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BUCKLING OF DOUBLE-T CONSTRUCTION ELEMENTS FOR BRIDGES IN CASE OF COMPLICATED LOADING

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Abstract. This paper analyzes the stability of double-T elements with complicated loading under bending moment and extra torsional moment. In case of simple bending, long elements and elements of small cross-section are under bending and torsion moments. Here, an extra torsional moment is also evaluated that can occur in constructions under external effects: unsymmetrical loads, wind and temperature. Classical solutions are analyzed in various papers and studies, and engineering solutions given in standards present no examples of such a complicated loading. Therefore, this paper suggests energy method to calculate critical bending forces and extra torsional moment values. Obtained analytical equations are tested by experiment and present to be acceptable.

Keywords: buckling, double-T element, complicated loading, bending, torsion.

1. Introduction

For bridge construction, various materials are used such as concrete, aluminium, composites; however, steel as one of the strongest materials is most popular. Support steel elements are main in most constructions. Support construction in girder bridges is bending element. Main support elements for girder bridges are designed from continuous rolled elements or complex welded elements.

Bending elements in bridge girders are replaced by axially loaded elements, connected one to another flexibly or rigidly. However, additional elements are necessary here – cross-beams to transfer uniformly distributed loads to girder joints in concentrated forces.

For a variety of bridge constructions, combined bridge constructions are to be mentioned; they contain both tensed, torsed and bended elements (Juozapaitis *et al.* 2008).

While analyzing system of bridges as constructional elements, the problem of separate element strength is encountered as well as the problem of separate element buckling. This problem is critical with longer elements.

Problems of buckling of compressed coloumn are discussed extensively (Duan, Wang 2008; Šapalas *et al.* 2005; Wang *et al.* 2004). However, problems of long bended element buckling are given rather less attention (Bradford 1990; Larue *et al* 2007; Plum, Svensson 1993; Samanta, Kumar 2006).

This article presents analysis of element stability for bridge constructions, as they suffer bending loads and external torsion.

2. Bending for long support double-T construction elements

During design and exploitation of bridge constructions, metal elements are suffering loads shown in Fig. 1.

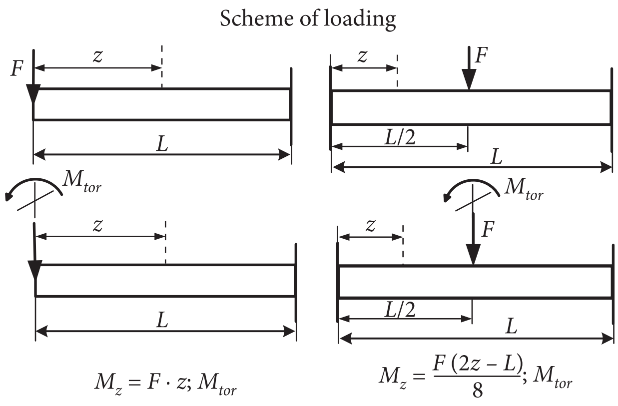

Fig. 1. Loading thin-walled constructional elements under complicated loading

While analyzing long support construction elements loaded by one cross-bending force on free end analytical values are obtained (Bradford 1990). Apart from bending, buckling is observed. However, adding extra momentum of torsion, i.e. acting on the beam with bending moment M_b and torsion moment M_{tor} analytical values cannot be obtained. For obtaining this value first of all support double-T beam loaded by concentrated force F must be analyzed (Fig. 2).

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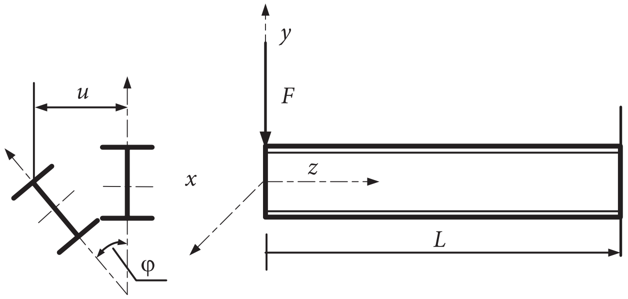

Fig. 2. Bending of support double-T beam

Conditions of differential balance are presented as follows

$$B_1 \frac{d^2u}{dz^2} = -\varphi Fz;$$

$$C\frac{du}{dz} - D\frac{h^2}{2}\frac{d^3u}{dz^3} = Fz\frac{du}{dz} - F(\delta - u), \quad (1)$$

where u – shift through turning; φ – angle of torsion; δ – vertical shift; D – shelf stiffness $\left(D \approx \frac{1}{2}B_1\right)$; h – profile height; L – element length; $B_1 = EI_y$ (E – modulus of elasticity, I_y – min moment of inertia); $C = GI_{tor}$ (G – shear modulus, I_{tor} – moment of inertia in torsion).

The solution presented in study Serna *et al.* (2006) for calculation of critical buckling moment M_{cr} is obtained with moment in one plane and requires estimating the empirical coefficients. Serna *et al.* (2006) presents the following formula for calculation of critical buckling moment:

$$M_{cr} = C_1 \frac{\pi^2 EI_y}{(kL)^2}\sqrt{\left(\frac{k}{k_x}\right)^2 \frac{I_x}{I_y} + \frac{(kL)^2 GI_{tor}}{\pi^2 EI_y}}, \quad (2)$$

where M_{cr} – critical bending moment; C_1 – coefficient of lateral bending conditions at end supports; k – bending coefficient; k_x – warping coefficient.

Manual of Steel Construction: Load and Resistance Factor Design of American Institute of Steel Construction Inc (AISC) also requires many empirical coefficients for estimation of M_{cr} critical moment. Also the formula presented is applied in loading profiles with one load.

AISC presents the lateral–torsional buckling by the following Eq:

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_y GI_s + \left(\frac{\pi E}{L_b}\right)^2 I_y I_x}, \quad (3)$$

where L_b – lateral buckling length; C_b – moment gradient coefficient, obtained from the Eq

$$C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2, \quad (4)$$

where M_1 and M_2 – absolute bending moment values at the ends of the beam. If $M_1 = -M_2$, then $C_b = 1$. The beam suffering several moments in one plane C_b is calculated as:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}, \quad (5)$$

where M_{max} – absolute max moment; M_A – absolute moment in 1st quarter of the beam; M_B – absolute moment in middle part of the beam; M_c – absolute moment in 3rd quarter of the beam.

The British code for steelworks in buildings *BS 5950-1:2000 Structural Use of Steelwork in Buildings. Code of Practice for Design. Rolled and Welded Sections* incorporates a formulation very similar to that of *AISC*.

Lateral–torsional buckling of beams is presented in *EN 1993-1-1 Eurocode 3: Design of Steel Structures. Part 1–1: General Rules and Rules for Buildings.* M_{cr} is based on gross cross-sectional properties and takes into account the loading conditions, the real moment distribution and restraints:

$$M_{cr} = \frac{kc}{L}\sqrt{EI_y\left(GI_{tor} + \frac{\pi^2}{L^2}EI_w\right)} = \frac{kc\pi EI_y}{L^2}\sqrt{\frac{I_w}{I_y} + \frac{L^2 GI_{tor}}{\pi^2 EI_y}}, \quad (6)$$

where k and c – coefficients dependent on load type, place of loading and restraint conditions; I_w – constant of torsion.

For solving Eq (1) the energetic method is applied (Reddy 2002).

Therefore, estimating shelf curves for turning beam equal to $u = 0.5\varphi h$, side energy can be put as:

$$(B_1 - 2D)\int_0^L \left(\frac{d^2u}{dz^2}\right)^2 dz + D\int_0^L \left(\frac{d^2u}{dz^2} + \frac{h}{2}\frac{d^2\varphi}{dz^2}\right)^2 dz + D\int_0^L \left(\frac{d^2u}{dz^2} - \frac{h}{2}\frac{d^2\varphi}{dz^2}\right)^2 dz = B_1\int_0^L \left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{Dh^2}{2}\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz. \quad (7)$$

Torsion energy is equal to

$$C\int_0^L \left(\frac{d\varphi}{dz}\right)^2 dz. \quad (8)$$

Action in buckling equals to

$$F\int_0^L \varphi \frac{d^2u}{dz^2} z dz. \quad (9)$$

With energy equal to the action fulfilled the Eq can be put as

$$F\int_0^L \varphi \frac{d^2u}{dz^2} z dz = B_1\int_0^L \left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{Dh^2}{2}\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz + C\int_0^L \left(\frac{d\varphi}{dz}\right)^2 dz. \quad (10)$$

After eliminating $\frac{d^2u}{dz^2}$ from Eq (1) is obtained

$$\frac{F^2}{B_1}\int_0^L \varphi^2 z^2 dz = C\int_0^L \left(\frac{d\varphi}{dz}\right)^2 dz + \frac{Dh^2}{2}\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz. \quad (11)$$

Function φ put as

$$\varphi = aL^2 - az^2, \quad (12)$$

where a – constant.

After integration is obtained the following:

$$\int_0^L \left(\frac{du}{dz}\right)^2 dz = \frac{4}{3}a^2 L^3; \quad (13)$$

$$\int_0^L \varphi^2 z^2 dz = \frac{8}{105}a^2 L^7; \quad (14)$$

$$\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz = 4a^2 L; \quad (15)$$

$$\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz = \frac{4}{3}a^2 L^3. \quad (16)$$

Therefore,

$$\frac{F^2}{B_1}\frac{8}{105}a^2 L^7 = C\frac{4}{3}a^2 L^3 + \frac{Dh^2}{2}4a^2 L; \quad (17)$$

thus

$$F_{cr}^2 = 17.5 B_1 \frac{CL^2 + 1.5Dh^2}{L^6} \quad (18)$$

and

$$F_{cr} = 4.18\sqrt{\frac{B_1\left(CL^2 + 1.5Dh^2\right)}{L^6}}. \quad (19)$$

As given in classical and standard solutions, stability cases for bridge constructions are analyzed under simple loads. Studies of complicated loading are not yet made. Therefore, in cases with main bending moment loads that give torsion, extra torsional moment is also active, and it is an important problem in practice. For obtaining such values, applications of energy method must be analyzed.

3. Energetic method for critical buckling loads in case of bending and torsion

With additional torsion load and bending energy increases in value

$$\frac{1}{2}\int_0^L \frac{M_{1,tor}^2}{C} dz, \quad (20)$$

where $M_{1,\,tor}$ – additional moment of torsion.

Action fulfilled increases in value

$$\int_0^L \frac{M_{1,tor}^2}{C} dz. \quad (21)$$

Comparing action to energy is obtained the following:

$$\frac{F^2}{B_1}\int_0^L \varphi^2 z^2 dz + \frac{1}{C}\int_0^L M_{1,tor}^2 dz = C\int_0^L \left(\frac{d\varphi}{dz}\right)^2 dz + \frac{Dh^2}{2}\int_0^L \left(\frac{d^2\varphi}{dz^2}\right)^2 dz + \frac{1}{2C}\int_0^L M_{1,tor}^2 dz; \quad (22)$$

with

$$\varphi = aL^2 - az^2, \quad (23)$$

where a – constant.

After integration the following equation is obtained:

$$\frac{F^2}{B_1}\frac{8}{105}a^2 L^7 - \frac{4}{3}a^2 L^3 C - \frac{Dh^2}{2}4a^2 L + \frac{1}{2C}M_{1,tor}^2 L = 0. \quad (24)$$

For estimating parameter a, Eq of action fulfilled can be used

$$F_{cr}\delta_{cr} = \frac{F_{cr}^2}{B_1}\int_0^L \varphi^2 z^2 dz. \quad (25)$$

Therefore,

$$\delta_{cr,t} = \frac{8}{105 B_1} F_{cr} a^2 L^7 \quad (26)$$

and

$$a = \sqrt{\frac{105 B_1 \delta_{cr,t}}{8 F_{cr} L^7}}. \quad (27)$$

Eq (24) presents the case of complicated loading as the beam is bended and turned with additional moment of torsion.

4. Experiment

For corroboration of theoretical solutions tests with double T profile No. 10 beam were performed in Fig. 3. Test results are given in Table. With the middle of parameter a value in Eq (24), having constant values of beam cross-section and length values F_{cr} and $M_{tor,cr}$ can be predicted. Diagram of such dependences is given in Fig. 4.

5. Conclusions

1. Analysis of buckling values in thin-walled constructional elements shows that in case of both bending and buckling loads no analytical answers can be obtained.
2. In case of complicated buckling loads, the energetic method is suggested that allows solving problems with sufficient engineer error of 5% accuracy comparing with accurate analytical methods.
3. After calculating constants for buckling Eq in experimental critical bending forces and moments of torsion, while buckling can be predicted.

Test results

Cross-section	Length, m	Moment of torsion M_{tor}, Nm	Middle of experimental critical force F_{cr}, N	Variation ratio of experimental critical force, %	Middle of shift δ_{cr}, m	Variation ratio of shift, %	Middle of parameter a	Variation ratio of parameter a, %
I No. 10	4	10	1910	6.776	0.036	9.235	0.023	1.337
	4	30	1798	7.256	0.034	11.723	0.023	2.577
	4	50	1658	7.586	0.031	5.100	0.023	4.741
	3.5	10	2422	6.196	0.018	7.207	0.023	4.169
	3.5	30	2314	6.966	0.017	6.869	0.023	2.004
	3.5	50	2170	7.027	0.016	9.388	0.023	2.848
	3	10	3370	6.267	0.008	10.203	0.023	4.500
	3	30	3196	7.780	0.008	8.839	0.023	3.709
	3	50	3004	7.052	0.007	15.408	0.023	4.769

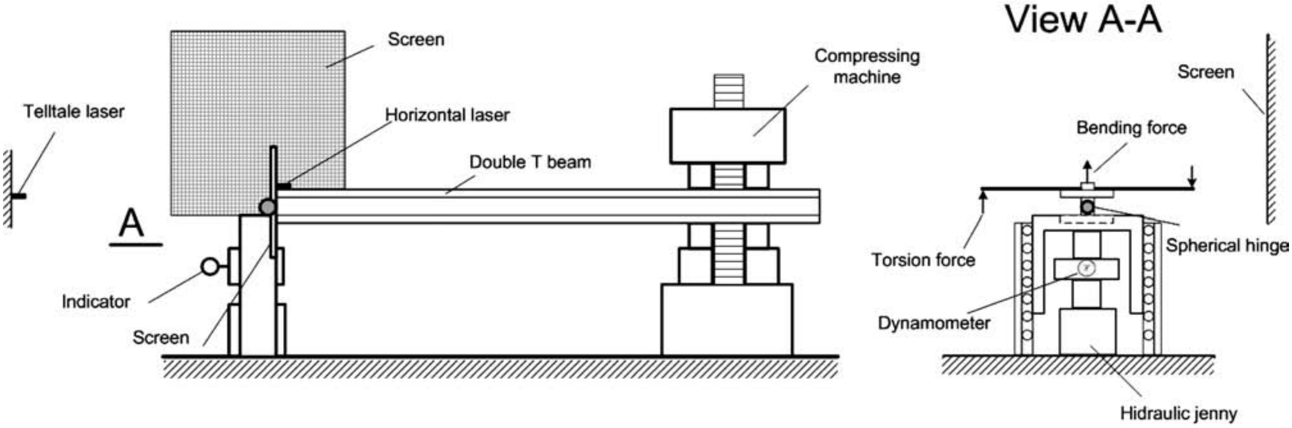

Fig. 3. Test diagram for supported beam

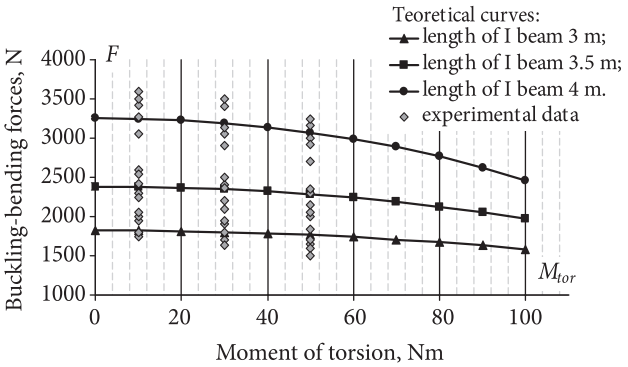

Fig. 4. Dependence of critical buckling-bending forces and moment of torsion

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