considered under Table 4:

Table 4. The considered weights for the indexes which affect the bridge importance

$\lambda_i$	$I_{IPW}$	$I_{BWC}$	$I_{UC}$
	0.25	0.25	0.5

Finally, the importance measurement index  $\alpha(e)$ , was calculated using Equation (13):

$$\alpha(j) = \frac{\sum_{i=1}^m \kappa_i \tilde{r}_{ij}}{\sum_{i=1}^m \kappa_i \tilde{r}_{ij}}. \quad (13)$$

The Equation (13) for the indexes used in this article was rewritten as follows Equation (14):

$$\alpha(e) = \frac{\kappa_{I_{IPW}} \cdot \tilde{I}_{IPW}(e) + \kappa_{I_{BWC}} \cdot \tilde{I}_{BWC}(e) + \kappa_{I_{UC}} \cdot \tilde{I}_{UC}(e)}{\sum_{j=1}^n (\tilde{I}_{IPW}(e) + \tilde{I}_{BWC}(e) + \tilde{I}_{UC}(e))}. \quad (14)$$

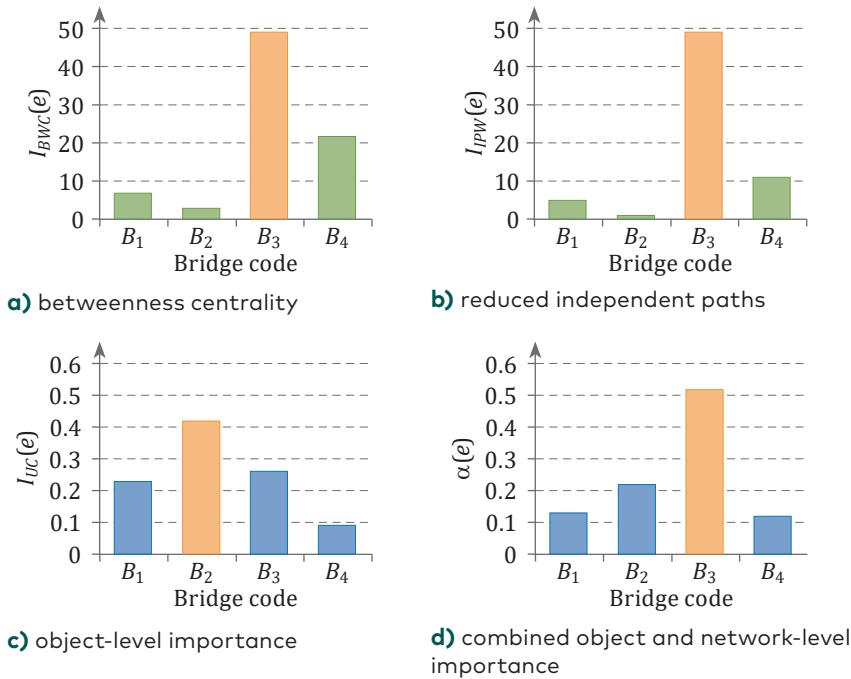


## 5. Numerical examples

In this section, three numerical examples have been presented to illustrate how  $\alpha(e)$  is used. In these examples, results of bridges prioritisation in the unique-characteristics-only state are compared to those obtained in the case of using  $\alpha(e)$ . The first example was related to the hypothetical network presented in section 3.

Figure 3 shows the results of calculating  $\alpha(e)$  for the four network bridges. In this example, it was also assumed that bridge  $B_2$  is more critical than other bridges based on its unique characteristics. However, based on the results obtained from using  $\alpha(e)$ , bridge  $B_3$  was selected as an essential bridge.

The second example was related to the prioritisation of four bridges ( $B_1$ ,  $B_2$ ,  $B_3$ , and  $B_5$ , as shown in Figure 4) that were studied in the Duchaczek and Skorupka (2016). Table 5 shows the unique characteristics of the Wroclaw city bridges.



**Figure 3.** The results of criteria used for importance ranking of the hypothetical bridge network

Table 5. Unique characteristics of the Wrocław city bridges\*

Bridge name	Bridge code	Bridge protection	Traffic volume	Length of the analysed span, m	Construction material	Spans construction	Access to the span bottom
Bolesława Chrobrego	B <sub>1</sub>	Poor	Low	3 × 25 + 48	Reinforced concrete	Arch	Very good
Bolesława Krzywoustego	B <sub>2</sub>	Good	Heavy	3 × 21	Reinforced concrete	Beam	Very good
Władysława Sikorskiego	B <sub>3</sub>	Good	Heavy	2 × 46.5	Steel	Truss	Poor
Polanowski	B <sub>4</sub>	Poor	Very low	30	Steel	Beam	Good
Grunwaldzki	B <sub>5</sub>	Very good	Very heavy	112.5	Steel	Suspension	Poor
Milenijny	B <sub>6</sub>	Very good	Very heavy	68 + 153 + 68	Reinforced concrete + steel cables	Cable-stayed	Good

Note: \*by Duchaczek and Skorupka (2013).

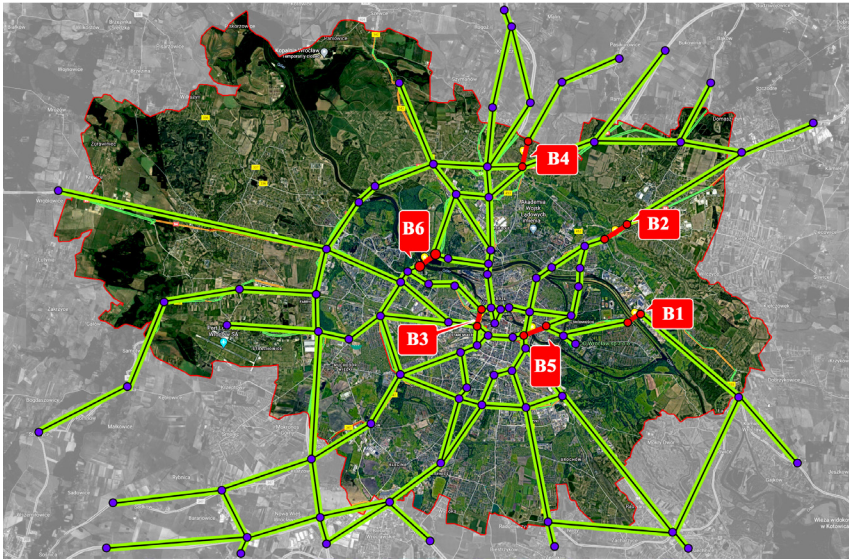
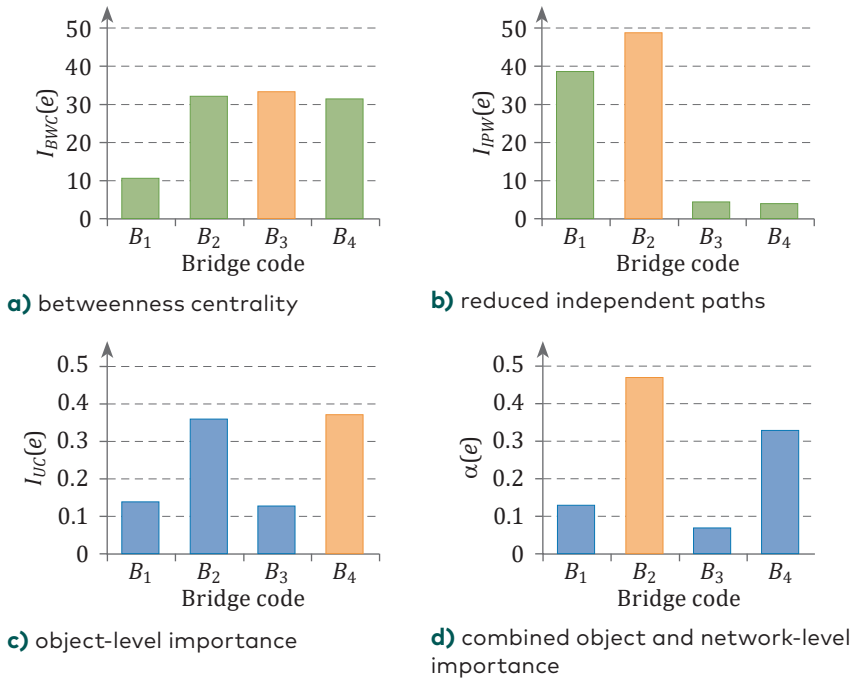


Figure 4. Location of the studied bridges in the Wrocław city

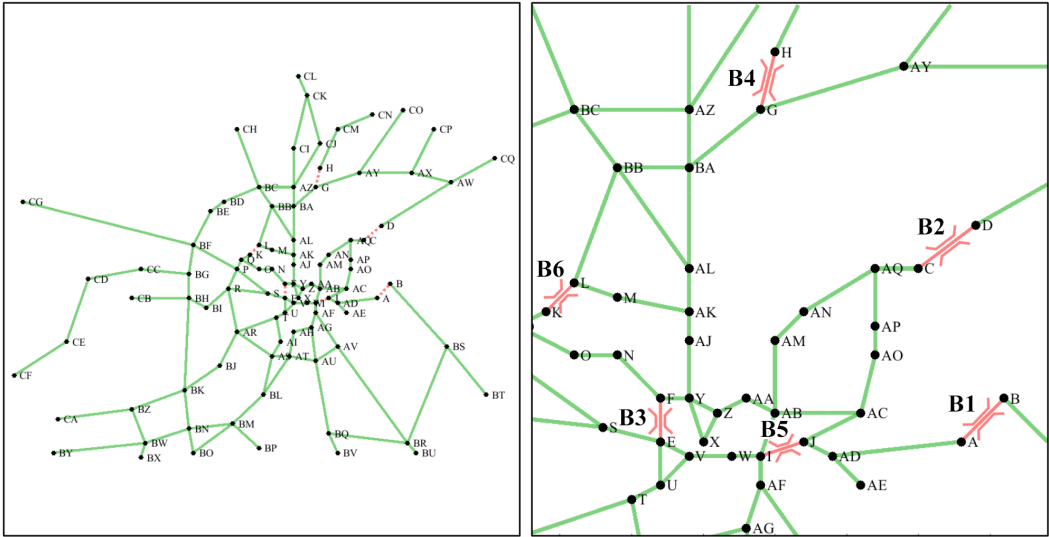
Based on its unique characteristics, the Grunwaldzki bridge (code:  $B_5$ ) was considered the essential bridge in agreement with prioritisation conducted by researchers. Figure 5 shows the results of calculating  $\alpha(e)$  for the four bridges in this example. Considering the location of the bridge in the Wroclaw city network and calculating  $\alpha(e)$ , the Bolesława Krzywoustego bridge (code:  $B_2$ ) was identified as the most critical bridge in this example.

In the third example, six studied bridges that were studied in the Duchaczek and Skorupka (2013) research work were compared to each other (Figure 6). Based on prioritisation conducted by the researchers, the Milenijny bridge (code  $B_6$ ) was the most critical network bridge based on its unique characteristics.

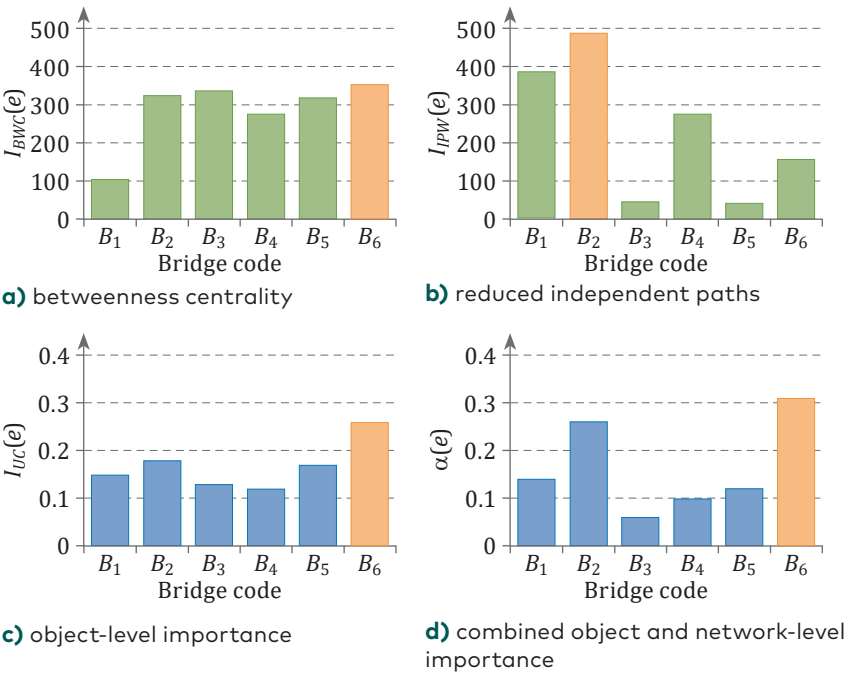
Results of calculating  $\alpha(e)$  for the bridges in this example show that bridge ( $B_6$ ), which was previously selected as the essential bridge based on its unique characteristics, was again selected as the most critical network bridge. These results show the importance of this bridge based on both individual and network characteristics. Figure 7 shows the results of calculating  $\alpha(e)$  for the six bridges in this example.



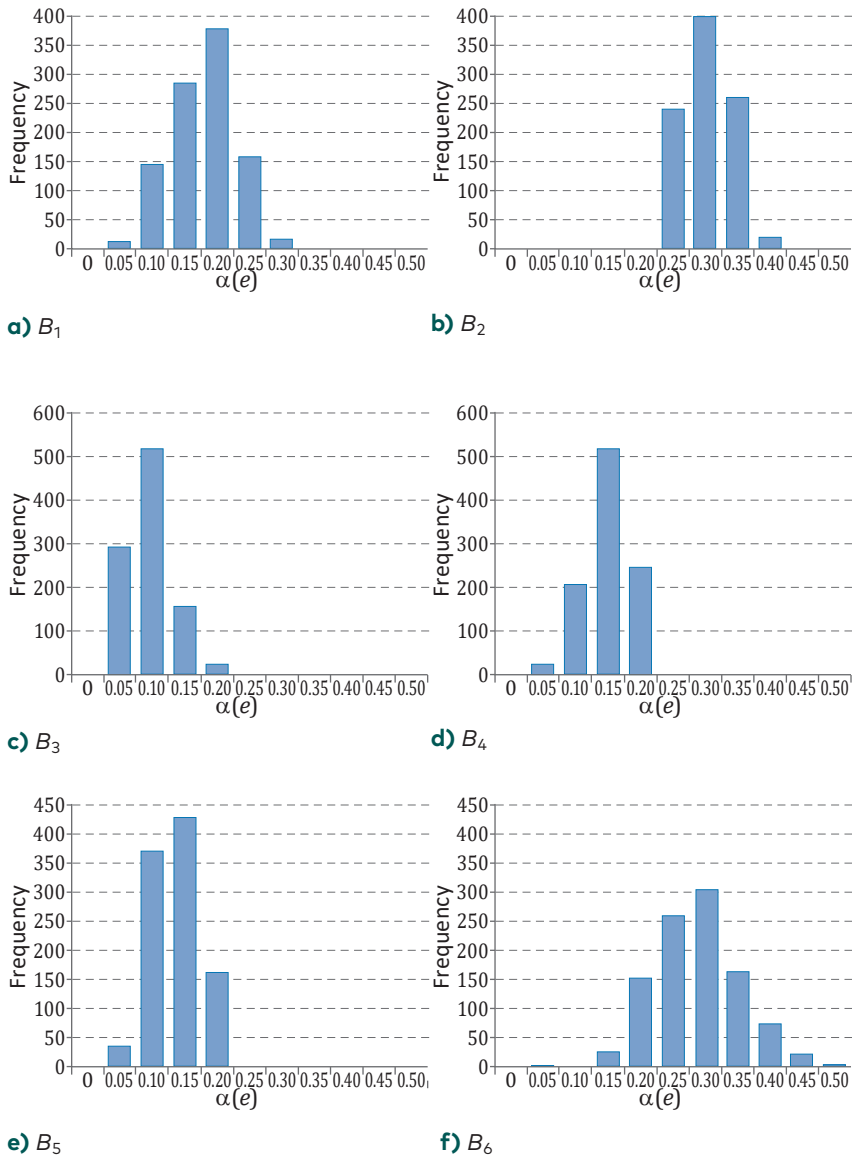
**Figure 5.** The results of criteria used for prioritisation of four bridges of Wroclaw city



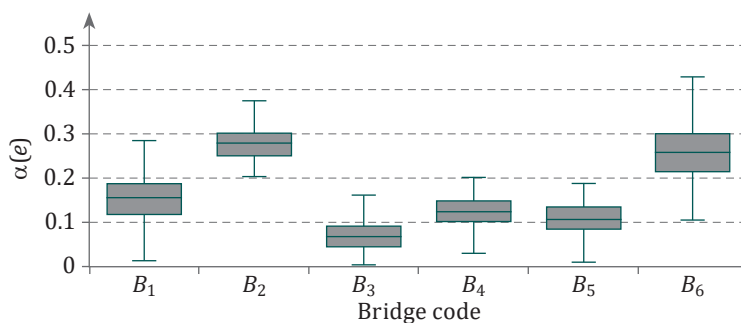
**Figure 6.** Location of the bridges of Wroclaw city in the simulated network



**Figure 7.** The results of criteria used to identify critical bridge among six bridges of Wroclaw city



**Figure 8.** Results of  $\alpha(e)$  for 1000 random values of  $\lambda_i$



**Figure 9.** The superiority of bridges  $B_2$  and  $B_6$  in all replications of Monte Carlo simulations

Moreover, the results of Monte Carlo analysis for the third example showed that bridge  $B_2$  in 60 per cent of repetitions and bridge  $B_6$  in 40 per cent of repetitions were selected as the most important bridges. Other network bridges were not selected as superior in any of the repetitions with different random weights of these indices. This shows that bridges  $B_2$  and  $B_6$  had always been necessary regardless of different opinions of network owners on the relative importance of the indicators. Figure 9 shows the superiority of bridges  $B_2$  and  $B_6$  over other bridges.

## 6. Conclusions

In this paper, the effect of considering bridge location and network topology in which a bridge is located was investigated on changing the bridges prioritisation. To do so,  $\alpha(e)$  was presented for measuring bridge importance by considering both the unique characteristics of a bridge and network characteristics.

Three numerical examples have been presented to understand how this index works. In these examples, the results of determining the importance of bridges in the unique-characteristics-only state were compared to those gained using the presented index  $\alpha(e)$ . The studied examples showed that topological characteristics and bridge location alter its importance and unique characteristics.

The unique characteristics of a bridge, such as its structure type, geometric dimensions, cost and time required for reconstruction, its symbolic value, are independent of the bridge location. Therefore, if bridges with similar unique characteristics are located in different network locations, they are equally important in these criteria.

It should be pointed out that, as opposed to other unique characteristics independent of the bridge location, the bridge traffic

varies based on its location. Although traffic is a proper criterion for identifying critical bridges, it is insufficient to reflect the network-level importance of the bridges. The results showed that some bridges with less traffic in normal conditions might be necessary for emergency response. For example, in the second example, it was seen that bridge  $B_5$ , which had more traffic than bridge  $B_2$  was more critical than this bridge under its unique criteria.

However, considering the location of the bridge  $B_2$  in the network showed that if this bridge is removed from the network, 488 independent routes be lost. However, the removal of the bridge  $B_5$  affects only forty independent routes in the network.

If only criteria related to the unique characteristics of bridges are used to prioritise them, bridges with high network-level and low object-level importance are considered without priority.

Furthermore, in this paper, betweenness centrality indices and a reduced number of independent paths were used to consider the network consequences of bridge failure. Other topological indicators can analyse the effect of branch removal from the network in future studies. This paper compares the bridges prioritisation based on topological indicators and criteria used by other researchers. In future studies, an integrated multi-criteria model can be developed by combining unique and network criteria of bridges. In addition, the present study has been only focused on the index of bridge importance and different methods to calculate it. Future research can examine other relevant parameters in assessing bridges risk and against man-made hazards.

## REFERENCES

- Al Kazimi, A., & Mackenzie, C. A. (2016, April). The economic costs of natural disasters, terrorist attacks, and other calamities: An analysis of economic models that quantify the losses caused by disruptions. In *2016 IEEE Systems and Information Engineering Design Symposium (SIEDS)* (pp. 32-37). IEEE. <https://doi.org/10.1109/SIEDS.2016.7489322>
- Aydin, N. Y., Duzgun, H. S., Wenzel, F., & Heinimann, H. R. (2018). Integration of stress testing with graph theory to assess the resilience of urban road networks under seismic hazards. *Natural Hazards*, 91(1), 37-68. <https://doi.org/10.1007/s11069-017-3112-z>
- Banerjee, S., Vishwanath, B. S., & Devendiran, D. K. (2019). Multihazard resilience of highway bridges and bridge networks: a review. *Structure and Infrastructure Engineering*, 15(12), 1694-1714. <https://doi.org/10.1080/15732479.2019.1648526>

- Berche, B., Von Ferber, C., Holovatch, T., & Holovatch, Y. (2009). Resilience of public transport networks against attacks. *The European Physical Journal B*, 71(1), 125-137. <https://doi.org/10.1140/epjb/e2009-00291-3>
- Bocchini, P., & Frangopol, D. M. (2012). Optimal resilience-and cost-based postdisaster intervention prioritisation for bridges along a highway segment. *Journal of Bridge Engineering*, 17(1), 117-129. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000201](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000201)
- Bocchini, P., & Frangopol, D. M. (2012). Restoration of bridge networks after an earthquake: multi-criteria intervention optimisation. *Earthquake Spectra*, 28(2), 427-455. <https://doi.org/10.1193/1.4000019>
- Brown, G. G., & Cox, Jr, L. A. (2011). How probabilistic risk assessment can mislead terrorism risk analysts. *Risk Analysis: an International Journal*, 31(2), 196-204. <https://doi.org/10.1111/j.1539-6924.2010.01492.x>
- Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O'Rourke, T. D., Reinhorn, A. M., ... & Von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake Spectra*, 19(4), 733-752. <https://doi.org/10.1193/1.1623497>
- Chang, L., Peng, F., Ouyang, Y., Elnashai, A. S., & Spencer Jr, B. F. (2012). Bridge seismic retrofit program planning to maximise postearthquake transportation network capacity. *Journal of Infrastructure Systems*, 18(2), 75-88. [https://doi.org/10.1061/\(ASCE\)IS.1943-555X.0000082](https://doi.org/10.1061/(ASCE)IS.1943-555X.0000082)
- Chipley, A., & Lasch, M. A. (2007) Site and urban design for security. Guidance against potential terrorist attacks: providing protection to people and buildings. *Risk Management Series*. FEMA 430. Federal Emergency Management Agency, Washington, DC.
- Chipley, M., Kaminskas, M., Lyon, W., Beshlin, D., & Hester, M. (2003). Reference manual to mitigate potential terrorist attacks against buildings: providing protection to people and building. *Risk Management Series*. FEMA. Federal Emergency Management Agency, Washington, DC.
- Chipley, M., Lyon, W., Smilowitz, R., Williams, P., Arnold, C., Blewett, W., Hazen, L., & Krimgold, F. (2012). Primer to Design Safe School Projects in Case of Terrorist Attacks and School Shootings. *Buildings and Infrastructure Protection Series*. FEMA-428/BIPS-07/January 2012. Edition 2. US Department of Homeland Security.
- Chopra, S. S., Dillon, T., Bilec, M. M., & Khanna, V. (2016). A network-based framework for assessing infrastructure resilience: a case study of the London metro system. *Journal of the Royal Society Interface*, 13(118), 20160113. <https://doi.org/10.1098/rsif.2016.0113>
- Cox, Jr, L. A. (2008). Some limitations of "Risk = Threat × Vulnerability × Consequence" for risk analysis of terrorist attacks. *Risk Analysis: an International Journal*, 28(6), 1749-1761. <https://doi.org/10.1111/j.1539-6924.2008.01142.x>
- Cox, Jr, L. A. (2009). Improving risk-based decision making for terrorism applications. *Risk Analysis: an International Journal*, 29(3), 336-341. <https://doi.org/10.1111/j.1539-6924.2009.01206.x>
- Davis, C., Sammarco, E., & Williamson, E. (2017). *Bridge security design manual* (No. FHWA-HIF-17-032).



- Decò, A., Bocchini, P., & Frangopol, D. M. (2013). A probabilistic approach for the prediction of seismic resilience of bridges. *Earthquake Engineering & Structural Dynamics*, 42(10), 1469-1487. <https://doi.org/10.1002/eqe.2282>
- Deng, L., Wang, W., & Yu, Y. (2016). State-of-the-art review on the causes and mechanisms of bridge collapse. *Journal of Performance of Constructed Facilities*, 30(2), 04015005. [https://doi.org/10.1061/\(ASCE\)CF.1943-5509.0000731](https://doi.org/10.1061/(ASCE)CF.1943-5509.0000731)
- Diaz, E. E. M., Moreno, F. N., & Mohammadi, J. (2009). Investigation of common causes of bridge collapse in Colombia. *Practice Periodical on Structural Design and Construction*, 14(4), 194-200. [https://doi.org/10.1061/\(ASCE\)SC.1943-5576.0000006](https://doi.org/10.1061/(ASCE)SC.1943-5576.0000006)
- Dillon, R. L., Liebe, R. M., & Bestafka, T. (2009). Risk-based decision making for terrorism applications. *Risk Analysis: an International Journal*, 29(3), 321-335. <https://doi.org/10.1111/j.1539-6924.2008.01196.x>
- Dong, Y., Frangopol, D. M., & Saydam, D. (2014). Pre-earthquake multi-objective probabilistic retrofit optimisation of bridge networks based on sustainability. *Journal of Bridge Engineering*, 19(6), 04014018. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000586](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000586)
- Duchaczek, A., & Skorupka, D. (2013). Evaluation of probability of bridge damage as a result of terrorist attack. *Archives of Civil Engineering*, 59(2). <https://doi.org/10.2478/ace-2013-001>
- Duchaczek, A., & Skorupka, D. (2016). A risk assessment method of bridge facilities damage in the aspect of potential terrorist attacks. *Periodica Polytechnica Civil Engineering*, 60(2), 189-198. <https://doi.org/10.3311/PPci.7627>
- Ezell, B. C., Bennett, S. P., Von Winterfeldt, D., Sokolowski, J., & Collins, A. J. (2010). Probabilistic risk analysis and terrorism risk. *Risk Analysis: an International Journal*, 30(4), 575-589. <https://doi.org/10.1111/j.1539-6924.2010.01401.x>
- Faturechi, R., & Miller-Hooks, E. (2014). Travel time resilience of roadway networks under disaster. *Transportation Research Part B: Methodological*, 70, 47-64. <https://doi.org/10.1016/j.trb.2014.08.007>
- Feng, Q., Cai, H., Chen, Z., Zhao, X., & Chen, Y. (2016). Using game theory to optimise allocation of defensive resources to protect multiple chemical facilities in a city against terrorist attacks. *Journal of Loss Prevention in the Process Industries*, 43, 614-628. <https://doi.org/10.1016/j.jlp.2016.07.010>
- Frangopol, D. M., & Bocchini, P. (2011). Resilience as optimisation criterion for the rehabilitation of bridges belonging to a transportation network subject to earthquake. In *Structures Congress 2011* (pp. 2044-2055). [https://doi.org/10.1061/41171\(401\)178](https://doi.org/10.1061/41171(401)178)
- Frangopol, D. M., Sause, R., & Kusko, C. S. (2010, July). Bridge maintenance, safety, management and life-cycle optimisation. In *Proceedings of the Fifth International IABMAS Conference, Philadelphia, USA* (pp. 11-15).
- Garcia, R. J., & von Winterfeldt, D. (2016). Defender-attacker decision tree analysis to combat terrorism. *Risk Analysis*, 36(12), 2258-2271. <https://doi.org/10.1111/risa.12574>

- Garg, R. K., Chandra, S., & Kumar, A. (2020). Analysis of bridge failures in India from 1977 to 2017. *Structure and Infrastructure Engineering*, 1-18.  
<https://doi.org/10.1080/15732479.2020.1832539>
- Girvan, M., & Newman, M. E. (2002). Community structure in social and biological networks. *Proceedings of the National Academy of Sciences*, 99(12), 7821-7826. <https://doi.org/10.1073/pnas.122653799>
- Giunta, M. (2017). Sustainability and resilience in the rehabilitation of road infrastructures after an extreme event: an integrated approach. *The Baltic Journal of Road and Bridge Engineering*, 12(3), 154-160.  
<https://doi.org/10.3846/bjrbe.2017.18>
- Greenberg, M., Haas, C., Cox Jr, A., Lowrie, K., McComas, K., & North, W. (2012). Ten most important accomplishments in risk analysis, 1980–2010. *Risk Analysis*, 32(5), 771. <https://dx.doi.org/10.1111/j.1539-6924.2012.01817.x>
- Guo, A., Liu, Z., Li, S., & Li, H. (2017). Seismic performance assessment of highway bridge networks considering post-disaster traffic demand of a transportation system in emergency conditions. *Structure and Infrastructure Engineering*, 13(12), 1523-1537.  
<https://doi.org/10.1080/15732479.2017.1299770>
- Hartmann, A. K. (2014). Large-deviation properties of resilience of transportation networks. *The European Physical Journal B*, 87(5), 1-10.  
<https://doi.org/10.1140/epjb/e2014-50078-4>
- Hinman, E. E., Rojahn, C., Smilowitz, R., Campi, D., Myers, R. J., Sauer, N., & Mork, P. A. (2003). Primer for design of commercial buildings to mitigate terrorist attacks: providing protection to people and buildings. *Risk Management Series*. FEMA. Federal Emergency Management Agency, Washington, DC.
- Hosseini, S., Barker, K., & Ramirez-Marquez, J. E. (2016). A review of definitions and measures of system resilience. *Reliability Engineering & System Safety*, 145, 47-61. <https://doi.org/10.1016/j.res.2015.08.006>
- Hsu, T. H., & Lin, L. Z. (2006). QFD with fuzzy and entropy weight for evaluating retail customer values. *Total Quality Management & Business Excellence*, 17(7), 935-958. <https://doi.org/10.1080/14783360600598223>
- Ip, W. H., & Wang, D. (2011). Resilience and friability of transportation networks: evaluation, analysis and optimisation. *IEEE Systems Journal*, 5(2), 189-198.  
<https://doi.org/10.1109/JSYST.2010.2096670>
- Issa, L. (2008). *Development of an inspection checklist for risk assessment of bridges in New Jersey* (Doctoral dissertation, Rutgers University-Graduate School-New Brunswick). <https://doi.org/10.7282/T35M6434>
- Kameshwar, S., Cox, D. T., Barbosa, A. R., Farokhnia, K., Park, H., Alam, M. S., & van de Lindt, J. W. (2019). Probabilistic decision-support framework for community resilience: incorporating multi-hazards, infrastructure interdependencies, and resilience goals in a Bayesian network. *Reliability Engineering & System Safety*, 191, 106568.  
<https://doi.org/10.1016/j.res.2019.106568>
- Karamlou, A., & Bocchini, P. (2014). Optimal bridge restoration sequence for resilient transportation networks. In *Structures Congress 2014* (pp. 1437-1447). <https://doi.org/10.1061/9780784413357.127>

- Keeney, R. L., & Von Winterfeldt, D. (2011). A value model for evaluating homeland security decisions. *Risk Analysis: an International Journal*, 31(9), 1470-1487. <https://doi.org/10.1111/j.1539-6924.2011.01597.x>
- Kennett, M. N., Letvin, E., Chipley, M., & Ryan, T. (2005). Risk assessment: a how-to guide to mitigate potential terrorist attacks against buildings. *Risk Management Series*. FEMA. Federal Emergency Management Agency, Washington, DC.
- Kim, K., & Lee, J. (2020). Fragility of Bridge Columns under vehicle impact using risk analysis. *Advances in Civil Engineering*, 2020. <https://doi.org/10.1155/2020/7193910>
- Krimgold, F. (2003). Insurance, finance, and regulation primer for terrorism risk management in buildings: providing protection to people and buildings. *Risk Management Series*. US Department of Homeland Security, FEMA.
- Kučas, A. (2015). Graph-based multi-attribute decision making: impact of road fencing on ecological network. *The Baltic Journal of Road and Bridge Engineering*, 10(2), 105-111. <https://doi.org/10.3846/bjrbe.2015.13>
- Leung, M., Lambert, J. H., & Mosenthal, A. (2004). A risk-based approach to setting priorities in protecting bridges against terrorist attacks. *Risk Analysis: an International Journal*, 24(4), 963-984. <https://doi.org/10.1111/j.0272-4332.2004.00500.x>
- Li, Y., Wang, T., Song, X., & Li, G. (2016). Optimal resource allocation for anti-terrorism in protecting overpass bridge based on AHP risk assessment model. *KSCE Journal of Civil Engineering*, 20(1), 309-322. <https://doi.org/10.1007/s12205-015-0233-3>
- Li, Z., Jin, C., Hu, P., & Wang, C. (2019). Resilience-based transportation network recovery strategy during emergency recovery phase under uncertainty. *Reliability Engineering & System Safety*, 188, 503-514. <https://doi.org/10.1016/j.ress.2019.03.052>
- Liao, T.-Y., Hu, T.-Y., & Ko, Y.-N. (2018). A resilience optimisation model for transportation networks under disasters. *Natural Hazards*, 93(1), 469-489. <https://doi.org/10.1007/s11069-018-3310-3>
- Liu, C., Fan, Y., & Ordóñez, F. (2009). A two-stage stochastic programming model for transportation network protection. *Computers and Operations Research*, 36(5), 1582-1590. <https://doi.org/10.1016/j.cor.2008.03.001>
- Liu, Y., McNeil, S., Hackl, J., & Adey, B. T. (2020). Prioritising transportation network recovery using a resilience measure. *Sustainable and Resilient Infrastructure*, 1-12. <https://doi.org/10.1080/23789689.2019.1708180>
- Lu, J., Atamturktur, S., & Huang, Y. (2016). Bi-level resource allocation framework for retrofitting bridges in a transportation network. *Transportation Research Record*, 2550(1), 31-37. <https://doi.org/10.3141/2550-05>
- Lu, L., & Zhang, M. (2013). Edge betweenness centrality. *Encyclopedia of Systems Biology*, 647-648. [https://doi.org/10.1007/978-1-4419-9863-7\\_874](https://doi.org/10.1007/978-1-4419-9863-7_874)
- Macek, D., & Mestanova, D. (2009). Multi-criteria evaluation of crash barrier systems types. *The Baltic Journal of Road and Bridge Engineering*, 4(3), 108-108. <https://doi.org/10.3846/1822-427X.2009.4.108-114>

- Merschman, E., Doustmohammadi, M., Salman, A. M., & Anderson, M. (2020). Postdisaster decision framework for bridge repair prioritisation to improve road network resilience. *Transportation Research Record*, 2674(3), 81-92. <https://doi.org/10.1177/0361198120908870>
- Nassif, H., Issa, L., Najm, H., & Davis, J. (2006). *Simple bridge security inspection: final report, September 2006* (No. FHWA NJ-2006-011). New Jersey Department of Transportation Bureau of Research and U. S. Department of Transportation Federal Highway Administration.
- National Infrastructure Advisory Council (US). (2009). *Critical infrastructure resilience: final report and recommendations*. National Infrastructure Advisory Council.
- National Research Council. (2010). *Review of the Department of Homeland Security's approach to risk analysis*. National Academies Press.
- Osei-Asamoah, A., & Lownes, N. E. (2014). Complex network method of evaluating resilience in surface transportation networks. *Transportation Research Record*, 2467(1), 120-128. <https://doi.org/10.3141/2467-13>
- Petersen, L., Lundin, E., Fallou, L., Sjöström, J., Lange, D., Teixeira, R., & Bonavita, A. (2020). Resilience for whom? The general public's tolerance levels as CI resilience criteria. *International Journal of Critical Infrastructure Protection*, 28, 100340. <https://doi.org/10.1016/j.ijcip.2020.100340>
- Ramazani, H., Shafahi, Y., & Seyedabrishami, S. (2011). A fuzzy traffic assignment algorithm based on driver perceived travel time of network links. *Scientia Iranica*, 18(2), 190-197. <https://doi.org/10.1016/j.scient.2011.03.028>
- Ray, J. C. (2007). Risk-based prioritisation of terrorist threat mitigation measures on bridges. *Journal of Bridge Engineering*, 12(2), 140-146. [https://doi.org/10.1061/\(ASCE\)1084-0702\(2007\)12:2\(140\)](https://doi.org/10.1061/(ASCE)1084-0702(2007)12:2(140))
- Richardson, H. W., Park, J., Moore II, J. E., & Pan, Q. (2014). *National economic impact analysis of terrorist attacks and natural disasters*: Edward Elgar Publishing.
- Rios, J., & Insua, D. R. (2012). Adversarial risk analysis for counterterrorism modeling. *Risk Analysis: an International Journal*, 32(5), 894-915. <https://doi.org/10.1111/j.1539-6924.2011.01713.x>
- Roberts, J., Kulicki, J. M., Beranek, D. A., Englot, J. M., Fisher, J. W., Hungerbeeler, H., ... & Witt, K. (2003). *Recommendations for bridge and tunnel security* (No. FHWA-IF-03-036). Federal Highway Administration (US).
- Rummel, T., Hyzak, M. D., & Ralls, M. L. (2002). Transportation security activities in Texas. In *Vital Links in Securing Our Mobility. 2002 International Bridge Conference. 19th Annual IBCEngineers' Society of Western Pennsylvania* (No. IBC-02-A3).
- Schintler, L. A., Kulkarni, R., Gorman, S., & Stough, R. (2007). Using raster-based GIS and graph theory to analyse complex networks. *Networks and Spatial Economics*, 7(4), 301-313. <https://doi.org/10.1007/s11067-007-9029-4>
- Singh, K., Gardoni, P., & Stochino, F. (2020). Probabilistic models for blast parameters and fragility estimates of steel columns subject to blast loads. *Engineering Structures*, 222, 110944. <https://doi.org/10.1016/j.engstruct.2020.110944>

- Smith, J. L., Swatzell, R., & Hall, B. (2004). *Prevention of Progressive Collapse-DoD Guidance and Application*, Building Design for Homeland Security-FEMA 426, Course no. E155.
- Smith, M., Rowshan, S., Krill Jr, S., Seplow, J., & Sauntry, W. (2002). *A Guide to Highway Vulnerability Assessment for Critical Asset Identification and Protection*.
- Testa, A. C. (2015). *Resilience of Transportation Infrastructure Systems to Climatic Extreme Events*. Masters Theses. 173. <https://doi.org/10.7275/6464188>
- Valeo, M., Nassif, H., Issa, L., Capers Jr, H., & Ozbay, K. (2012). Analytic hierarchy process to improve simple bridge security checklist. *Transportation Research Record*, 2313(1), 201-207. <https://doi.org/10.3141/2313-21>
- Vugrin, E. D., Turnquist, M. A., & Brown, N. J. (2014). Optimal recovery sequencing for enhanced resilience and service restoration in transportation networks. *International Journal of Critical Infrastructures*, 10(3-4), 218-246. <https://doi.org/10.1504/IJCIS.2014.066356>
- Williamson, E. B., & Winget, D. G. (2005). Risk management and design of critical bridges for terrorist attacks. *Journal of Bridge Engineering*, 10(1), 96-106. [https://doi.org/10.1061/\(ASCE\)1084-0702\(2005\)10:1\(96\)](https://doi.org/10.1061/(ASCE)1084-0702(2005)10:1(96))
- Yi, Z., Agrawal, A. K., Ettouney, M., & Alampalli, S. (2014). Blast load effects on highway bridges. I: Modeling and blast load effects. *Journal of Bridge Engineering*, 19(4), 04013023. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000547](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000547)
- Yu, R., Chen, L., Fang, Q., & Huan, Y. (2018). An improved nonlinear analytical approach to generate fragility curves of reinforced concrete columns subjected to blast loads. *Advances in Structural Engineering*, 21(3), 396-414. <https://doi.org/10.1177/1369433217718986>
- Zhang, C., & Ramirez-Marquez, J. E. (2013). Protecting critical infrastructures against intentional attacks: a two-stage game with incomplete information. *IIE Transactions*, 45(3), 244-258. <https://doi.org/10.1080/0740817X.2012.676749>
- Zhang, L., & Reniers, G. (2016). A game-theoretical model to improve process plant protection from terrorist attacks. *Risk Analysis*, 36(12), 2285-2297. <https://doi.org/10.1111/risa.12569>
- Zhang, N., & Alipour, A. (2020). Two-stage model for optimised mitigation and recovery of bridge network with final goal of resilience. *Transportation Research Record*, 2674(10), 114-123. <https://doi.org/10.1177/0361198120935450>
- Zhang, W., & Wang, N. (2016). Resilience-based risk mitigation for road networks. *Structural Safety*, 62, 57-65. <https://doi.org/10.1016/j.strusafe.2016.06.003>
- Zhang, W., Wang, N., & Nicholson, C. (2017). Resilience-based post-disaster recovery strategies for road-bridge networks. *Structure and Infrastructure Engineering*, 13(11), 1404-1413. <https://doi.org/10.1080/15732479.2016.1271813>
- Zhang, W., Wang, N., Nicholson, C., & Tehrani, M. H. (2018). A stage-wise decision framework for transportation network resilience planning. *arXiv preprint arXiv:1808.03850*.

- Zhang, X., Miller-Hooks, E., & Denny, K. (2015). Assessing the role of network topology in transportation network resilience. *Journal of Transport Geography*, 46, 35-45. <https://doi.org/10.1016/j.jtrangeo.2015.05.006>
- Zhou, Y., Wang, J., & Yang, H. (2019). Resilience of transportation systems: concepts and comprehensive review. *IEEE Transactions on Intelligent Transportation Systems*, 20(12), 4262-4276. <https://doi.org/10.1109/TITS.2018.2883766>