IMPROVEMENT OF CURVATURE CHANGE RATE (PARAMETER) IN TWO-LINE RURAL ROADS

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Abstract. The studies in various countries show that one of the main causes of accidents on rural roads still remains the drivers’ errors in choosing safe speed, which can be attributed to the errors of perceiving the road alignment. The current practice shows that the engineering safety improvement measures as well as accident prevention measures such as intersection design, traffic signs and equipment, facilities for vulnerable road users, access management, and human behaviour are not always effective. Probably, one of the alternatives to solve this problem is a design methodology that is used in the European Union countries and based on road safety criteria and road safety module. This methodology is aimed at ensuring design consistency between the elements of road alignment in a way that the driver intuitively chooses safe speed or the probability of errors in perceiving the road alignment is minimum. The article...
presents experimental research carried out in Vilnius Gediminas Technical University, the objective of which is to determine the accuracy of road safety module and to suggest possible alternatives for its improvement.

**Keywords:** curvature change rate of road alignment, geometric road design, horizontal curves, road safety, road safety criteria, road safety module.

## Introduction

Road alignment is a linear object made of tangents and horizontal curves. Horizontal curves are especially important elements of road alignment – the horizontal curve of a road strongly affects the safety and comfort of travel, and that is why the curvature profiles are widely used in road design and road alignment analysis. Three kinds of geometric elements are used in designing horizontal alignment of roads: tangents, circular curves, and spiral transitions (Camacho-Torregrosa et al., 2015; Colonna et al., 2018; Elvik, 2019; Intini et al., 2020; Bogenreif et al., 2012). Improperly selected parameters of horizontal curves can be critical for road safety and travelling comfort.

Geometric design consistency has a significant effect on safety of rural motorways. Previous studies on the relationship between geometric design consistency and crash frequency focused on two-lane rural highways since these highways have higher crash rates and are generally characterised by considerable inconsistencies. Studies have shown that operating speed, vehicle stability, alignment indices, and driver’s workload are the common consistency measures that might affect safety (Al-Sahili & Dwaikat, 2019).

The study by Gemechu and Tulu (2021) was based on the evaluation of existing alignments. The authors observed that the more geometrically inconsistent the alignment, the greater number of crashes occurred than it was expected. They suggested three different models that incorporated individual measures of design consistency and could be used to predict crash frequency on a two-lane rural highway segment having nearly similar environmental factors to the segment considered in this study. The models developed in this study depend on the design consistency measures used and are limited to horizontal curves and tangents only. This may limit the applicability of the developed models to sections that are combined with vertical curves.

A study by Montella and Imbriani (2015) clearly highlights that the achievement of proper geometric design consistency is a key design element also on motorways due to the safety consequences of design inconsistencies. The driver usually chooses speed in order to feel safe and this speed depends on a general condition of road, a separate
element of road alignment, road environment, meteorological conditions, and also on a sequence of further actions (Bosurgi & D’Andrea, 2012). In cases where design solutions fail to guarantee consistency between the different elements of road alignment, there is a growing probability that further actions of the driver will be inadequate to the existing situation, i.e., there is an increasing probability of errors in the perception of road.

Khan et al. (2012) evaluated safety of horizontal curves with respect to curve geometric characteristics and sign data. The focus was made on collecting a good-quality large dataset to develop models and to explore the relationship between safety on horizontal curves and sign types, specifically curves and turn signs. The dataset included curves on different road types to determine the difference in safety characteristics, which had never been examined in the literature. The results identified variables with large significance and described curve characteristics in greater detail. The crash prediction models were used in safety performance functions for horizontal curves.

A safety evaluation tool called TARVA has been created for the evaluation of the current safety situation (to select locations for safety treatments) and safety effects of road safety improvements (to select cost-effective safety measures). A safety evaluation tool called ONHA has been created for the analysis of all accident data, vehicles involved in those accidents and their occupants, as well as pedestrians (Peltola et al., 2012). However, this methodology has an essential disadvantage – it is based on accident records, i.e., a high accident concentration section or black spot is determined only where the number of accidents has reached the limit values.

The EU countries use design methodology suggested by the German scientist R. Lamm (Lamm et al., 2007), which enables already in a design stage to determine the potentially dangerous road sections, which in future can become high accident concentration sections or black spots. This methodology is based on road safety criteria and road safety module.

Analysis of road reconstruction project solutions shows that certain countries acquire a mistaken practice, which could be described as follows – a design road axis in road sections to be reconstructed shall have the least possible deviation from the existing axis. This practice is motivated by the aim to minimize reconstruction costs or to satisfy the interests of third parties, or to avoid conflict situations caused by changing a roadway position. There is a strong concern that this mistaken practice is being incorporated in road design standards of Lithuania, which provide for more and more exceptions related to road reconstruction projects.
Looking at the results of these reconstruction projects, it became necessary to make a study and to check the quality of design solutions at an early design stage of the reconstruction project. The aim was also to check the compliance of reconstruction projects with the requirements to the visual perception of road quality that were set in the design standards of certain countries, such as the USA, Germany, Sweden and Switzerland (AASHTO, 2018; Swedish National Road Administration, 1986; Trafikverket, 2015; Swiss Association of Road Specialists, 1991). In road design standards of Germany (Forschungsgesellschaft..., 2012), in order to make the route smooth the requirements are set to not only the radii of horizontal circular curves (compared to Lithuanian design standards) but also to the minimum length of curves.

Road curvature, describing parameters of horizontal curves, is one of criteria allowing to determine how much the road section is dangerous. The German scientist R. Lamm (Lamm et al. 2007) developed the method of determining road safety criteria and road safety module based on road tortuosity, which is presented in more detail in Section 1.

The aims of this article are to examine the methodology used in the EU countries to improve the level of traffic safety on two-lane country roads and based on traffic safety criteria and the traffic safety module; to identify possible shortcomings of the methodology and to propose possible alternatives to improve this methodology.

1. **Curvature change rate, road safety criteria and road safety module**

1.1. **Curvature change rate**

In road construction or reconstruction projects, it is important to not only properly design the different elements of road alignment but to also ensure a consistency between each other. Literature suggests that the geometric design parameter, which mostlu affects road safety, is the horizontal curvature or the Curvature Change Rate (Montella & Imbriani, 2015).

The studies made in Western Europe and the USA (Lamm et al., 1995; Lamm et al., 1996; Lamm et al., 2007; Bauer & Harwood, 2013; Easa & Mehmood, 2008) have determined that the main cause of accidents on two-lane rural roads is a sudden change in vehicle speeds between the different elements of road alignment. The radius of curvature is the geometric feature used to determine speed at which a curve is negotiated (Shao et al., 2014).
Figure 1. Calculation of $CCR_S$ in a general case

Figure 2. Calculation of $CCR_S$ for a circular curve

Figure 3. Calculation of $CCR_S$ in case when transition curves are of different length
According to different studies (Oña et al., 2014), speed reduction is considered one of the major factors contributing to road safety. For that reason, several guidelines have been recommended for maximum desirable speed reductions from tangents to horizontal curves and for maximum differentials between design and operating speeds on horizontal curves.

It was decided to suggest a reasonable and reliable design methodology allowing to avoid sudden changes in speeds and to guarantee effective design solutions for road alignment. Investigations have demonstrated (Lamm et al., 2007) that from various geometric design parameters there is a correlation between operating speed and accident rate, and it can be described by the Curvature Change Rate of the single curve ($CCR_S$). The $CCR_S$ is applied for a separate design element (horizontal curve) as well as for the overall road alignment. The Curvature Change Rate of the horizontal curve is a ratio between the turning angle of alignment and length of curve. The Curvature Change Rate of the road alignment is a ratio between the sum of absolute values of the alignment turning angles and the length of road section.

Figures 1–3 give different examples of calculating $CCR_S$.

$CCR_S$ – the Curvature Change Rate; $L = L_{P1} + L_A + L_{P2}$ – overall length of curve, m; $L_A$ – length of circular curve, m; $R$ – radius of circular curve, m; $L_{P1}$, $L_{P2}$ – lengths of the first and the second transition curves, m; $\gamma$ – reduced angle of road alignment.

### 1.2. The safety criteria and safety module

In this article, the road alignment has been studied as a sequence of successive interrelated elements based on quantitative criteria that are called safety criteria (Lamm et al., 1999; Lamm et al., 2007):

- Criterion I – consistency of the design speed $V_P$;
- Criterion II – consistency of the 85th-percentile speed $V_{85}$;
- Criterion III – dynamic consistency on curves.

The design speed $V_P$ means technically and economically justified speed determined for design purposes, taking into account the category and purpose of the road, the conditions of road construction or reconstruction. Consistency of the speed $V_P$ means that the design speed shall remain constant on as long road sections as possible. Consistency of the design speed is considered good when the difference between the 85-th percentile speed $V_{85}$ and the design speed $V_P$ is less than 10 km/h.

The 85th percentile speed $V_{85}$ means the speed at which 85% of vehicles will drive in free flow under normal driving conditions, clean and dry road pavement. The speed $V_{85}$ in a certain road section is related to the $CCR$ of this road section and is calculated by the general
regression equation of the speed $V_{85}$. When the gradient $i \leq 6\%$ and the $CCR \leq 1600$ gon/km, $V_{85}$ is calculated by Equation (1):

$$V_{85} = 105.31 + 2 \times 10^{-5} \times CCR^2 - 0.071 \times CCR,$$

where $CCR$ – the overall Curvature Change Rate of existing alignment.

$CCR$ is calculated by Equation (2):

$$CCR = \frac{\sum_{i=1}^{n}(CCR_{Si} \cdot L_i)}{\sum_{i=1}^{n}L_i},$$

where $CCR_{Si}$ – the Curvature Change Rate of the $i$-th design element; $L_i$ – length of the $i$-th design element.

Safety Criterion II reflects the speed differences $[V_{85i} - V_{85,i+1}]$ between the 85th percentile speeds with the changing road alignment. A well-balanced speed between successive design elements (curves and tangents) allows achieving constant, safe and cost-effective traffic.

A risk of accident on curve increases with the decreasing skid resistance. Therefore, the designer must clearly know the required friction force between tyres and road surface, especially on curves. Consistency in driving dynamics on curves is reflected by Safety Criterion III. It compares the coefficient of assumed side friction $f_{RA}$ with the coefficient of demanded side friction $f_{RD}$ for the vehicles travelling on curve at a speed $V_{85}$ (Colonna et al., 2016). The assumed side friction $f_{RA}$ is related to the design speed $V_p$ and calculated by Equation (3):

$$f_{RA} = \frac{V_p^2}{127R} - e,$$

where $R$ – a radius of horizontal curve; $e$ – a super elevation rate.

The demanded side friction $f_{RD}$ is related to the 85th percentile speed $V_{85}$ (operating speed):

$$f_{RD} = \frac{V_{85}^2}{127R} - e.$$

Difference between the assumed side friction $f_{RA}$ and the demanded side friction $f_{RD}$, indicating “poor” road section, is $-0.04$ (Table 1).

For a general safety analysis of road alignment, the three safety criteria are combined into an overall safety module. Safety Criteria I to III indicate that roadway sections could exhibit different design safety levels with respect to the individual safety criteria. The reason for this is that each of the safety criteria represents a separate safety aspect in highway geometric design. It may happen, for example, that the
transition section between an independent tangent and a curve could correspond to poor design according to Safety Criterion II, whereas Safety Criterion I with respect to the design speed or Safety Criterion III with respect to side friction assumed, or both, provide acceptable values for the observed curved site. Therefore, for a fast, comprehensive overview of new or existing (old) alignment or road networks, Safety Criteria I to III will be combined into a safety module. The safety module establishes three design levels (Table 1):

- **Good** design level – weight coefficient +1, no corrections are necessary for the road section;
- **Fair** design level – weight coefficient 0, the engineering safety improvement measures are recommended for the road section;
- **Poor** design level – weight coefficient −1, the road section needs corrections.

The design levels are determined for both driving directions, then their arithmetic mean is calculated and according to the above given limits the final evaluation is carried out.

The idea of safety criteria and safety module is definitely innovative, and the methodology created on the basis of this idea is logically complete and recognised in many countries. Assuming that the safety module gives the results, the accuracy level of which corresponds to reality, the use of this methodology offers a number of significant advantages:

- To justify by calculations a design speed and thereby to avoid substantial errors in road reconstruction or construction projects;
- To identify potentially dangerous locations already in the stage of design solutions as well as on the existing road where due to random circumstances the black spots have not been determined yet.

This research is aimed at determining whether the assumption – the accuracy of safety module is sufficient – is correct.

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**Table 1. Classification of design levels**

(source: Lamm et al., 1999; Lamm et al., 2007)

<table>
<thead>
<tr>
<th>Safety criteria</th>
<th>Good (+)</th>
<th>Fair (0)</th>
<th>Poor (−)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permissible difference</td>
<td>$</td>
<td>\text{CCR}<em>{S,i} - \text{CCR}</em>{S,i+1}</td>
<td>\leq 180$</td>
</tr>
<tr>
<td>$</td>
<td>V_{85,i} - V_p</td>
<td>\leq 10$</td>
<td>$10 &lt;</td>
</tr>
<tr>
<td>$</td>
<td>V_{85,i} - V_{85,i+1}</td>
<td>\leq 10$</td>
<td>$10 &lt;</td>
</tr>
<tr>
<td>$f_{RA} - f_{RD} \geq 0.01$</td>
<td>$-0.04 \leq f_{RA} - f_{RD} &lt; 0.01$</td>
<td>$f_{RA} - f_{RD} &lt; -0.04$</td>
<td></td>
</tr>
</tbody>
</table>
2. **Research methodology**

The experiment was carried out by passing the test sections by a passenger car under normal meteorological conditions (i.e., when the surface is dry, when it is not raining or snowing). The research sample consisted of 10 two-lane rural gravel road sections (A road section is a segment of a road or in individual cases, an entire road. The division of roads into sections can be determined by purely practical purposes, e.g., build a new road, repair or reconstruct a section of an existing road, etc.) (Table 2), which were reconstructed in 2002–2016, i.e., gravel roads were paved. The total length of the selected road sections was 33.05 km. The sampling procedure was based on a principle of correct sampling where the less difference between the values to be measured, the more samples have to be selected in order to determine reliability of their difference. Thus, 10 sections were selected with larger possible differences between each other: different lengths, different frequency of horizontal curves, number of fatal and injury accidents before and after road reconstruction, etc. (Sections selected for research are directly related to practical purposes. The lengths of all sections are sufficient for the traffic safety module to function adequately.) Traffic accident statistics are evaluated in the year before and after the reconstruction, traffic accidents are evaluated that are related to failure to ensure the geometric parameters.

The research was carried out in four stages:

**Stage I.** For each study road section, the Curvature Change Rate \( CCR_s \) of separate horizontal curves was calculated and the overall Curvature Change Rate of the road section. Then, the values of safety criteria were determined and the design levels of safety module of the existing alignment were identified for each section. In cases when the design level was poor, the sections were attributed to potentially dangerous. The research was carried out in both traffic directions.

**Stage II.** The situation was simulated where there were no design solutions and it was difficult to identify the transition curves on the existing road. An alternative road alignments were designed by replacing combined horizontal curves with simple circular curves in a way that deviations of alternative alignment from the designed one were insignificant i.e., less than 30 cm. The safety criteria and the design levels of safety module for the alternative road alignment were determined in the same manner as in **Stage I**.

**Stage III.** A natural experiment was conducted with a random driver passing test sections by car at predetermined speeds and recording the driver's reactions and actions in horizontal curves.
Stage IV. All sections were recalculated using the same algorithm of the traffic safety module, but replacing the \( CCR_s \) and \( CCR \) parameters with alternative parameters \( DCR_s \) and \( DCR \). The mathematical expressions are presented in Section 3 of the article.

From the ten samples, one road section was selected as an example on two-lane rural road No. 4610 from 0.400 to 6.120 km (marked green in Table 2). The length of study road section is 5.720 km, width – 7 m, width of carriageway – 3.0 m, shoulders – 0.5 m each, design speed \( V_p \) – 90 km/h.

The overall Curvature Change Rate was obtained \( CCR_s = 120 \text{ gon/km} \), and according to Equation (1) the 85-th percentile speed on this road section was \( V_{85} = 97 \text{ km/h} \).

Making an assumption that the Curvature Change Rate was calculated correctly, when making design solutions for this road section the design speed had to be \( V_p = 90 \text{ km/h} \), though this was not implemented in practice based on the currently valid design standards of Lithuania.

Table 2. The research sample (set up by the authors)

<table>
<thead>
<tr>
<th>No.</th>
<th>Road No.</th>
<th>The beginning of road section</th>
<th>The end of road section</th>
<th>Length of road section</th>
<th>Number of curves</th>
<th>Fatal and injury accidents before reconstruction (one year before reconstruction)</th>
<th>Fatal and injury accidents after reconstruction (one year after reconstruction)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>km</td>
<td>km</td>
<td>km</td>
<td>units</td>
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<td>units</td>
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<tr>
<td>1</td>
<td>2711</td>
<td>1.990</td>
<td>7.840</td>
<td>5.850</td>
<td>9</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2716</td>
<td>0.890</td>
<td>3.000</td>
<td>2.110</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3210</td>
<td>8.692</td>
<td>9.597</td>
<td>0.905</td>
<td>12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4604</td>
<td>12.470</td>
<td>15.650</td>
<td>3.180</td>
<td>11</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>4604</td>
<td>17.700</td>
<td>22.870</td>
<td>5.170</td>
<td>10</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>4606</td>
<td>21.100</td>
<td>23.050</td>
<td>1.950</td>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>4610</td>
<td>0.400</td>
<td>6.120</td>
<td>5.720</td>
<td>19</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>4613</td>
<td>3.970</td>
<td>7.000</td>
<td>3.030</td>
<td>5</td>
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<tr>
<td>9</td>
<td>4625</td>
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<td>2.565</td>
<td>2.565</td>
<td>9</td>
<td>0</td>
<td>1</td>
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<tr>
<td>10</td>
<td>4637</td>
<td>0.000</td>
<td>2.565</td>
<td>2.565</td>
<td>10</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total: 33.045</td>
<td>93</td>
<td>12</td>
<td>18</td>
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<td></td>
</tr>
</tbody>
</table>
Table 3 gives the analysis of a study road alignment according to the safety criteria:

- Based on Safety Criterion I – no poor sections were determined on the road;
- Based on Safety Criterion II – the design level is poor – there are several design elements not well-balanced between each other with the speed difference of more than 20 km/h;
- Based on Safety Criterion III – the road alignment is dangerous – on horizontal curves there is a high risk for vehicles to run off the road;
- Based on the safety module (combining all the safety criteria) – the road section has three potentially dangerous segments in one or in both driving directions.

In summary, the results of experiment show that the obtained values of different safety criteria and overall safety module in certain sections are either doubtful or do not reflect the real situation:

- The undoubtedly dangerous section (from 3.617 to 3.824 km) was determined by the safety module only in one driving direction (Figure 4);

<table>
<thead>
<tr>
<th>Curve No.</th>
<th>Section, km</th>
<th>R, m</th>
<th>CCR_{st}, mgon/km</th>
<th>Safety Criterion I</th>
<th>Safety Criterion II</th>
<th>Safety Criterion III</th>
<th>Direction</th>
<th>Design levels of the safety module</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.553 - 2.684</td>
<td>250</td>
<td>255</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.50 -0.67</td>
</tr>
<tr>
<td>8</td>
<td>3.617 - 3.722</td>
<td>150</td>
<td>424</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-0.33 -0.67</td>
</tr>
<tr>
<td>10</td>
<td>3.741 - 3.824</td>
<td>250</td>
<td>225</td>
<td>-1</td>
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<td>0</td>
<td>0</td>
<td>0.00 0.00</td>
</tr>
<tr>
<td>12</td>
<td>3.957 - 3.984</td>
<td>120</td>
<td>531</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.33 -0.33</td>
</tr>
<tr>
<td>22</td>
<td>4.831 - 4.918</td>
<td>220</td>
<td>289</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33 0.33</td>
</tr>
<tr>
<td>24</td>
<td>4.928 - 4.995</td>
<td>150</td>
<td>424</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0.00 -0.67 0.00</td>
</tr>
<tr>
<td>28</td>
<td>5.170 - 5.281</td>
<td>200</td>
<td>318</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33 0.00</td>
</tr>
<tr>
<td>30</td>
<td>5.314 - 5.366</td>
<td>250</td>
<td>255</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33 0.33</td>
</tr>
<tr>
<td>32</td>
<td>5.415 - 5.473</td>
<td>160</td>
<td>398</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33 0.33 -0.50</td>
</tr>
<tr>
<td>36</td>
<td>5.857 - 5.938</td>
<td>120</td>
<td>531</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-0.33 -0.67</td>
</tr>
</tbody>
</table>

Table 3. Example of analysis of road alignment by the safety criteria
Two sections (from 4.928 km to 4.995 km and from 5.857 km to 5.938 km) were determined by the safety module as dangerous, though in reality this might not be true (Figures 5 and 6). Thus, the results of experiment are ambiguous – the safety module works, but the accuracy of results is insufficient. In certain cases, the module fails to adequately evaluate horizontal curve in an overall context of the road.

**Figure 4.** The road alignment from 3.617 km to 3.824 km (8, 10 curves)

**Figure 5.** The road alignment from 4.928 km to 4.995 km (24 curve)

**Figure 6.** The road alignment from 5.857 km to 5.938 km (36 curve)
3. Results of experimental research

Alternative approach to the Curvature Change Rate

The results of experimental research showed that the results of Stage I and Stage II were ambiguous – although the safety module worked, the accuracy of its results was limited because it often contradicted reality (a natural experiment in Stage III).

The main parameter on which the first and the second criteria are based as well as the overall safety module is the Curvature Change Rate in the single curve \( CCR_s \). According to the current methodology, the mathematical expression of the \( CCR_s \) of each horizontal curve in case of combined horizontal curve (or in general case) is as follows (Lamm et al., 2007):

\[
CCR_s = \frac{L_{P1}}{2R} + \frac{L_A}{R} + \frac{L_{P2}}{2R} \times \frac{200}{\pi} \times 10^3 = \frac{\gamma \times 63700}{L}, \text{ gon/km.} \quad (5)
\]

In case where there is only circular bend, Equation (5) is significantly simplified:

\[
CCR_s = \frac{L_A \times 63700}{R} = \frac{63700}{R}, \text{ gon/km.} \quad (6)
\]

The \( CCR_s \) values show that the larger \( R \), the smaller Curvature Change Rate. However, it is obvious that when using Equation (6) for calculating the same radius \( R \), the same \( CCR_s \) value is always obtained, i.e., the turning angle of alignment has no influence on the obtained value of \( CCR_s \).

For example, a vehicle speed is 90 km/h, there are two circular horizontal curves with 120 m radius each; however, the first one is...
entered into 3-degree turning angle, the second one—into 30-degree turning angle. Though the Curvature Change Rate calculated by Equation (6) for both curves is the same \( CCR_S = 531 \text{ gon/km} \), it is obvious that in an overall context of the road these curves are dangerous in a different way when driving at the same speed. The question is: Whether the car driving at a speed of 90 km/h and successfully having entered both curves will exit them successfully as well?

Mathematically, the quantity \( 1/R \) is called curvature and its unit of measurement is 1/m. Equation (6) shows that when curvature is multiplied by a certain coefficient the angular quantity is obtained with the units of measurement gon/km and it is not reliable. Thus, the current equations of \( CCR_S \) are not suitable to simple circular curves, since the calculated quantities or parameters do not evaluate these curves in an overall context of the road. It can be stated that the \( CCR \) suggested by R. Lamm is not usable to simple circular curves. For circular curves, the Curvature Change Rate is not calculated, since curvature is only a quantity inversely proportional to radius. If the equation of \( CCR_S \) is not accurate in a separate case, it is likely that it is not accurate also in general. A closer look at Equation (5) shows that it could be also partly simplified. It can be stated that in these equations a deciding segment can be curvature multiplied by coefficient:

\[
CCRS = \frac{L_{p_1} + L_A + L_{p_2}}{2R} = \frac{L_{p_1} + L_{p_2} + L_A}{2R} \times 63700 = \frac{L_{p_1} + L_{p_2} + L_A + L_A}{2R} \times 63700
\]

\[
= \left( \frac{1}{2R} + \frac{L_A}{2RL} \right) \times 63700, \text{ gon/km.} \quad (7)
\]

The largest dilemma is related to the determination of the first and the second safety criteria, which are based on the calculated \( CCR_S \). When safety criteria are determined inaccurately, the whole safety module fails. The current experiment exactly showed the results of such inaccurate modelling—the apparently dangerous locations were not determined and vice versa—a danger was determined at those locations where it was not in fact.

From Figure 7, an assumption can be made that the quantity \( CCR_S \) can result from the analysis of geometric elements of the combined horizontal curve. However, Equations (6) and (7) show that the elements of curve, used in the current \( CCR_S \) calculation equations, partly or wholly eliminate each other. The R. Lamm method ignores all the Points of Intersection (PI), which have no curves.

Analysis of curve geometry leads to a deadlock: a curve of any radius nicely rounds the angle, with the increasing radius all the curve elements increase in proportion (Figures 7 and 8). Here, the question arises: What...
would happen if the curves of such radii were transferred to a real road? On the road the curves of such radii are blocked by the third safety criterion, but this is all about the Curvature Change Rate (i.e., about the first and partly the second criterion).

In summary, it is true to say that the Curvature Change Rate is insufficiently justified mathematically; therefore, it is possible to suggest new definitions and new mathematical expressions for calculating horizontal curves.

3.1. Alternative expression of the Curvature Change Rate

The adequate Curvature Change Rate is impossible without a good understanding of vehicle physics and driver’s behaviour. From the driver’s point of view a curve always tempers a turn of alignment, i.e., always tempers the angle $\gamma$. The new definitions are as follows:

Curvature Change Rate is a ratio between the sum of absolute values of reduced angles in the Points of Intersection of road alignment and the length of road section.

Direction’s Change Rate in the single curve (DCRS) is a reduced angle in the Point of Intersection of road alignment (imaginary or real) where the direction of road alignment changes:

$$DCR_S = \gamma - \Delta\gamma, \text{gon,}$$

where $\gamma$ – a reduced angle of road alignment, $\Delta\gamma$ – a conditional angle or angle correction in the PI.

Curvature Change Rate of the road section is a ratio between the sum of absolute values of reduced angles in the Points of Intersection of road alignment and the length of road section, described by Equation (9):

$$DCR = \frac{\sum \limits_{i=1}^{i=n} (DCR_{Si} \cdot L_i)}{\sum \limits_{i=1}^{i=n} L_i}.$$  

Figure 9. Geometrical deadlock
In Equation (8), $\Delta \gamma$ is a conditional angle or angle correction (reduction) in the PI due to the effect of horizontal curve. The problem is – where and how to see this angle, from where and in which direction to look? Looking from any point in the curve itself, i.e., without considering the car and the driver, we find ourselves in the deadlock (Figure 9).

Conditional angle $\Delta \gamma$ must be a function of curve radius, the driver’s reaction time and vehicle speed. In case of combined horizontal curve, the arguments of a function must be supplemented with the lengths of transition curves.

In the first case – taking into consideration the driver’s behaviour and vehicle physics it is necessary to determine two points (from which and to which to look), knowing which Equation (8) could be used.

In the second case – conditional angle $\Delta \gamma$ could be described by regression function, the main argument of which is the radius $R$ and which is based on the following assumptions:

- When there is no radius, the Curvature Change Rate is $\gamma$;
- When the radius approaches infinity, the Curvature Change Rate becomes zero, i.e., a quasi-tangent section is obtained.

Mathematically, this assumption can be written as:

$$\lim_{R \to \infty} (\gamma - \Delta \gamma) = 0,$$  \hspace{1cm} (10)

A regression function is expressed as:

$$\Delta \gamma = \frac{2}{\pi} \text{arctg}(k R \gamma).$$  \hspace{1cm} (11)

The Curvature Change Rate based on this assumption is expressed by a regression function, one of the possible expressions of which is:

$$DCR_s = \gamma - \frac{2}{\pi} \text{arctg}(k R \gamma).$$  \hspace{1cm} (12)

Equation (12) is suitable only for a circular horizontal curve. For a combined curve, the expression would be more complicated and this would require additional research.

In a regression function (Equation (12)), the arctangent was chosen due to the fact that any horizontal curve of road was first of all the arc of a circle with which this elementary mathematical function was inseparably related. An important argument of the suggested regression function is coefficient $k$, which determines regression intensity. This coefficient can be, in its turn, a function of curve radius, the driver’s reaction time and vehicle speed. The approximate (initial) value of the coefficient $k$ was determined based on the results of Stage III natural experiment. It is not presented in the article yet because it is necessary to significantly increase the sample of the analogous experiment.
After changing the parameters $CCR, CCR_S$ to $DCR, DCR_S$ and repeating the calculations, it was found that not only did the safety module work, but significantly more accurate results were obtained, compared to Stage III natural experiment. Therefore, the above theoretical assumptions are true with a significant probability.

**Conclusions**

Until now, in Lithuania, the reconstruction of automobile roads was not a good practice, i.e., to deviate as little as possible from the existing road axis in horizontal and vertical position. Therefore, it can be assumed that this practice was not adequately justified both from a technical and an economic point of view. Design standards for automobile roads are characterised by the fact that they allow for a free interpretation of the requirements set forth in them. Therefore, it can be stated that there may be inaccuracies in the road design norms currently in force in Lithuania that must be corrected. It is necessary to supplement them with new regulations, which allow one to identify sections of the road that are not properly designed from the point of view of traffic safety, and to correct errors in the design solutions of the road route at the design stage.

Theoretical research showed that insufficient accuracy of the safety module was determined by insufficient mathematical justification of the parameter called Curvature Change Rate. The Curvature Change Rate calculated by the currently used equations does not relate or only partly relates horizontal curves with the turning angles of alignment, though namely this is the purpose of this parameter.

The current methodology of safety criteria and safety module could be improved by alternative definition of the Curvature Change Rate and its mathematical expression, which could possibly ensure an adequate accuracy of methodology. The proposed alternative potentially solves this problem.

The aim of further research is to suggest a mathematical expression of the coefficient $k$ or to come as close to this expression as possible in a way to create assumptions for other researchers in finding an adequate expression of the coefficient by theoretical and experimental studies.
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