

PERFORMANCE EVALUATION OF STONE COLUMNS IN FINE SOIL CONDITIONS: A FEM-BASED CASE STUDY

BAHMAN ZARAZVAND*, JANA FRANKOVSKA

*Department of Geotechnics, Faculty of Civil Engineering, Slovak University of Technology,
Bratislava, Slovakia*

Received 24 September 2024; accepted 10 March 2025

Abstract. This paper presents a comprehensive case study on the numerical analysis of stone columns as a ground improvement technique for an expressway embankment. The primary objective is to assess the effectiveness of stone columns in enhancing the performance of predominantly fine-grained soils using Finite Element Method (FEM) analysis. To achieve the objective, detailed numerical models are developed in both three-dimensional (3D) and two-dimensional (2D) plane strain configurations to simulate embankment conditions accurately. Key geotechnical parameters, including the modulus of elasticity and hydraulic conductivity of the stone column material, are incorporated to account for the improved stiffness and drainage effects. The installation process considers critical factors such as vibration-induced changes and horizontal displacement to capture the evolution of soil stress conditions. A staged construction approach is implemented to realistically simulate the sequential embankment construction process and its impact over time. To ensure model reliability, validation is performed by comparing numerical results with field measurements obtained from horizontal inclinometers installed beneath the embankment. The analysis focuses on key performance indicators such as settlement behaviour, the generation and dissipation of excess pore water pressure, and overall stability assessments. The results demonstrate a strong correlation between numerical predictions and field observations, confirming the accuracy of the developed models. This study provides valuable insights

* Corresponding author. E-mail: bahman.zarazvand@stuba.sk

Bahman ZARAZVAND (ORCID ID 0000-0003-1634-3537)
Jana FRANKOVSKA (ORCID ID 0000-0003-4683-1400)

Copyright © 2025 The Author(s). Published by RTU Press

This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

into the performance of stone column-reinforced embankments, highlighting significant improvements in load-bearing capacity, reduction in settlement, and overall ground stability. By evaluating the role of stone columns in accelerating consolidation and enhancing the stiffness, strength, and stability of fine-grained soil layers, the research contributes to the optimisation of design and construction methodologies for ground improvement. Additionally, a comparative assessment of 3D and 2D plane strain numerical models is conducted to evaluate their predictive capabilities in representing real embankment behaviour. The findings support the advancement of safer and more resilient infrastructure solutions.

Keywords: consolidation settlement, embankment, ground improvement, numerical analysis, stone columns.

Introduction

Stone columns, also known as granular piles, are vertical inclusions of compacted aggregate strategically installed to improve soil stability, enhance load-bearing capacity, facilitate drainage, and increase shear strength. Their construction involves drilling a cavity into the ground, which is then filled with gravel or crushed stone, followed by systematic compaction in layers to ensure structural integrity and efficient load transfer (Barksdale & Bachus, 1983; Ambily & Gandhi, 2007). The effectiveness of stone columns largely depends on the installation technique, which varies based on soil conditions and project-specific requirements. Among the most widely adopted methods are vibro-replacement and the dry bottom-feed technique, both of which provide significant improvements in ground performance by increasing soil density and optimizing load distribution (Barksdale & Bachus, 1983; Priebe, 1995). The installation of stone columns requires specialized machinery, including vibratory probes for compaction and drilling rigs for cavity formation, ensuring precise placement and effective densification of the aggregate (Hausmann, 1990; Baez & Martin, 1993). The primary materials used for stone columns include gravel and crushed stone, selected for their high strength, durability, and drainage capacity. In some cases, recycled aggregates are also utilised as a cost-effective and sustainable alternative, provided they meet the required mechanical and geotechnical performance criteria (Smith & Collis, 2001; Elias & Welsh, 2002). Natural aggregates are generally preferred for stone columns due to their high strength, durability, and compaction efficiency. However, recycled aggregates are gaining attention as a sustainable and cost-effective alternative, provided they meet the necessary engineering and performance standards (Rajesh & Ghosh, 2021; Black et al., 2007). Over time, ground improvement techniques have evolved significantly, integrating traditional approaches such as soil replacement, lime and cement stabilisation, and geosynthetics to enhance soil properties and improve stability in complex geotechnical conditions (Hausmann, 1990; Elias & Welsh, 2002). These

advancements have refined stone column technology, establishing it as a widely adopted and effective solution for reinforcing weak soils and ensuring the stability of critical infrastructure projects.

Stone columns are widely recognised as an effective ground improvement technique, offering multiple benefits for enhancing the engineering properties of weak or soft soils. Their installation densifies the surrounding soil, significantly increasing bearing capacity and improving overall stability (Black et al., 2007). By enhancing soil stiffness, stone columns effectively reduce both immediate and long-term settlements, making them a reliable solution for controlling deformation under structural loads (Balaam & Booker, 1981). Additionally, they increase shear strength while reducing soil plasticity, thereby minimising the risk of excessive deformation and instability (Alkhorshid et al., 2018).

One of the key advantages of stone columns is their high permeability, which allows them to function as efficient vertical drainage paths, particularly in fine-grained soils. This characteristic accelerates consolidation, significantly reducing settlement time and enhancing soil behaviour under applied loads (Priebe, 1995). Additionally, stone columns play a vital role in seismic hazard mitigation, particularly in liquefaction-prone regions, by increasing soil density and improving drainage capacity, enabling the rapid dissipation of excess pore water pressure during earthquakes (Baez & Martin, 1993). Beyond liquefaction resistance, they also help mitigate lateral spreading, a major concern in earthquake-prone areas that can threaten the stability of embankments and structures (Hausmann, 1990).

The load-bearing capacity of soils is significantly enhanced through the use of stone columns, allowing them to support heavier structures and accommodate higher design loads efficiently (Barksdale & Bachus, 1983). This improvement also results in a more uniform settlement profile, minimising differential settlement, which could otherwise lead to structural distress (Mitchell & Huber, 1985). When integrated into slopes or embankments, stone columns further enhance overall stability by reinforcing the soil mass and increasing shear strength, thereby reducing the risk of slope failure (Ghazavi et al., 2008). From a construction perspective, stone columns accelerate the consolidation process, enabling faster ground preparation and reducing project delays associated with long-term soil settlement (Broms, 1999). Their cost-effectiveness makes them an attractive alternative to deep foundations and other expensive ground improvement methods, particularly in large-scale infrastructure projects. Additionally, the use of locally available granular materials for stone column installation minimises material importation costs, further reducing expenses and environmental impact (Krishna et al., 2004).

A key advantage of stone columns is their versatility, as they can be effectively applied in a wide range of soil conditions, from soft cohesive clays to loose sandy deposits, making them a flexible and adaptable solution for various geotechnical

challenges (Goughnour & Bayuk, 1979). Their use also reduces the required thickness of soil improvement layers, leading to savings in construction materials and overall project costs (Priebe, 1995). In areas prone to uplift forces caused by groundwater fluctuations or seismic activity, stone columns provide additional resistance, enhancing the overall stability of foundation systems (Vesic, 1972). Their design adaptability allows for customised configurations based on site-specific requirements, ensuring optimal performance in different ground conditions (Van Impe, 1989). Over the long term, stone columns contribute to lower post-construction maintenance costs, as their ability to improve soil properties minimises the need for costly repairs and structural interventions (Raju et al., 2002). Their durability and sustained performance make them a reliable and integral ground reinforcement technique, ensuring the longevity and resilience of infrastructure projects (Alkhorshid et al., 2018).

Numerous analytical models have been developed to assess the performance of stone columns, offering valuable insights into their effectiveness as a ground improvement technique. One of the most influential contributions in this field is the work of Priebe (1995), who introduced an analytical method for predicting settlement reduction in stone column-treated ground. Another significant contribution was made by Balaam and Booker (1981), who developed an analytical solution for evaluating the bearing capacity of stone columns. Their model incorporates critical design parameters, including column diameter, spacing, and soil characteristics, enabling a more precise assessment of the load transfer mechanisms. This research established a theoretical foundation for understanding the complex interactions between stone columns and the surrounding soil, playing a key role in advancing the design and optimisation of stone column-reinforced foundations.

Building on previous analytical approaches, Barksdale and Bachus (1983) developed a comprehensive methodology for the design and construction of stone columns, emphasising the critical role of soil properties and column material in optimising performance. Their research has had a lasting impact, shaped modern geotechnical design practices and influencing contemporary methodologies. Expanding on these foundations, Mitchell and Huber (1985) provided deeper insights into load transfer mechanisms within stone columns, detailing stress distribution and deformation patterns around the columns. Their study laid the groundwork for subsequent research efforts and practical applications, significantly enhancing the understanding of stone column behaviour under various loading conditions. Similarly, Goughnour and Bayuk (1979) investigated the effectiveness of stone columns in silty clays, developing analytical solutions to predict improvements in shear strength and settlement reduction.

In recent years, numerical modelling has become an essential tool in geotechnical engineering, enabling a deeper understanding of soil behaviour and the optimisation

of ground improvement techniques. Techniques such as the Finite Element Method (FEM), Finite Difference Method (FDM), and Discrete Element Method (DEM) (Han & Ye, 2001; Indraratna et al., 2015) have enabled detailed simulations of the complex interactions between stone columns and surrounding soil under various conditions. These models provide insights that are often difficult to achieve through analytical methods alone, offering a more precise assessment of settlement behaviour, load distribution, and stress transfer mechanisms (Raithel & Kempfert, 2000; Castro & Sagaseta, 2011).

Numerical modelling has proven to be an effective tool for evaluating the performance of stone columns across various soil conditions. Castro (2017) conducted a comparative analysis using numerical models to assess stone columns in soft clays, demonstrating their ability to capture complex soil-structure interactions under different loading scenarios. Similarly, Choobbasti et al. (2011) employed FEM simulations to investigate stone column behaviour in varying soil conditions, highlighting the importance of accurate input parameters for predicting settlement and load transfer more reliably than traditional analytical methods. Further advancements have been made in the numerical modelling of geosynthetic-encased stone columns. Alkhorshid et al. (2018) explored their behaviour through numerical simulations, revealing their enhanced performance in soft soils, particularly in cases where lateral confinement plays a crucial role.

In addition to static loading conditions, recent developments in numerical modelling have extended to evaluating stone columns under dynamic and seismic loading, providing deeper insights into their stability and effectiveness in mitigating ground deformation during earthquakes. Cengiz & Guler (2018) conducted a series of numerical analyses to evaluate the performance of stone columns during seismic events, making a significant contribution to the understanding of their dynamic behaviour. Expanding on this research, Arefpanah & Sharafi (2024) investigated the response of stone columns in liquefiable soils, offering crucial insights into their effectiveness in mitigating seismic-induced ground failure.

Beyond seismic applications, numerical studies have played a key role in assessing the long-term performance and consolidation behaviour of stone columns. Han & Ye (2001a) employed numerical models to examine the role of stone columns in accelerating ground consolidation and enhancing overall soil stability. Similarly, Castro & Sagaseta (2011) used three-dimensional finite element analysis to investigate the interaction between stone columns and surrounding soil, providing valuable insights into stress distribution and deformation mechanisms.

Further research has focused on optimising construction techniques and design parameters to enhance stone column performance. Nav et al. (2020) used numerical models to assess the impact of different installation techniques, providing practical recommendations for improving construction efficiency. Similarly, Kelesoglu &

Durmus (2022) investigated load transfer mechanisms under various loading conditions using FEM, contributing to a more refined understanding of stress distribution and settlement behaviour. Lu & Li (2023) conducted a parametric study evaluating the effects of column spacing and diameter on overall ground performance, offering valuable guidance for design optimisation. Additionally, Tandel et al. (2020) explored the challenges of using stone columns in expansive soils, identifying effective solutions for improving ground stability in such conditions.

Recent advancements have also investigated the combined use of stone columns and geosynthetics. Rajesh & Ghosh (2021) developed a comprehensive numerical model to analyse the synergistic effect of stone columns and geosynthetics in reinforcing soft soils, providing practical design recommendations for improving performance. Collectively, these studies highlight the critical role of numerical modelling in refining design methodologies, predicting long-term behaviour, and enhancing the effectiveness of stone columns across a range of geotechnical applications.

The primary objective of this study is to evaluate the effectiveness of stone columns in accelerating consolidation and enhancing the stiffness, strength, and stability of fine soil layers beneath embankments. To achieve this objective, a comparative numerical analysis is conducted using both three-dimensional (3D) and two-dimensional (2D) plane strain models, allowing for an assessment of their predictive accuracy in simulating embankment behaviour. By examining the capabilities and limitations of these numerical approaches, this study provides an evaluation of stone column performance in ground improvement applications. To ensure the reliability and accuracy of the numerical models, a rigorous validation process is undertaken. The first numerical model is based on an embankment constructed along the R2 highway in Slovakia, where settlement-time graphs from simulations are compared with field measurements from horizontal inclinometers installed on-site (GEOFOS, 2016). This comparison serves as a critical step in validating the model's predictive capabilities. Beyond the base model, additional scenarios are simulated to further investigate the effectiveness of stone columns. Model 2 examines the minimum time required for full consolidation, analysing the generation and dissipation of excess pore water pressure and predicting settlement behaviour under traffic loads. Model 3, on the other hand, represents a scenario without ground improvement, serving as a baseline to isolate and quantify the direct impact of stone columns on embankment performance. By comparing these models, the study provides a clear assessment of the role of stone columns in reducing settlement and enhancing stability. Key factors analysed include the compaction effects of stone columns, the influence of drainage paths, soil-column interaction mechanisms, and the impact of geostatic stress and traffic-induced loads. This holistic approach offers valuable insights into the engineering applications of stone

columns, reinforcing their significant role in improving soil properties. Ultimately, the findings emphasise the importance of stone columns in enhancing embankment performance and durability, contributing to the development of safer and more resilient infrastructure.

1. Material

This study considers a combination of embankment fill, stone column material, and fine-grained subsoil, which significantly influence the settlement and stability of the embankment. These materials are selected based on geotechnical investigations conducted along the R2 expressway in Slovakia (GEOFOS, 2016). The embankment is constructed using coarse-grained soil, ensuring adequate compaction and load distribution. Its high shear strength and permeability facilitate stability under traffic loads while minimising the risk of excessive settlement. The stone columns are composed of crushed stone, chosen for its superior stiffness and drainage capacity. The high permeability of this material allows for rapid dissipation of excess pore water pressure, accelerating the consolidation process. Additionally, the stone columns enhance the load-bearing capacity of the underlying soil by reinforcing weak layers and improving stress distribution. The natural subsoil consists of three distinct layers, with sandy clay (CS) forming the uppermost layer, followed by medium plastic clay (CI), and underlain by a clayey gravel (GC) layer. The sandy clay exhibits slightly higher permeability than the underlying clay but still retains a relatively low drainage capacity. Beneath this layer, the medium plastic clay is more compressible, with lower permeability, leading to prolonged consolidation periods and increased settlement potential. The lowest clayey gravel layer provides improved strength and transitional stiffness. The groundwater table is located 1.4 m below the ground surface, significantly influencing pore water pressure dissipation during consolidation. The limited permeability of the upper fine-grained subsoil layers restricts natural drainage, prolonging settlement unless vertical drainage elements, such as stone columns, are introduced to accelerate the process.

To accurately represent the behaviour of these materials under loading conditions, numerical modelling incorporates the Hardening Soil Model (HSM), which accounts for non-linear stress-strain relationships and stress-dependent stiffness variations. The geotechnical parameters used in the simulations, including unit weight, stiffness moduli, shear strength, and permeability, are summarised in Table 1. These values were derived from laboratory tests and in-situ investigations to ensure realistic modelling of embankment performance.

Table 1. Soil properties

Parameters	Units	Medium plastic clay (CI) with firm consistency	Sandy clay (CS)	Clayey Gravel (GC)	Stone Column Material	Embankment Material
γ	kN/ m ³	19.80	19.8	19.5	17.4	20
γ_{sat}	kN/ m ³	20.10	20.2	20	19.5	21
ϕ'	°	18.80	21.7	30	40	32
c'	kPa	21.00	14	4	3	5
$E_{\text{oed}} = E_{50}$	MPa	8.57	6.02	67.31	82	32
E_{ur}	MPa	25.71	18.06	201.9	247	96
M	-	0.7	0.60	0.5	0.5	0.5
k	m/ day	1.46E-03	0.235E-03	0.59E-03	86.4	86.4

2. Methodology

2.1. Compacting effects adjacent to stone columns

Installing stone columns induces vibration and horizontal displacement in the surrounding soil, leading to an increase in lateral earth pressure. This effect is particularly significant in vibro-installation techniques, where the surrounding soil undergoes stress redistribution and densification. To account for these effects, researchers have proposed using a modified coefficient of lateral earth pressure (K^*), which is greater than the at-rest lateral earth pressure coefficient (K_0) (Priebe, 1976; Pitt et al., 2003; Elkasabgy, 2005; Elshazly et al., 2006). Elshazly et al. (2006) established a strong correlation between inter-column spacing and K^* , emphasising its importance in capturing stress redistribution and improving ground stability.

In this study, the stone columns are installed with a spacing of 2 m, a configuration that significantly influences lateral earth pressure and soil compaction effects. Based on established empirical relationships and applying conservative lower-bound estimates, the value of K^* is determined to be 0.92.

This adjustment accounts for the increased lateral earth pressure caused by soil compaction during installation, leading to enhanced soil density, stiffness, and load-bearing capacity. The relationship between inter-column spacing and K^* is a critical parameter in optimising stone column performance, ensuring the stability of the treated soil, and enhancing the overall effectiveness of ground improvement.

2.2. Drainage path effects

In the numerical modelling process, the drainage function of stone columns is explicitly incorporated by considering their significantly higher permeability than the surrounding fine-grained soil. The permeability contrast between the stone column material and the native soil facilitates the rapid dissipation of excess pore water pressure, thereby accelerating consolidation and enhancing soil stability (Han & Ye, 2001). This mechanism is particularly crucial for fine-grained subsoils, where limited natural drainage leads to extended settlement periods without improvement. To accurately simulate this drainage function, the numerical model explicitly represents the hydraulic conductivity of the stone columns, ensuring a realistic assessment of settlement behaviour, load transfer mechanisms, and time-dependent consolidation (Castro, 2017). This study evaluates the role of stone columns in dissipating excess pore water pressure under various loading conditions, including staged embankment construction, cyclic loading, and sustained traffic loads. The comparison between models with and without stone columns provides a quantitative measure of their effectiveness in reducing settlement and improving embankment performance. By enhancing vertical drainage efficiency, stone columns significantly reduce consolidation time, leading to faster stabilization of embankments and improving their long-term reliability (Han & Ye, 2002). The integration of drainage effects in numerical simulations ensures a comprehensive understanding of their role in ground improvement, contributing to the development of more effective and sustainable embankment reinforcement solutions.

2.3. Sequential phase analysis

The staged construction technique in numerical modelling is pivotal for accurately simulating embankment construction sequences, enabling a realistic assessment of embankment behaviour during various construction phases. This method facilitates the progressive addition of structural elements and incremental load applications, capturing the gradual development of stresses, strains, and consolidation effects. In

geotechnical engineering, particularly for large-scale embankment projects, time-dependent consolidation analysis is essential for predicting settlement behaviour and ensuring stability throughout the construction process. Staged modelling reflects the gradual increase in overburden pressure, allowing for an accurate assessment of pore pressure dissipation and settlement progression over time.

Several studies have demonstrated the efficacy of staged construction modelling. For instance, Ali & Ansary (2020) conducted a numerical analysis of staged embankment construction on soft soil, dividing the process into multiple phases with consolidation periods between each stage to allow excess pore pressure dissipation and settlement determination. Similarly, Tran (2001) utilised finite element modelling to study the behaviour and stability of an earth fill dam constructed in stages on soft subsoil, evaluating stress-strain behaviour and pore-water pressure at each construction phase.

2.4. 3D finite element modelling (FEM)

The development of a three-dimensional (3D) finite element model is crucial for accurately simulating embankments reinforced with stone columns. This advanced modelling approach allows for a detailed representation of soil stratification, embankment geometry, and material properties, ensuring a high-fidelity simulation of ground behaviour. By incorporating the spatial arrangement of stone columns, including their precise dimensions and spacing, the model effectively captures the complex interactions between the reinforcement elements and the surrounding soil. This enables a realistic assessment of stress distribution, soil-structure interaction, and deformation patterns under various loading conditions. Castro (2017) conducted a comprehensive review of numerical modelling techniques for stone columns, emphasising the critical role of accurate geometric representation in finite element analyses. The study demonstrated that 3D modelling provides a more reliable prediction of embankment performance by precisely simulating the complex interplay between stone columns and the surrounding soil matrix. The findings highlight the necessity of incorporating true geometric configurations to enhance model accuracy and predictive capabilities.

In this study, a high-resolution 3D finite element model was meticulously developed to simulate the embankment, taking into account variations in soil stratification, embankment geometry, and material properties. The model explicitly represents the

spatial distribution of stone columns, each measuring 5 m in length, 0.6 m in diameter, and spaced at 2-meter intervals. To balance computational efficiency with model accuracy, the domain extends 2 m in the y-direction, as depicted in Figure 1. The stone columns are modelled in their true cylindrical form, with half-columns incorporated along the boundaries to accurately capture their interaction with adjacent soil.

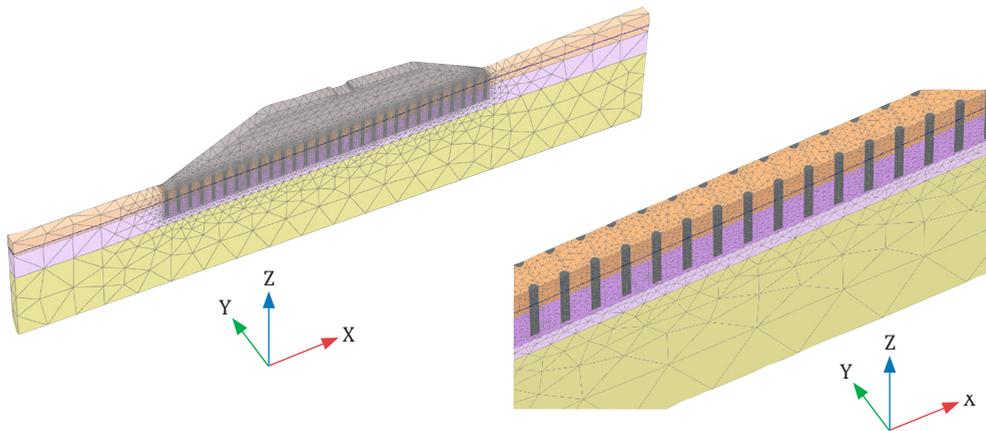


Figure 1. Conceptual representation of stone column simulation in the 3D numerical model

2.5. Conversion from 3D to 2D plain strain model

Given the considerable length of the embankment in comparison to its width and thickness, a plane strain modelling approach is appropriate for numerical analysis. The plane strain assumption simplifies the computational process while retaining accuracy by assuming negligible strain variations along the longitudinal direction. This method has been widely adopted in geotechnical engineering for analysing stone column-reinforced foundations (Tan & Oo, 2005; Tan et al., 2008). The initial efforts to develop a plane strain modelling approach for stone column foundations were introduced by Tan & Oo (2005), who proposed a conversion scheme that transforms an axisymmetric unit cell model into an equivalent plane strain model. Their methodology ensured that composite stiffness and permeability were matched between the 3D and 2D representations, making the numerical analysis more computationally efficient while preserving mechanical behaviour. Building on this foundation, Tan et al. (2008) refined the approach by introducing two simplified methods to improve the plane strain representation of a unit cell. Their work focused on maintaining geometric and permeability equivalence, thereby facilitating practical numerical modelling with enhanced accuracy.

To represent a group of stone columns in a 2D plane strain model, the equivalent wall (strip) method is employed. This approach ensures that the cross-sectional area of the stone column strip in the 2D model is equivalent to that of the actual stone columns in 3D space. The conversion process involves deriving the thickness of the equivalent wall using the following equations (Tan et al., 2008).

$$A_{\text{column.strip}} = A_{\text{wall}}, \quad (1)$$

$$\pi d^2/4 = t \cdot s, \quad (2)$$

$$t = \pi d^2/4s, \quad (3)$$

where

d is the diameter of the stone columns;

S is the spacing between the stone columns;

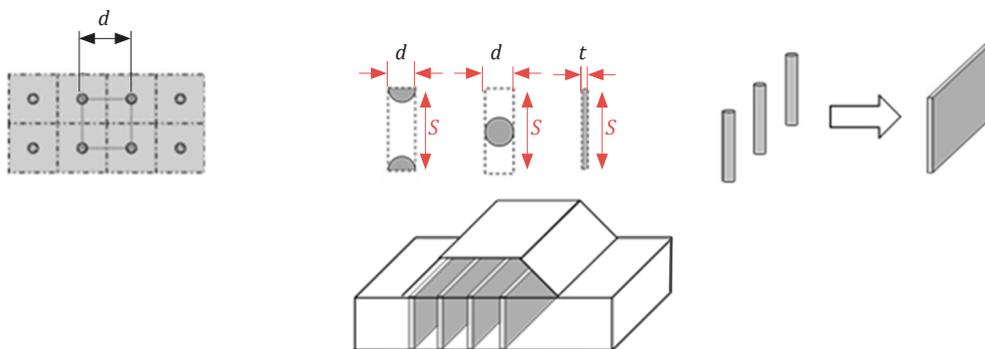
t is the thickness of the equivalent wall.

Substituting the values from the case study, the equivalent wall thickness is calculated as follows:

$$t = (\pi d^2)/4S = [\pi \times (0.6)^2] / (4 \times 2) = 0.14 \text{ m}.$$

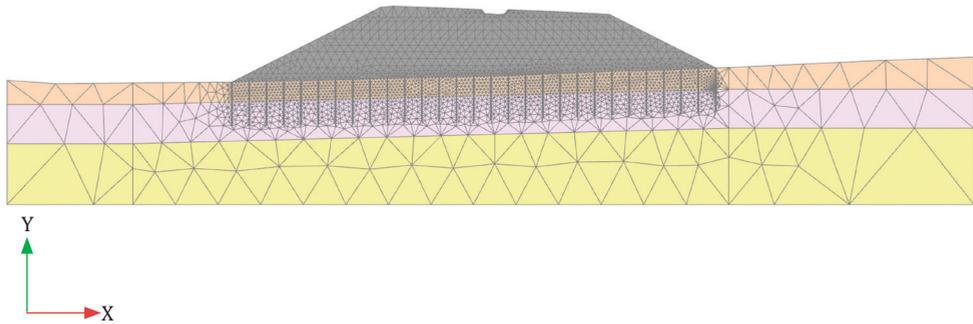
Thus, in the 2D plane strain model, the equivalent wall thickness representing the stone column reinforcement is approximately 0.141 m. This transformation ensures that the mechanical behaviour, stress distribution, and settlement effects of the stone columns are accurately captured in numerical analysis.

Figure 2(a) illustrates the idealized simulation of a square pattern of stone columns with the equivalent wall method, while Figure 2(b) presents the implementation of the equivalent stone column strip in a 2D plane strain model in PLAXIS. This transformation from 3D to 2D modelling enhances computational efficiency while preserving the critical interaction effects between stone columns and surrounding soil.



(a) Idealization of stone column arrangement using the equivalent wall method

Figure 2. Conceptual representation of stone column simulation in the 2D-plane strain numerical model



(b) 2D plane strain simulation of stone columns in PLAXIS

Figure 2. Conceptual representation of stone column simulation in the 2D-plane strain numerical model

3. Numerical models

The embankment studied in this research was constructed along the R2 highway in Slovakia, which serves as a critical transportation link between the cities of Trenčín and Košice. A schematic representation of the embankment, along with its geological profile, is provided in Figure 3. To monitor settlement behaviour, a horizontal inclinometer was installed 1.5 m beneath the embankment surface. To enhance the stability and load-bearing capacity of the subsoil, stone columns were incorporated as a ground improvement measure. These stone columns have a diameter of 0.6 m and are spaced 2 m apart (centre-to-centre). This configuration is designed to improve soil stiffness, accelerate consolidation, and mitigate excessive settlement, ensuring the long-term structural integrity of the embankment.

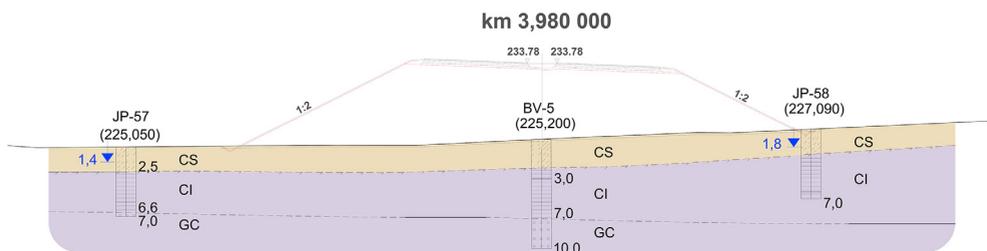


Figure 3. Schematic representation of the embankment on the R2 expressway

3.1. Model 1: As-built model for process of validation

The as-built model replicates the embankment as constructed, including the installed stone columns. The primary objective of this model is to verify and validate the numerical simulation by comparing the computed settlement-time behaviour with actual field measurements obtained from the horizontal inclinometer. To achieve this objective, the staged construction approach was implemented, reflecting the real sequence of embankment construction. Each construction phase was modelled in accordance with the actual timeline and loading conditions, ensuring an accurate simulation of soil behaviour under progressive loading. The details of the staged construction process are summarised in Table 2.

Table 2. Staged construction phases of Model 1

Phases	Phases	Type of Analysis	Time interval, day	Total time, day
1	Implementation of stone columns and applying densification	Consolidation	1	1
2	Construction of embankment (1 st Part)	Consolidation	1	2
3	Construction of embankment (2 nd Part)	Consolidation	1	3
4	Construction of embankment (3 rd Part)	Consolidation	1	4
5	Construction of embankment (4 th Part)	Consolidation	1	5
6	Construction of embankment (5 th Part)	Consolidation	1	6
7	Construction of embankment (6 th Part)	Consolidation	1	7
8	Construction of embankment (7 th Part)	Consolidation	1	8
-	Time required for full consolidation for construction of embankment	Consolidation/ Minimum excess pore water pressure	To be calculated	To be calculated
-	Stability analysis of embankment	Safety	-	-
9	Applying traffic load	Plastic	1	To be calculated
-	Time required for full consolidation for applying traffic load	Consolidation/ Minimum excess pore water	To be calculated	To be calculated

3.2. Model 2: Time interval-independent investigation

During this phase of the investigation, the focus is on isolating the time required for full consolidation, independent of the actual construction duration. Therefore, the staged construction approach is again utilised, with each phase assigned a fixed duration of seven days. This method allows for a systematic evaluation of the role of stone columns as vertical drainage elements, expediting excess pore water pressure dissipation.

At the final stage, the time required to reach 90% consolidation settlement is determined using numerical simulations. This calculation provides critical insights into the effectiveness of stone columns in reducing consolidation time. The breakdown of the construction stages for Model 2 is provided in Table 3.

Table 3. Staged construction phases of Model 2

Phases	Phases	Type of Analysis	Time interval, day	Total time, day
0	Initial phase	Gravity Loading	0	0
1	Implementation of stone columns and applying densification	Consolidation	1	1
2	Construction of embankment (1 st Part)	Consolidation	1	2
3	Construction of embankment (2 nd Part)	Consolidation	1	3
4	Construction of embankment (3 rd Part)	Consolidation	1	4
5	Construction of embankment (4 th Part)	Consolidation	1	5
6	Construction of embankment (5 th Part)	Consolidation	1	6
7	Construction of embankment (6 th Part)	Consolidation	1	7
8	Construction of embankment (7 th Part)	Consolidation	1	8
-	Time required for full consolidation for construction of embankment	Consolidation/ Minimum excess pore water pressure	To be calculated	To be calculated
-	Stability analysis of embankment	Safety	-	-
9	Applying traffic load	Plastic	1	To be calculated
-	Time required for full consolidation for applying traffic load	Consolidation/ Minimum excess pore water	To be calculated	To be calculated
-	Stability analysis of embankment after applying traffic load	Safety	-	-

By eliminating construction time variability, this model provides a direct assessment of the stone column impact on consolidation rates.

3.3. Model 3: Analysis without soil improvement

Model 3 examines the behaviour of the embankment in the absence of any ground improvement measures, serving as a baseline for evaluating the effectiveness of stone columns. By simulating the embankment without stone columns, this model enables a direct and quantitative comparison of key performance parameters, including settlement reduction, excess pore water pressure dissipation, and overall stability. The staged construction sequence for Model 3 follows the same structure as Model 2, as detailed in Table 3, with the only distinction being the omission of the stone column installation and densification phase. This modification ensures a clear assessment of how soil improvement influences embankment behaviour under identical loading and construction conditions. Through this comparative analysis, the study effectively quantifies the extent to which stone columns contribute to reducing settlement, highlighting their role in enhancing subsoil stiffness and load-bearing capacity. Additionally, it evaluates the rate of excess pore water pressure dissipation, providing insights into the improved drainage efficiency facilitated by stone columns. The findings from Model 3 further demonstrate the impact of stone columns on embankment stability, particularly in terms of mitigating differential settlement and maintaining structural integrity under sustained and cyclic loading conditions. By isolating the effects of soil improvement, this analysis reinforces the importance of stone columns in embankment construction, offering valuable guidance for optimising their design and implementation in similar geotechnical applications.

4. Results and discussion

4.1. Model 1: As-built model for validation

The validation of the numerical model was conducted by comparing the settlement recorded from a horizontal inclinometer installed at a depth of 1 m below the embankment surface with the settlement predicted by numerical simulations. The primary validation metric was the magnitude of settlement, with the results presented in Figure 4. Over a period of approximately 200 days, all three datasets – the 2D model, 3D model, and field measurements – exhibited a decreasing settlement trend, indicating progressive consolidation and soil compression under applied loads. Initially, the settlement rate was rapid, then gradually slowed as consolidation progressed. Both the 2D and 3D models followed similar trends, with minor variations in predicted settlement values. By day 194, both numerical models estimated a final settlement of approximately -72 mm to -73 mm, indicating

strong consistency between the two numerical approaches. However, the field measurements displayed a lower settlement magnitude, stabilising at approximately -52 mm after 200 days. This discrepancy of 28% suggests that the numerical models overestimated settlement, particularly in the later stages of consolidation.

In the initial 40 days, the field settlement closely followed the numerical predictions, but diverged after day 50, highlighting factors not fully captured in the simulations. One key factor influencing this disparity is the nature of the field measurement itself. The inclinometer recorded maximum horizontal settlement at a depth of 1 meter in the centre of the embankment, meaning it reflects settlement at a single point within the soil body. Since soil deformation varies spatially, relying on one measurement point may not accurately represent the overall embankment settlement behaviour, leading to potential inaccuracies when compared to numerical predictions. Additionally, uncertainties in field instrumentation must be considered. Variability in the installation depth, precision of sensor calibration, and data recording intervals could introduce errors in measured values, contributing to deviations between field data and model results.

From a numerical modelling perspective, a major factor contributing to the discrepancy is the treatment of soil stratigraphy. The numerical model assumed uniform soil conditions between the three boreholes shown in Figure 3, with linear interpolation of soil properties. However, in reality, soil properties vary significantly between boreholes, exhibiting spatial heterogeneity that is challenging to fully capture in numerical simulations. Numerical models typically idealize soil as a continuous, homogeneous, isotropic material, whereas in reality, soils are porous, non-homogeneous, and anisotropic. The assumption of uniform conditions may overestimate settlement, as the model does not consider the presence of stiff soil inclusions, which could reduce total settlement, or localized weak zones, which could cause greater deformation in specific areas.

Another important factor affecting settlement predictions is the choice of material properties and the soil model used in the numerical analysis. The soil parameters were derived from laboratory tests on borehole specimens, which, despite strict testing protocols, might not fully reflect in-situ conditions due to sample disturbance and differences in field-scale behaviour. Variability in stiffness, compressibility, and permeability across the site can significantly impact settlement predictions.

Load conditions in the numerical model also significantly influence settlement predictions. The unit weight of the embankment, which governs the load applied to the subsoil layers, was assumed to remain constant throughout all staged construction phases in the model. A relatively conservative high value was adopted, as specified in Table 1. However, in reality, soil conditions are more complex, and several factors contribute to variations in unit weight over time. One key factor is the degree of saturation, which is not constant throughout the embankment.

Depending on weather conditions, drainage efficiency, and construction sequence, soil moisture content may fluctuate, leading to changes in bulk and effective unit weight. Additionally, soil particles are inherently non-homogeneous, meaning that compaction, grain size distribution, and material composition can vary across different embankment layers. These variations influence the actual load transferred to the subsoil layers, potentially causing localized differences in stress distribution and settlement behaviour. By assuming a constant unit weight, the numerical model may oversimplify the actual loading conditions, potentially leading to deviations in predicted settlements. Future improvements in modelling could incorporate spatially variable unit weight values based on measured field moisture content and compaction variability, providing a more accurate representation of real-world conditions.

The settlement distribution obtained from the numerical simulations is illustrated in Figures 5 and 6, representing the 2D and 3D settlement contours after full consolidation. The 2D model estimated a maximum settlement of 12.47 mm, while the 3D model predicted a maximum settlement of 12.38 mm. Despite minor variations, both numerical approaches exhibited similar settlement patterns, reaffirming the consistency of predictions. However, due to the inherent differences in stress distribution, the 3D model provided a slightly more refined representation of the embankment behaviour.

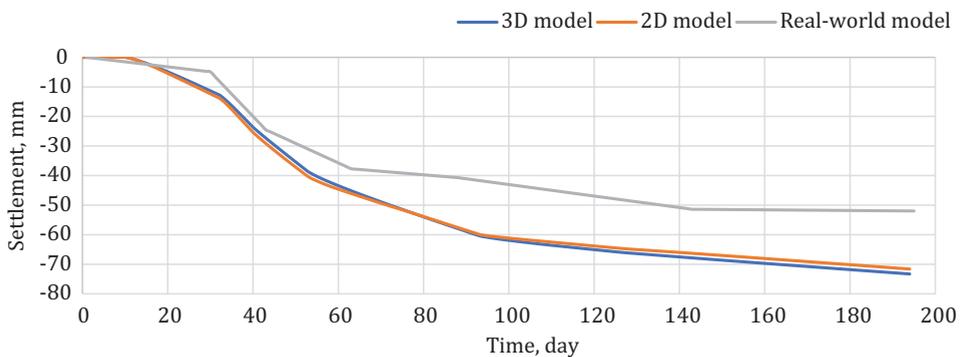


Figure 4. Comparative graphs of time-settlement relationships for numerical models and field measurements

Figures 5 and 6 show the settlement contours in the two-dimensional and three-dimensional models, respectively, after full consolidation. The calculated maximum settlement values are 12.47 mm and 12.38 mm for the 2D and 3D models, respectively.

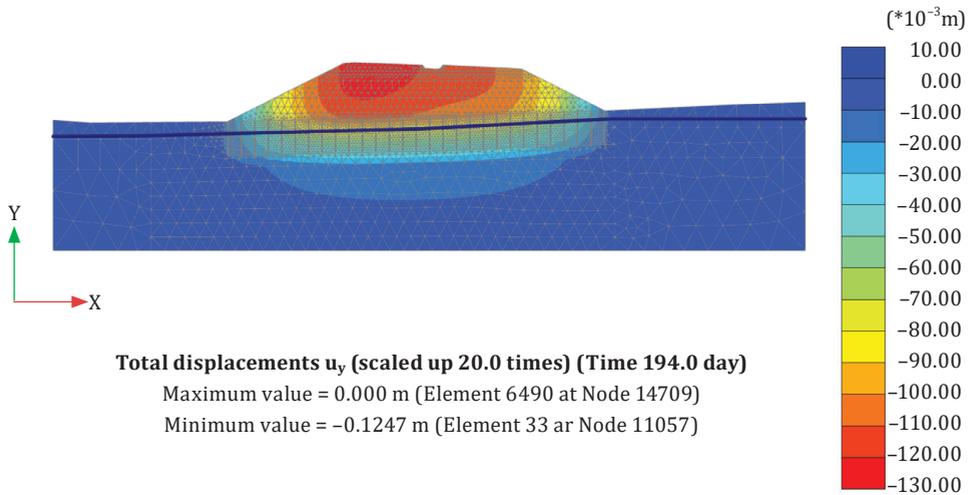


Figure 5. The magnitude of settlement estimated by the plane strain model (2D)

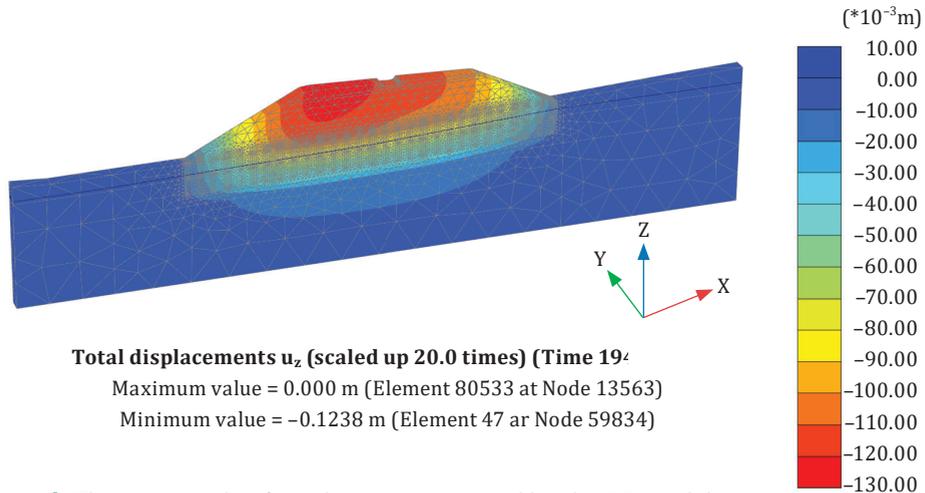


Figure 6. The magnitude of settlement estimated by the 3D model

4.2. Models 2 and 3

The numerical analysis for Models 2 and 3 was conducted to compare four distinct scenarios, assessing the influence of soil improvement (stone columns) versus no improvement in both 2D and 3D numerical models. The analysis focused on settlement behaviour, excess porewater pressure generation and dissipation,

and overall embankment stability to evaluate how stone columns improve ground performance under operational conditions.

4.3. Settlement analysis

The settlement results reveal significant differences between the improved (with stone columns) and unimproved (without stone columns) cases, as well as notable variations between the 2D and 3D models. In the improved case, where stone columns were implemented, the 2D model predicted a final settlement of 72 mm, while the 3D model estimated 73 mm. The close agreement between these models confirms that stone columns effectively reduce settlement by enhancing soil stiffness and providing additional load-bearing support.

In contrast, in the unimproved case, the settlement increased significantly, reaching 103 mm in both the 2D and 3D models. This represents a 43% increase compared to the improved scenario, highlighting the critical role of stone columns in mitigating settlement and enhancing the embankment performance.

When comparing the 2D and 3D models, the results show a 1 mm difference in the improved case, with the 3D model predicting a slightly higher settlement than the 2D model. However, in the unimproved case, both models produced identical results. This close agreement suggests that both modelling approaches effectively capture overall settlement trends. However, 3D modelling provides a more refined representation of stress distribution and soil behaviour, particularly in cases where complex soil-structure interactions play a crucial role. While both 2D and 3D models yield comparable results for total settlement, 3D modelling remains superior in capturing localized effects, making it a more accurate tool for geotechnical analysis involving soil reinforcement methods such as stone columns.

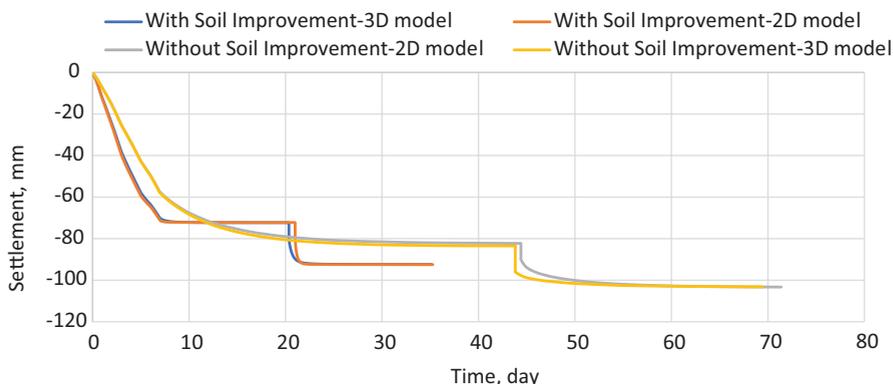


Figure 7. Comparative graphs of settlement versus time of 2D and 3D models

4.4. Generation and dissipation of excess porewater pressure

The analysis of excess porewater pressure (P_{excess}) was conducted to compare the performance of improved (with stone columns) and unimproved (without stone columns) embankments and to assess differences between 2D and 3D numerical models. Figure 8 shows the variation of maximum porewater pressure over time under both construction loading and subsequent traffic loads.

During the initial construction phase, the improved scenario benefits from enhanced drainage due to the presence of stone columns. As a result, the peak porewater pressures are relatively low: the 2D model predicts a peak of 42.89 kPa, while the 3D model shows 48.41 kPa. In contrast, the unimproved case experiences much higher peak pressures, with values of 113.14 kPa in the 2D model and 112.64 kPa in the 3D model. Furthermore, the full dissipation of porewater pressure is achieved more rapidly in the improved scenario – 31 days for the 2D model and 37 days for the 3D model – compared to 44.4 days (2D) and 45.29 days (3D) in the unimproved case. This shorter consolidation period highlights the efficiency of stone columns in promoting drainage and reducing initial pressure buildup.

Immediately after the construction phase, traffic loads are applied. Under these conditions, the improved embankment exhibits a secondary increase in porewater pressure. The 2D model indicates a peak of 131.25 kPa, while the 3D model records a significantly higher peak of 174.61 kPa. Conversely, the unimproved case shows only modest increases under traffic loading, with peak pressures reaching 48.92 kPa (2D) and 37.34 kPa (3D). The higher transient pressures observed in the improved scenario under traffic loads can be attributed to the increased compaction of the reinforced soil, which, while promoting faster drainage overall, produces a more pronounced short-term pressure response.

Differences between the 2D and 3D models are evident throughout the analysis. In the improved case, the 3D model consistently predicts higher peak porewater pressures than the 2D model. This is likely due to the 3D model's more accurate representation of the cylindrical geometry of stone columns, whereas the 2D model approximates them as rectangular walls, thereby underestimating localized pressure concentrations and overestimating drainage efficiency. In the unimproved scenario, however, both modelling approaches yield similar results, reflecting the more uniform behaviour of soil without stone columns.

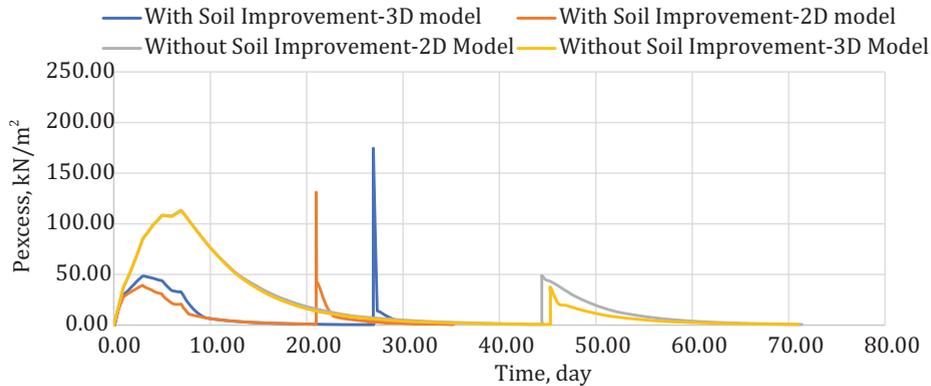


Figure 8. Comparative graphs of time-excess porewater pressure

4.5. Stability analysis

Stability analysis was conducted for all four scenarios to evaluate the safety factor (SF) of the embankment under traffic loading. Figure 9 presents the calculated safety factors for both 2D and 3D models, considering improved (with stone columns) and unimproved (without stone columns) soil conditions.

For the improved case (with stone columns), the 2D model predicts a safety factor of 1.58, while the 3D model provides a slightly higher value of 1.65. The increase in SF observed in the 3D model reflects its more accurate representation of soil-structure interaction, particularly in terms of stress distribution around stone columns. The 3D model better captures localized effects, such as radial drainage and load transfer mechanisms, which the 2D model tends to simplify due to geometric approximations.

For the unimproved case (without stone columns), the safety factor values for the 2D and 3D models are 1.57 and 1.63, respectively. The minimal difference between the improved and unimproved cases suggests that stone columns have a relatively minor effect on global embankment stability. While they improve load distribution and increase soil stiffness, their primary contribution lies in settlement reduction and porewater pressure dissipation rather than significantly enhancing structural stability. The slight increase in SF observed with stone columns indicates some improvement in soil strength and load-bearing capacity, but it does not drastically alter the overall stability of the embankment. Further analysis of shear failure mechanisms in the limit state is illustrated in Figure 10, which shows the shear failure surfaces for both the 2D and 3D models.

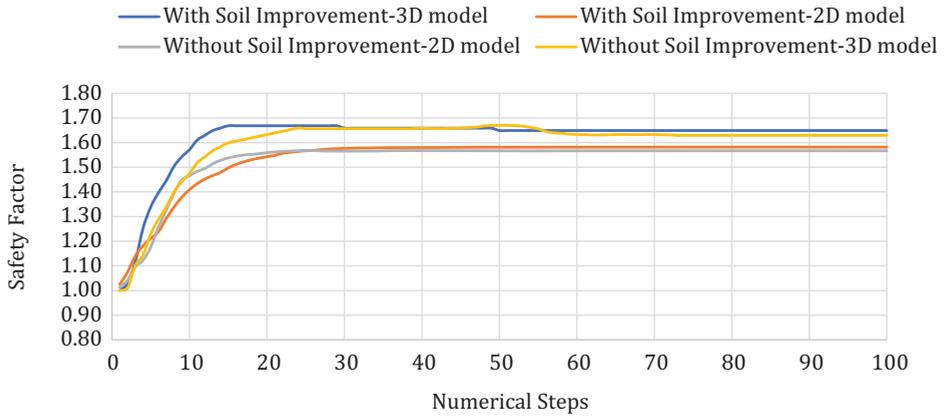


Figure 9. Safety factor of stability

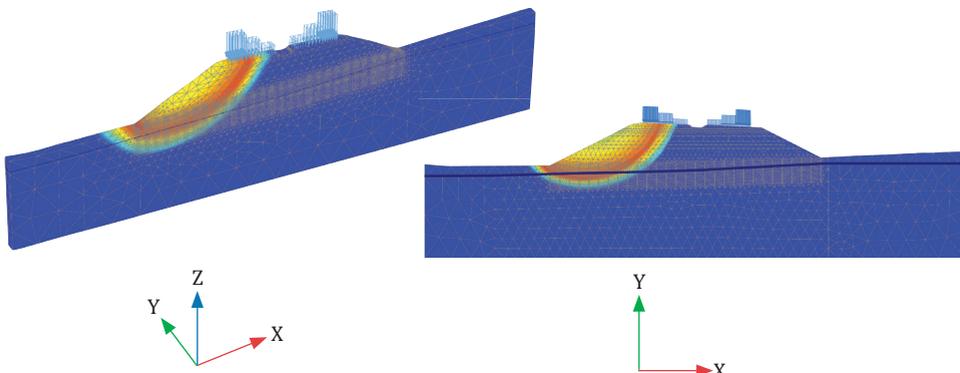


Figure 10. Shear failure surface highlighted by red contours

Conclusions

This study presents a finite element analysis to evaluate the effectiveness of stone columns as a ground improvement technique in fine soil conditions beneath embankments. The research compared both 2D and 3D modelling approaches to assess key performance parameters, including settlement reduction, excess pore water pressure dissipation, and overall stability under traffic loading. The main conclusions drawn from the investigation are presented below.

Model validation and limitations: Validation against field measurements revealed that the numerical models overestimated settlement by approximately 28% during

later consolidation stages. Despite this discrepancy, the numerical settlement curves closely follow the overall trend observed in field measurements, particularly in the initial and intermediate consolidation phases. The deviation in later stages is primarily attributed to idealizations in soil stratigraphy and uniform material properties within the models. However, this overestimation is on the safe side, ensuring a conservative approach in predicting settlement behaviour.

Settlement reduction: The incorporation of stone columns significantly reduces settlement. Numerical models indicate that, under improved conditions, the final settlement is approximately 43% lower than in unimproved scenarios. This reduction is primarily attributed to the enhanced soil stiffness and improved load distribution provided by the stone columns.

Enhanced drainage and accelerated consolidation: Stone columns act as efficient vertical drainage paths, accelerating the dissipation of excess pore water pressure. In improved scenarios, full consolidation is achieved roughly 30% faster than in cases without stone columns. This expedited consolidation is a critical benefit, reducing the time required for the embankment to reach stable conditions.

Stability and stress distribution: While the overall global stability of the embankment is only marginally affected by stone columns, the 3D numerical model shows an improvement in the safety factor of about 4% compared to the 2D model. This modest increase reflects a more accurate representation of localized stress distribution and load transfer mechanisms, underscoring the advantage of using 3D modelling to capture complex soil–structure interactions.

Acknowledgements

The authors are grateful for the financial support provided by the Grant Agency of the Ministry of Education, Science, Research and Sport of the Slovak Republic. The project presented in this article is supported by grant project VEGA 1/0745/21.

Funding

This work was supported by the VEGA under Grant 1/0745/21.

Disclosure Statement

The authors declare that they have no competing financial, professional, or personal interests that could have influenced the work presented in this manuscript.

REFERENCES

- Abdelbaset, A., & Arafa, I. T. (2023). Improvement of consolidation settlement beneath square footings using uncased and encased stone columns. *Structures*, 57, Article 105609. <https://doi.org/10.1016/j.istruc.2023.105609>
- Ali, M., & Ansary, M. A. (2020). Numerical analysis of staged construction of an embankment on soft soil. *International Journal of Geotechnical Engineering*, 14(6), 582–594.
- Alkhorshid, N. R., Araújo, G. L. S., & Palmeira, E. M. (2018). Behaviour of geosynthetic-encased stone columns in soft clay: Numerical and analytical evaluations. *Soils and Rocks*, 41(3), 333–343. <https://doi.org/10.28927/SR.413333>
- Ambily, A. P., & Gandhi, S. R. (2007). Behavior of Stone Columns Based on Experimental and FEM Analysis. *Journal of Geotechnical and Geoenvironmental Engineering*, 133(4), 405–415. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2007\)133:4\(405\)](https://doi.org/10.1061/(ASCE)1090-0241(2007)133:4(405))
- Arefpanah, S., & Sharafi, A. (2024). Analytical and experimental study on shaking effects for improved stone column foundations. *Proceedings of the Institution of Civil Engineers – Ground Improvement*, 177(4), 255–272. <https://doi.org/10.1680/jgrim.22.00004>
- Ashour, S., Ghataora, G., & Jefferson, I. (2022). Behaviour of model stone column subjected to cyclic loading. *Transportation Geotechnics*, 35, Article 100777. <https://doi.org/10.1016/j.trgeo.2022.100777>
- Baez, J. I., & Martin, G. R. (1993). Advances in the design of vibro systems for improving liquefaction resistance. *Proceedings of the Symposium on Ground Improvement*, Vancouver, Canada, 111–118.
- Balaam, N. P., & Booker, J. R. (1981). Analysis of rigid rafts supported by granular piles. *International Journal for Numerical and Analytical Methods in Geomechanics*, 5(4), 379–403. <https://doi.org/10.1002/nag.1610050405>
- Barksdale, R. D., & Bachus, R. C. (1983). *Design and construction of stone columns* (FHWA/RD-83/026). Federal Highway Administration, Washington, DC. <https://apps.itd.idaho.gov/apps/manuals/Materials/Materials/20References/FHWA-RD-83-026.pdf>
- Black, J. A., Sivakumar, V., Madhav, M. R., & Hamill, G. A. (2007). Reinforced stone columns in weak deposits: Laboratory model study. *Journal of Geotechnical and Geoenvironmental Engineering*, 133(9), 1154–1161. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2007\)133:9\(1154\)](https://doi.org/10.1061/(ASCE)1090-0241(2007)133:9(1154))
- Broms, B. B., & Boman, P. (1979). Stabilization of soil with lime columns. *Ground Engineering*, 12(4), 539–555.
- Castro, J. (2017). Modelling stone columns. *Materials*, 10(7), Article 782. <https://doi.org/10.3390/ma10070782>
- Castro, J., & Sagaseta, C. (2011). Consolidation around stone columns: Influence of column deformation. *International Journal for Numerical and Analytical Methods in Geomechanics*, 35(16), 1733–1752. <https://doi.org/10.1002/nag.745>
- Cengiz, C., & Guler, E. (2018). Seismic behaviour of geosynthetic encased columns and ordinary stone columns. *Geotextiles and Geomembranes*, 46(1), 40–51. <https://doi.org/10.1016/j.geotextmem.2017.10.001>

- Choobbasti, A. J., Zahmatkesh, A., & Noorzad, R. (2011). Performance of stone columns in soft clay: Numerical evaluation. *Geotechnical and Geological Engineering*, 29, 675–684. <https://doi.org/10.1007/s10706-011-9409-x>
- Das, A. K., & Deb, K. (2019). Response of stone column-improved ground under $c-\phi$ soil embankment. *Soils and Foundations*, 59(3), 617–632. <https://doi.org/10.1016/j.sandf.2019.01.003>
- Ehsaniyamchi, A., & Ghazavi, M. (2019). Short-term and long-term behaviour of geosynthetic-reinforced stone columns. *Soils and Foundations*, 59(5), 1579–1590. <https://doi.org/10.1016/j.sandf.2019.07.007>
- Elias, V., & Welsh, J. P. (2002). *Ground improvement technical summaries* (Report No. FHWA-SA-98-086). Federal Highway Administration.
- Elkasabgy, M. A. (2005). *Performance of stone columns reinforced grounds*. M.Sc. Thesis, Zagazig University, Faculty of Engineering at Shobra, Cairo.
- Elshazly, H.A., Hafez, D., & Mosaad, M. (2006). Back calculating vibro-installation stresses in stone columns reinforced grounds. *Journal of Ground Improvement*, 10(2), 47–53. <https://doi.org/10.1680/grim.2006.10.2.47>
- Evans, J. C., Ruffing, D., & Elton, D. (2021). Introduction to ground improvement engineering. In *Fundamentals of Ground Improvement Engineering* (1st ed., pp. 1–20), CRC Press. <https://doi.org/10.1201/9780367816995-1>
- GEOFOS, s.r.o. (2016). *Measurement of subsoil settlement under embankments and measurement of pore pressures*. Rýchlostná cesta R2 Ruskovce – Pravotice, GTM. Národná diaľničná spoločnosť, a.s.
- Geramian, A., Castro, J., Ghazavi, M., & Miranda, M. (2022). Installation of groups of stone columns in clay: 3D Coupled Eulerian Lagrangian analyses. *Computers and Geotechnics*, 151, Article 104931. <https://doi.org/10.1016/j.compgeo.2022.104931>
- Ghanizadeh, A. R., Ghanizadeh, A., Asteris, P. G., Fakharian, P., & Armaghani, D. J. (2022). Developing bearing capacity model for geogrid-reinforced stone columns improved soft clay utilizing MARS-EBS hybrid method. *Transportation Geotechnics*, 34, Article 100906. <https://doi.org/10.1016/j.trgeo.2022.100906>
- Ghazavi, M., Yamchi, A. E., & Afshar, J. N. (2018). Bearing capacity of horizontally layered geosynthetic reinforced stone columns. *Geotextiles and Geomembranes*, 46(3), 312–318. <https://doi.org/10.1016/j.geotexmem.2018.01.002>
- Ghorbani, A., Hosseinpour, I., & Shormage, M. (2021). Deformation and stability analysis of embankment over stone column-strengthened soft ground. *KSCCE Journal of Civil Engineering*, 25(2), 404–416. <https://doi.org/10.1007/s12205-020-0349-y>
- Goughnour, R. R., & Bayuk, A. A. (1979). Analysis of stone column-soil matrix interaction under vertical load. *Proceedings of the International Conference on Soil Reinforcement*, 2, 663–679.
- Greenwood, D. A. (1970). Mechanical improvement of soils below ground surface. *Proceedings of the Ground Engineering Conference*, Institution of Civil Engineers, 11–22.
- Han, J., & Ye, S. L. (2001). Recent advances in ground improvement techniques. *Proceedings of the International Conference on Soil Mechanics and Geotechnical Engineering*, 4, 429–432.
- Han, J., & Ye, S.-L. (2001). Simplified method for consolidation rate of stone column reinforced foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(7), 597–603. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2001\)127:7\(597\)](https://doi.org/10.1061/(ASCE)1090-0241(2001)127:7(597))

- Han, J., & Ye, S.-L. (2002). A theoretical solution for consolidation rates of stone column-reinforced foundations accounting for smear and well resistance effects. *International Journal of Geomechanics*, 2(2), 135–151.
[https://doi.org/10.1061/\(ASCE\)1532-3641\(2002\)2:2\(135\)](https://doi.org/10.1061/(ASCE)1532-3641(2002)2:2(135))
- Hausmann, M. R. (1990). *Engineering principles of ground modification*. McGraw-Hill.
- Hughes, J. M. O., & Withers, N. J. (1974). Reinforcing soft soils with stone columns. *Ground Engineering*, 7(3), 42–49.
- Indraratna, B., Ngo, N. T., Rujikiatkamjorn, C., & Sloan, S. W. (2015). Coupled discrete element-finite difference method for analysing the load-deformation behaviour of a single stone column in soft soil. *Computers and Geotechnics*, 63, 267–278.
<https://doi.org/10.1016/j.compgeo.2014.10.002>
- Kelesoglu, M. K., & Durmus, C. (2022). Numerical plane-strain modelling of stone columns: Installation process, single and group column behaviour. *KSCE Journal of Civil Engineering*, 26(8), 3402–3415. <https://doi.org/10.1007/s12205-022-1671-3>
- Kirsch, F., & Bell, A. (2012). *Deep vibratory compaction of granular soils*. CRC Press.
- Krishna, H., Raju, V. R., & Wegner, R. (2004). Ground improvement using vibro replacement in Asia 1994 to 2004: A 10-year review. *Proceedings of the 5th International Conference on Ground Improvement Techniques*, Kuala Lumpur, Malaysia.
- Lu, X., & Li, C. (2023). An analytical solution for nonlinear consolidation of composite foundations improved by stone columns and vertical drains. *Computers and Geotechnics*, 162, Article 105598. <https://doi.org/10.1016/j.compgeo.2023.105598>
- Mert, A. C., & Onalp, A. (2020). Analysis of a prefabricated vertical drain (PVD) soil improvement project. *Teknik Dergi*, 33(1), 11521–11541.
<https://doi.org/10.18400/tekderg.641218>
- Miranda, M., Fernández-Ruiz, J., & Castro, J. (2021). Critical length of encased stone columns. *Geotextiles and Geomembranes*, 49(5), 1312–1323.
<https://doi.org/10.1016/j.geotextmem.2021.05.003>
- Mitchell, J. K., & Huber, T. R. (1985). Performance of a stone column foundation. *Journal of Geotechnical Engineering*, 111(2), 205–223.
[https://doi.org/10.1061/\(ASCE\)0733-9410\(1985\)111:2\(205\)](https://doi.org/10.1061/(ASCE)0733-9410(1985)111:2(205))
- Mitchell, J. K., & Soga, K. (2005). *Fundamentals of soil behaviour* (3rd ed.). John Wiley & Sons.
- Moseley, M. P., & Kirsch, K. (Eds.). (2004). *Ground improvement* (2nd ed.). CRC Press.
<https://doi.org/10.1201/9780203489611>
- Nav, M. A., Rahnavard, R., Noorzad, A., & Napolitano, R. (2020). Numerical evaluation of the behaviour of ordinary and reinforced stone columns. *Structures*, 25, 481–490.
<https://doi.org/10.1016/j.istruc.2020.03.021>
- Pandey, B. K., Rajesh, S., & Chandra, S. (2021). Performance enhancement of encased stone column with conductive natural geotextile under k_0 stress condition. *Geotextiles and Geomembranes*, 49(5), 1095–1106. <https://doi.org/10.1016/j.geotextmem.2021.03.004>
- Pandey, B. K., Rajesh, S., & Chandra, S. (2022). Time-dependent behaviour of embankment resting on soft clay reinforced with encased stone columns. *Transportation Geotechnics*, 36, Article 100809. <https://doi.org/10.1016/j.trgeo.2022.100809>
- Pitt, J. M., White, D. J., Gaul, A., & Hoevelkamp, K. (2003). *Highway applications for rammed aggregate piers in Iowa soils* (Iowa DOT Project TR-443, CTRE Project 00-60), USA.

- Priebe, H. J. (1995). The design of vibro replacement. *Ground Engineering*, 28(10), 31–37. https://cdn.ca.emap.com/wp-content/uploads/sites/13/1995/12/1995-12_Pages_31-37.pdf
- Priebe, H. J. (1976). Abschätzung des Setzungsverhaltens eines durch Stopfverdichtung verbesserten Baugrundes. *Die Bautechnik*, 54, 160–162. (in German)
- Raithel, M., & Kempfert, H.-G. (2000). Calculation models for dam foundations with geotextile coated sand columns. *Proceedings of the International Conference on Geotechnical & Geological Engineering GeoEng 2000*, 785–792. https://www.kup-geotechnik.com/media/2000_raithel_geoengmelbourne.pdf
- Rajesh, S., & Ghosh, P. (2021). Numerical analysis of geosynthetic-encased stone columns in soft soils. *Geotextiles and Geomembranes*, 49(2), 256–269.
- Raju, V. R. (2002). Vibro replacement for high earth embankments and bridge abutment slopes in Putrajaya, Malaysia. *International Conference on Ground Improvement Techniques, Malaysia*, 607–614.
- Riccio, M. V., Almeida, M. S. S., Vasconcelos, S. M., Pires, L. G. S., & Nicodemos, L. F. (2022). Embankment supported by low area replacement ratio stone columns, monitoring and numerical studies. *KSCFE Journal of Civil Engineering*, 26(2), 619–629. <https://doi.org/10.1007/s12205-021-0540-9>
- Satyanarayana, P. V. V., & Suvvari, S. (2016). Improvement of soft soil performance using stone columns improved with circular geogrid discs. *Indian Journal of Science and Technology*, 9(30), 1–6. <https://doi.org/10.17485/ijst/2016/v9i30/99186>
- Singh, I., & Sahu, A. K. (2019). A review on stone columns used for ground improvement of soft soil. *Proceedings of the 4th World Congress on Civil, Structural, and Environmental Engineering*, Rome, Italy. <https://doi.org/10.11159/icgre19.132>
- Smith, M. R., & Collis, L. (Eds.). (2001). *Aggregates: Sand, gravel, and crushed rock aggregates for construction purposes* (3rd ed.). Geological Society of London.
- Tan, S. A., & Oo, K. K. (2005). Plane strain modelling of stone column foundations. *Proceedings of the International Conference on Soil Mechanics and Geotechnical Engineering*, 1, 1425–1428.
- Tan, S. A., Tjahyono, S., & Oo, K. K. (2008). Simplified plane-strain modelling of stone-column reinforced ground. *Journal of Geotechnical and Geoenvironmental Engineering*, 134(2), 185–194. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2008\)134:2\(185\)](https://doi.org/10.1061/(ASCE)1090-0241(2008)134:2(185))
- Tandel, Y. K., Solanki, C. H., & Desai, A. K. (2020). Numerical modelling of stone columns in expansive soils. *Geotechnical and Geological Engineering*, 38(3), 2973–2987.
- Togati, N. V. V. K., Rao, K. M., & Noolu, V. (2023). Behaviour of stone columns based on 2⁴ factorial experiments with uniformly graded materials. *Materials Today: Proceedings*. <https://doi.org/10.1016/j.matpr.2023.03.183>
- Touma, J. F., Sadek, S., & Najjar, S. S. (2017). Rapid load testing of stone columns. *Geotechnical Frontiers 2017 Conference Proceedings*. <https://doi.org/10.1061/9780784480441.020>
- Tran, D. Q. (2001). Numerical modelling of an embankment on soft subsoil: A case study. *Computers and Geotechnics*, 28(6–7), 419–435. <https://doi.org/10.1088/1757-899x/136/1/012021>
- Van Impe, W. F. (1989). *Soil improvement techniques and their evolution*. CRC Press.
- Vesic, A. B. (1972). Expansion of cavities in infinite soil mass. *Journal of Soil Mechanics & Foundations Div*, 98(3), 265–290. <https://doi.org/10.1061/JSFEAQ.0001740>

- Watts, K. S., & Johnson, D. (2004). Vibro stone columns in ground improvement. In M. P. Moseley & K. Kirsch (Eds.), *Ground improvement* (2nd ed., pp. 97–133). CRC Press.
- Xu, F., Moayedi, H., Foong, L. K., Moghaddam, M. J., & Zangeneh, M. (2021). Laboratory and numerical analysis of geogrid-encased stone columns. *Measurement*, *169*, Article 108369. <https://doi.org/10.1016/j.measurement.2020.108369>
- Zhang, L., Peng, B., Xu, Z., & Zhou, S. (2022). Shear performance of geosynthetic-encased stone column based on 3D-DEM simulation. *Computers and Geotechnics*, *151*, Article 104952. <https://doi.org/10.1016/j.compgeo.2022.104952>
- Zheng, G., Yu, X., Zhou, H., Wang, S., Zhao, J., He, X., & Yang, X. (2020). Stability analysis of stone column-supported and geosynthetic-reinforced embankments on soft ground. *Geotextiles and Geomembranes*, *48*(3), 349–356. <https://doi.org/10.1016/j.geotexmem.2019.12.006>
- Zhou, Y., Kong, G., Zheng, J., Wen, L., & Yang, Q. (2021). Analytical solutions for geosynthetic-encased stone column-supported embankments with emphasis on nonlinear behaviours of columns. *Geotextiles and Geomembranes*, *49*(5), 1107–1116. <https://doi.org/10.1016/j.geotexmem.2021.03.005>

NOTATIONS

Variables and Functions

- γ – unit weight of soils in natural condition
 γ_{sat} – saturated unit weight of soils
 Φ' – effective internal angle friction of the soils
 c' – effective cohesion of the soils
 E_{oed} – oedometric modulus of elasticity
 E_{50} – secant stiffness modulus
 E_{ur} – unloading/reloading stiffness modulus
 M – stress-dependent stiffness parameter
 k – permeability of the soil

Abbreviations

- FEM – Finite Element Method
km – kilometre
kN – kilonewton
kPa – kilopascal
MPa – Megapascal