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AN ITERATIVE CALCULATION METHOD FOR SUSPENSION BRIDGE'S CABLE SYSTEM BASED ON EXACT CATENARY THEORY

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Abstract. In this paper, a flexible iterative method capable of considering the effects of slip between the main cable and saddles is presented for the analysis of the cable system in the suspension bridge. In the proposed procedure, nonlinear governing equations were first linearized based on the first-order Taylor expansion, then the tangent stiffness matrix was derived using appropriate numerical methods. Using the proposed flexible iterative procedure which is built upon the framework of Newton-Raphson method, the main cable's unstrained length and equilibrium forces which satisfy the configuration and mechanical property under bridge's completion state is obtained according to the main cable's initial geometry parameters, saddles parameters and hangers arrangement. Based on form-finding analysis, the method is also proposed to calculate the main cable's internal forces and displacements during the erection of stiffening girder; the reliability and efficiency of the method is demonstrated by two typical numerical examples. Furthermore, the proposed method is used as a pro-processing tool in the finite element analyses of a cable structure. Finally, a numerical example (Yingwuzhou Yangtze River Bridge) is reported to illustrate the advantages of the proposed method, including the accurate predictions of the main cable's unstrained length and the excursion of the saddles, which is crucial for choosing appropriate saddles parameters.

Keywords: elastic catenary element, Newton-Raphson method, form-finding, sliding element, suspension bridge, construction stage analysis, pre-processing.

1. Introduction

Suspension bridge has become one of the main choices for the construction of long span bridges mainly due to its aesthetic appearance, efficient utilization of structural materials and other well-known advantages. With the advance of structural analysis and construction technology, a suspension bridge with a main span length reaching almost 2000 m is now possible (Gimsing 1997; Kim *et al.* 2001). In recent years, as suspension bridges become larger and longer, the main cable's safety factor has decreased from initial 3.0~4.0 to about 2.5, and this means that more precise analysis techniques are required to predict the behaviours of the suspension bridges (Fan *et al.* 1999). Cable system, being composed of main cable, hangers and saddles, is a main load-bearing part in suspension bridge, and an important part to be designed for engineers. As a result, accurate calculation/prediction of the construction parameters regarding the cable system, such as, the unstrained lengths of both the main cable and the hangers, the configuration of main cable during construction, the fixed position for erecting hangers and the excursion of saddles, is necessary conditions to ensure the finished main cable configuration which meets the design requirements, and thus is the first step in the structural analysis/design of suspension bridge and the implementation of construction control.

To date, a number of methods have been developed for the calculation of cable system which is a flexible structure, including equivalent elastic modulus method, multi-node link element, multi-node curvy element, beam element and the exact catenary element method now being widely used. O'Brien was one of the pioneers contributing to the catenary theory. In 1964, he first derived a two-dimensional numerical solution of the catenary element, subsequently, generalized it to solve the spatial suspension cable under uniformly distributed load (UDL) and concentrated load. The segmental catenary method and the iterative flexibility procedure were proposed originally (O'Brien *et al.* 1964, 1967). With the development of Finite Element Method (FEM), the analytical catenary theory appeared to be combined with FEM. The cable element tangent stiffness matrix derived from catenary theory has been used to analyse the static and dynamic problems of the cable structures (Irvine

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1981; Jayaraman, Knudson 1981; Thai, Kim 2011). The catenary element has been used in lots of engineering applications to model the cable structures, such as form-finding for suspension bridges, the sliding effect between cable and saddle and the Yongjiong Grand Bridge (Korea) – a self-anchored bridge with spatial cable-shape (Kiisa 2012; Kim, Lee 2001; Kim *et al.* 2002; Luo 2004).

Another key issue in analyzing the cable system is the sliding effect between the main cable and saddles which was always neglected. Under bridge finished state, the sliding effect is very small by introducing the main cable theory intersection, but under the construction state and operation state, it cannot be ignored (Luo *et al.* 2005). The similar research about the sliding effect between the main cable and saddles originally began in 1988. McDonald proposed a pulley element simplified computations by automatically adjusting the proportion of cable on each side of a sheave in order to maintain equilibrium (McDonald, Peyrot 1988). Zhou *et al.* (2004) and Chung *et al.* (2011) also have done the similar work which was rarely used to analyse the suspension bridge. In recent three decades, a large number of long-span suspension bridges were built. Chinese researchers had done a lot of work on the sliding element. Luo (2005) developed a two-node saddle-cable element employed to model the saddle and its jacking conveniently. Wei and Liu (2006) derived a three-node saddle element and the longitudinal curve of saddle consists of two arcs with different radius. Qi *et al.* (2011) established a three-node combination element, including anchorage, splay saddle and anchor span strands, which improves the construction control accuracy of anchor span strands in computation of suspension bridges by FEM (Luo *et al.* 2005; Qi *et al.* 2011; Wei, Liu 2006).

In summary, the analyzing methods of cable system are attributed to analytical method based on catenary theory and FEM. The former has the advantages of simple pro-processing, high computational efficiency and accuracy, but it was used to only analyse the cable members and need FEM to determine the initial conditions. The latter has the advantages of good generality but is simultaneously accompanied with computation complexity and low accuracy. The purpose of this paper is combining the advantages of the two methods to solve the cable structure problems which also consider the sliding effect.

2. Form-finding procedure considering sliding effect

To obtain more accurate results, in this paper, the catenary cable element was adopted to model the suspension bridges' main cable system, the sliding effect was considered in the form-finding procedure based on the assumption that the main cable on the saddles' aspect is tangent to the saddles' surface, and then, the calculation procedure for form-finding analysis under bridge finished state and tangent stiffness matrix for the flexibility iterative procedure were given and derived, respectively, the form-finding analysis for other construction stages was also discussed. Finally, the flow chart describing the iterative procedure was given.

2.1. Catenary cable element

To accurately simulate the realistic behaviour of cable members, a catenary cable element based on the exact analytical expressions of an elastic catenary is employed to model the main cables as well as the hanger cables. It is assumed that the cable is perfectly flexible, with the self-weight distributed along its length. Consider an elastic catenary cable as shown in Fig. 1. The projected lengths of the cable can be derived as follows (Thai, Kim 2011).

$$l_x = -\frac{F_1 L_0}{EA} - \frac{F_1}{w}\left\{ \ln\left[\sqrt{F_1^2 + F_2^2 + \left(wL_0 - F_3\right)^2} + wL_0 - F_3 \right] - \ln\left(\sqrt{F_1^2 + F_2^2 + F_3^2} - F_3 \right) \right\}, \tag{1a}$$

$$l_y = -\frac{F_2 L_0}{EA} - \frac{F_2}{w}\left\{ \ln\left[\sqrt{F_1^2 + F_2^2 + \left(wL_0 - F_3\right)^2} + wL_0 - F_3 \right] - \ln\left(\sqrt{F_1^2 + F_2^2 + F_3^2} - F_3 \right) \right\}, \tag{1b}$$

$$l_z = -\frac{F_3 L_0}{EA} + \frac{wL_0^2}{2EA} + \frac{1}{w}\left\{ \sqrt{F_1^2 + F_2^2 + \left(wL_0 - F_3\right)^2} - \sqrt{F_1^2 + F_2^2 + F_3^2} \right\}, \tag{1c}$$

$$F_4 = -F_1;\ F_5 = -F_2;\ F_6 = -F_3 + wL_0, \tag{2}$$

where L_0 – the unstressed length of the cable; E and A – the elastic modulus and cross-sectional area of the cable; w – the self-weight per unit length of the cable, F_1~F_6 denote the cable element's node forces in different direction as shown in Fig.1.

Eq (1) is the exact catenary element's mathematical expression, it includes the nonlinear effect caused by the self-weight distributed along its length. The iterative flexibility procedure mentioned above proposed by O'Brien (1964, 1967) was based on the Eqs (1) and (2) and treated L_0, F_1, F_2, F_3 as known constants. To the shape–finding for suspension bridges, the main cable's unstrained length is unknown variable, but hangers' longitudinal arrangement lx and main cable's sag at mid-span are the cable system's critical control parameters which are treated as known constants. Thus, the first step of the proposed flexibility iterative procedure in this paper is adopting the soft package Matlab to solve Eq (1a) to obtain the unstrained length L_0,

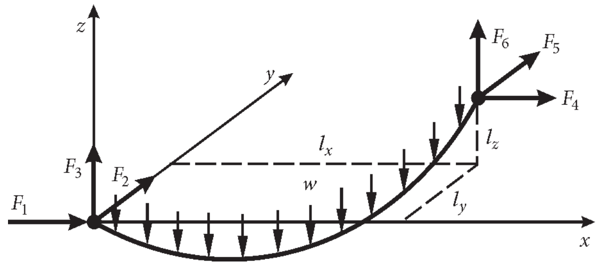

Fig. 1. Three-dimensional catenary cable

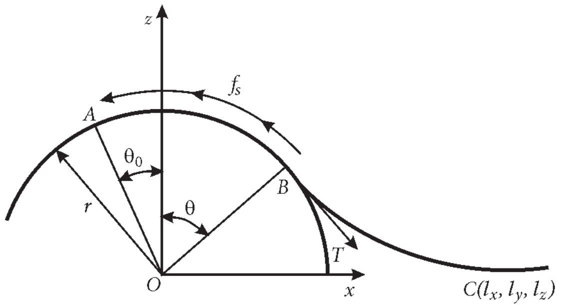

Fig. 2. Sliding effect between main cable and saddle

and the other steps follow the method derived by O'Brien (1964, 1967).

2.2. Sliding effect between main cable and saddle

Saddle is a unique component of a cable-supported bridge, composed of tower saddles and splay saddles, aimed to lead the main cable continuously through the pylons in curved tubes, to transfer the main cable tensile force. Main cable is a flexible structure, the tangent point between main cable and saddles will move along the surface of the saddle under different load conditions. The main cable is always fixed with the saddles at some locations to limit main cable's longitudinal movement under non-symmetrical load conditions to ensure the bridge keep in steady-state. In order to analyze the sliding effect between main cable and saddle, take the plane-shaped cable for example as shown in Fig. 2. The saddle's radius is r, O is the circle centre, the main cable and saddle fixed at A, B is the tangent point. θ_0 and θ are the angles as shown in Fig. 2. The cable's ABC unstrained length is l_s. The unit defined in this paper as: counter clockwise is positive and tension is positive, the others are the same as the coordinate system.

The tensile force at point B is T. OB's tangential direction is the same as T, neglecting the friction force between the main cable and saddle. Then:

$$l_{ab} = \frac{r\left(\theta+\theta_0\right)}{\dfrac{T}{EA}+1} = \frac{EAr\left(\theta+\theta_0\right)}{T+EA}, \tag{3a}$$

$$l_{bc}=l_s-l_{ab}, \tag{3b}$$

where l_{ab}, l_{bc} – cable AB and cable BC's unstrained length; E and A – the elastic modulus and cross-sectional area of the cable, respectively.

The same as single cable element, in the analysis of the sliding effect, if the cable's BC equilibrium state is known under self-weight load case, then:

$$\theta = -\arctan\left(\frac{F_x}{F_z}\right) \quad \mathbf{T}=\left[F_x, F_z\right]^T \quad T=\sqrt{F_x^2+F_z^2}, \tag{4}$$

$$x_a = x_b - r\left(\sin\theta + \sin\theta_0\right), \tag{5a}$$

$$z_a = z_b - r\left(\cos\theta - \cos\theta_0\right). \tag{5b}$$

According to Eqs (3)–(5), the sliding effect analysis is determined by Eqs (1)–(2).

2.3. Form-finding analysis under bridge finished state

In order to satisfy the bridge's function requirements, the designers always choose a proper main cable's rise-span ratio $\frac{f}{l}$ (f – the sag at mid-span, l – main span), hangers' arrangement, bridge deck's longitudinal line-shape to control the structure's internal forces and geometry information under bridge finished state. According to the above parameters, main cable's unstrained length, geometrical shape and hangers' unstrained lengths are known. As shown in Fig. 4, the following initial conditions are obtained:

1) saddles' information: saddles' radius r_1, r_2; saddles' vertices (x_1, y_1, z_1), (x_n, y_n, z_n); the angle between the fixed points and saddles' vertices are θ_0^L, θ_0^R;

2) bridge deck's line-shape: hangers' axial force F_i^h, not included self-weight, acquired from the bridge deck's line-shape, where, i = 2~n–1, n is the total number of the nodes;

3) the hangers' anchoring points on the main cable and stiffening girder are x_i^c, x_i^b, z_i^b, where, i = 2~n–1, c and b represent the main cable and stiffening girder;

4) Z-coordinate of the sag at mid-span is z_{sag};

5) the hangers are vertical.

2.3.1. Left saddle

As shown in Fig. 3a, $X_1=x_2^c-x_1$, according to section 2.2, import two variables $\mathbf{V} = [v_1, v_2]^T$, the saddle's equilibrium state will be acquired.

The variables may be assumed in different styles, such as $\mathbf{V}=\left[\theta_0^L, T\right]^T$ or $\mathbf{V}=\left[T_x, T_z\right]^T$. Different styles will lead to different convergence property. T_x and T_z have the similar magnitude, and not sensitive to rounding errors in the iterative procedure, therefore, this paper proposes $\mathbf{V}=\left[T_x, T_z\right]^T$ as the variables. It is worth noting that an improper variables choice, such as $\mathbf{V}=\left[\theta_0^L, T\right]^T$, will obtain a singular tangent stiffness matrix leading to a failure of iterative procedure.

If a set of proper initial values have been assigned to $\mathbf{V}$, to the cable B_L^2, the tensile force at B_L is $\mathbf{T}_{BL}=\left[-T_x, -T_z\right]^T$, and then the cable's unstrained length s_1 of the left saddle element, the tensile components F_4^1, F_6^1 at node 2 and z_2^c will be obtained. The initial values of $\mathbf{V}$ are calculated by assuming that the main cable-shape is parabolic curve under dead load.

2.3.2. Intermediate cable

To the element 2–3, as shown in Figs 3c, 3d:

$X_2 = x_3^c - x_2^c$, the No. 1 hanger's length is $l_1^h = z_2^c - x_2^b$, and its unstrained length, l_0^l, is calculated by the following equation:

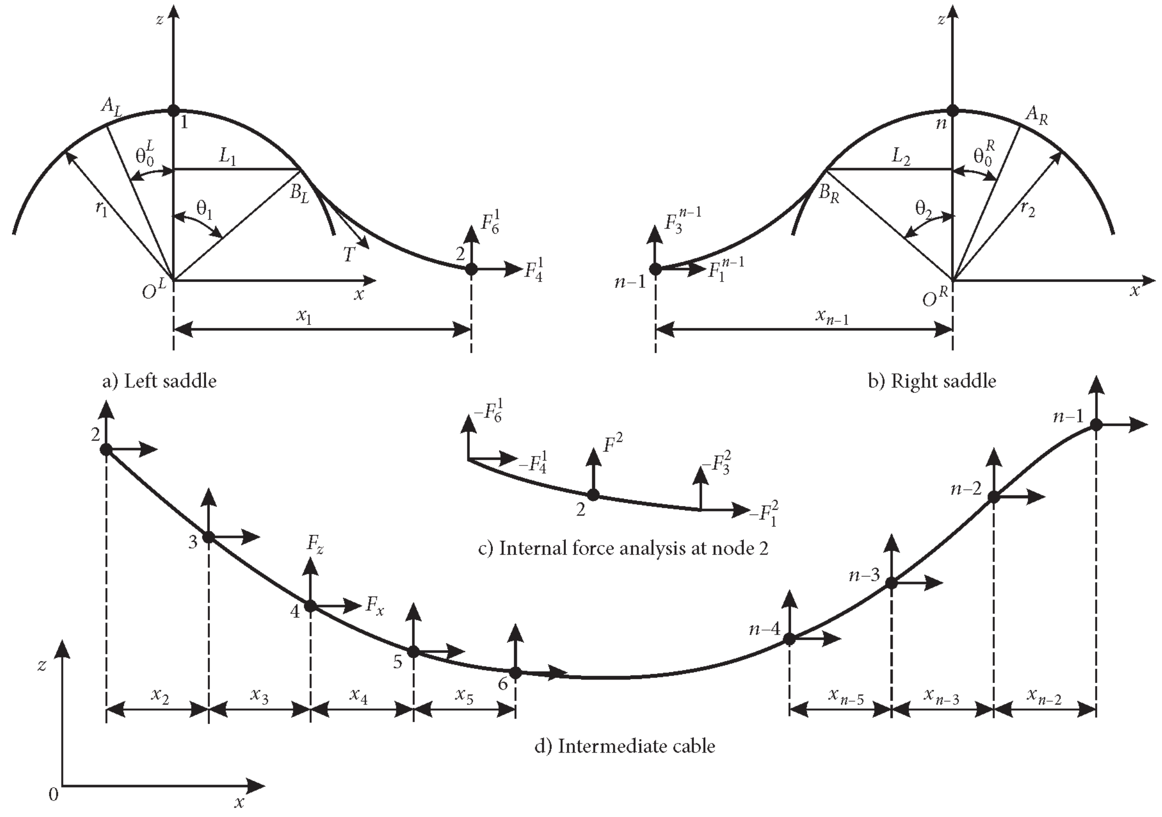

Fig. 3. The analysis of the main cable

$$l_i^0 = \frac{-\left(F_i^h + E_h A_h\right) + \sqrt{\left(F_i^h + E_h A_h\right)^2 + 2 w_h E_h A_h l_i^h}}{w_h}, \quad (6a)$$

$$F^i = F_i^h - w_h l_i^0, \quad (6b)$$

where w_h – represents hanger's weight per unit length; E_h – the modulus of elasticity; A_h – the hanger's cross section area; i – the No. of the hangers; F^i – denotes the hangers' axial force, including hangers' self-weight.

According to the equilibrium state at node 2:

$$F_1^2 = -F_4^1, \;\; F_3^2 = -F_6^1 + F^i. \quad (7)$$

Set $F_1 = F_1^{\,2}$, $F_3 = F_3^{\,2}$, $l_x = X_2$ as the initial values, the unstrained length s_2 of element 2–3, $F_4^{\,2}$, $F_6^{\,2}$ at node 2 and $z_3^{\,c}$ are obtained by Eqs (1)–(2). And then, go to the next step, until all the intermediate cable segments have been calculated.

2.3.3. Right saddle

$F_1^{\,n-1}$, $F_3^{\,n-1}$, X^{n-1} have been acquired in section 2.3.1, as shown is Fig. 3b, the initial value of θ_2^0 is the main cable's angle by assuming that the main cable shape is parabolic under dead load. The following relation for the non-contact cable (n–1)–BR is obtained.

$$l_x = X_{n-1} - r_2 \cos\theta_2^0, \quad (8)$$

The unstrained length of cable (n–1)–BR, s_{n-1}^0, the tensile components $F_4^{\,b}$, $F_6^{\,b}$ at node B are calculated by Eqs (1)–(2). If $R = \left|\theta_2^0 - \arctan\left(\frac{F_6^b}{F_4^b}\right)\right|$ does not meet the convergence criteria, adopting the Newton-Raphson method to iterative until R meets the convergence criteria. According to Eq (3), the unstrained cable length s_{n-1} of the right saddle element is acquired.

2.4. Tangent stiffness matrix

According to section 2.3, the calculation of the main cable has been finished, however, the variables **V** are not the exact values, therefore an iterative procedure will be needed to acquire the exact solution.

Set **U**=$[z_{sag}, z_n]^T$ as the goal, **V**=$[T_x, T_z]^T$ as the variables and function **F**(**V**)=**U**, where **F**(**V**)=$[f_1, f_2]^T$, **U**=$[z_{sag}, z_n]^T$.

$$\text{Set function } \boldsymbol{\varphi} = \mathbf{F} - \mathbf{U} = \left[f_1 - z_{sag}, f_2 - z_n\right]^T = 0. \quad (9)$$

Linearization of the nonlinear function $\boldsymbol{\varphi}$ j by first-order Taylor expansion at (T_x^i, T_z^i):

$$\varphi\left(T_x, T_z\right) \approx \varphi\left(T_x^i, T_z^i\right) + \frac{\partial\varphi}{\partial T_x^i}\left(T_x - T_x^i\right) + \frac{\partial\varphi}{\partial T_z^i}\left(T_z - T_z^i\right), \quad (10)$$

and,

$$\mathbf{F}\left(T_x^i, T_z^i\right) = \mathbf{U}^i = \left[z_{sag}^i, z_n^i\right]^T. \tag{11}$$

According to Eqs (9)–(11):

$$\left[\frac{\partial \varphi}{\partial T_x^i}, \frac{\partial \varphi}{\partial T_z^i}\right]\begin{bmatrix} T_x - T_x^i \\ T_z - T_z^i \end{bmatrix} = \begin{bmatrix} z_{sag} - z_{sag}^i \\ z_n - z_n^i \end{bmatrix}. \tag{12}$$

Simplified Eq (12),

$$\begin{bmatrix} z_{sag} \\ z_n \end{bmatrix} = \begin{bmatrix} z_{sag}^i \\ z_n^i \end{bmatrix} + \left[J^i\right]^{-1}\begin{bmatrix} T_x - T_x^i \\ T_z - T_z^i \end{bmatrix} \quad \left[J^i\right] = \begin{bmatrix} \dfrac{\partial f_1}{\partial T_x^i} & \dfrac{\partial f_1}{\partial T_z^i} \\ \dfrac{\partial f_2}{\partial T_x^i} & \dfrac{\partial f_2}{\partial T_x^i} \end{bmatrix}.$$

Thus, the iterative equation is expressed as the following function:

$$\begin{bmatrix} z_{sag}^{i+1} \\ z_n^{i+1} \end{bmatrix} = \begin{bmatrix} z_{sag}^i \\ z_n^i \end{bmatrix} + \left[J^i\right]^{-1}\begin{bmatrix} T_x - T_x^i \\ T_z - T_z^i \end{bmatrix}. \tag{13}$$

The residual error $R = \left[z_{sag} - z_{sag}^{i+1} \;\; z_n - z_n^{i+1}\right]^T$, and R's 2-norm is adopted as the convergence criterion.

$[J^i]$ is the tangent stiffness matrix. f_1, f_2 are very complex, it's difficult to obtain the exact expressions. Thus, the best way to obtain $[J^i]$ is adopting numerical method. The calculation steps are as follows:

1) set the initial values T_x, T_z, z_{sag}^i, z_n^i are acquired according to section 2.3;

2) gives T_x an increment ΔT_x, z_{sag}^{11}, z_n^{21} are calculated corresponding to the initial values $T_x + \Delta T_x$, T_z;

3) gives Tz an increment ΔT_z, z_{sag}^{12}, z_n^{22} are calculated corresponding to the initial values T_x, $T_z + \Delta T_z$;

4) thus, the tangent stiffness matrix $K=[J^i]^{-1}$ is expressed as,

$$K(1.1) = \frac{\left(z_{sag}^{11} - z_{sag}^i\right)}{\Delta T_x}, \quad K(2.1) = \frac{\left(z_n^{21} - z_n^i\right)}{\Delta T_x},$$

$$K(1.2) = \frac{\left(z_{sag}^{12} - z_{sag}^i\right)}{\Delta T_z}, \quad K(2.2) = \frac{\left(z_n^{22} - z_n^i\right)}{\Delta T_z},$$

the increments ΔT_x, ΔT_z should have the similar magnitude corresponding to T_x, T_z to avoid the computer rounding errors and the nonlinear effects. This paper recommends taking the original values 0.01 times;

5) after **K** is acquired, adopting Newton-Raphson method or modified Newton-Raphson method to implement the iterative procedure. By comparison, modified Newton-Raphson method is more efficient.

As shown in Fig. 4, after the main cable-shape in the main span is obtained, and with the hypothesis of that the tower is in zero-moment state under bridge finished state, the horizontal component of the side main cable tensile is able to be acquired. When calculating the side span's main cable-shape, removing the constraint of the sag and assume that the horizontal component of the cable tensile is known, the calculating degraded as a single variable single-objective optimization problem and the calculating steps are the same as Fig. 4.

After the form-finding analysis, the main cable's unstrained length has been acquired, treating l_x as unknown variables, unstrained length L_0 as known, the calculating steps shown in Fig. 4 is used for the suspension bridge erection process analysis.

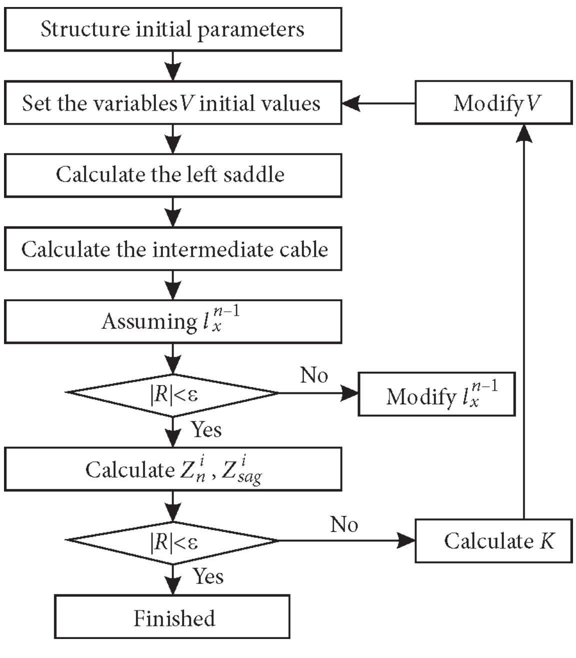

Fig. 4. Flow chart describing the iterative procedure

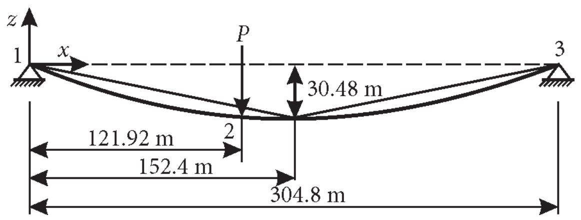

Fig. 5. Isolated cable under self-weight

3. Analytical verification

Example 1. The first example is a cable spanning of 304.8 m between two supports at the same elevation, where the sag at the mid-span is 30.48 m. The cable with modulus of elasticity E = 1.31×1011 N/m^2, cross section area A = 5.48×10^{-4} m^2, and weight per unit length w = 46.11 N/m.

As shown in Fig. 5, the saddles' radii are 0. Adopting the method proposed in this paper, the cable is divided into 20 catenary elements without considering the hangers and the concentrated force P = 0. The examples adopt R's 2-norm as the convergence criterion and convergence error is 10^{-5}. The results obtained by the proposed method and previous research are compared in Table 1. A good agreement is obtained.

Example 2. As shown in Fig. 5, P =35.586 kN, the cable-shape shown in Fig. 5 is the equilibrium configuration under self-weight. Many researchers have studied this issue, it was first considered by Michalos and Birnstiel (1962), and

Table 1. Comparison of coordinate, unstressed and horizontal force of isolated cable under self-weight

Researcher	Thai and Kim (2001)	Karoumi (1999)	Present work
Sag under self-weight at point 2	29.276 m	–	29.276 m
Unstressed cable length of section 1–2	125.88 m	–	125.85 m
Unstressed cable length of section 2–3	186.85 m	–	186.86 m
Total unstressed cable length	312.73 m	312.7 m	312.70 m
Horizontal force of the cable	–	1.7794×10^4 N	1.7793×10^4 N

Table 2. Comparison of displacements of isolated cable under concentrated load

Researcher	Element type	Displacements, m	
		vertical	horizontal
Michalos and Birnstiel	Elastic straight	–5.472	–0.845
O'Brien and Francis	Elastic catenary	–5.627	–0.860
Jayaraman and Knudson	Elastic straight	–5.471	–0.845
Jayaraman and Knudson	Elastic catenary	–5.626	–0.859
Tibert	Elastic catenary	–5.626	–0.859
Andreu *et al.*	Elastic catenary	–5.626	–0.860
Yang and Tsay	Elastic catenary	–5.626	–0.859
Thai and Kim	Elastic catenary	–5.626	–0.859
Present work	Elastic catenary	–5.626	–0.859

later analyzed by O'Brien and Francis (1964), Jayaraman and Knudson (1981), Tibert (1998), Andreu *et al.* (2006) and Yang and Tsay (2007). Some of them adopted catenary element and others adopted straight type element. The analysis model and convergence criterion and convergence error are the same as in Example 1. Table 2 shows a comparison of the vertical and horizontal displacements obtained by the present work and those predicted by the other researchers.

These two number examples show that the present work acquire high accuracy results through a few iterative steps, besides, they also show their two applications: (1) suspension bridge shape-finding analysis under bridge finished state, (2) analysis of the main cable during erection. The former corresponds with Example 1, treating the arrangement of the hangers and the suspension bridge's geometric parameters as initial conditions. The latter corresponds with Example 2, based on the shape-finding analysis, the unstrained lengths of each cable element are initial conditions, by the load increment method, the main cable's internal forces and displacements during the stiffness girder erection process is able to be obtained, this method is independent of the before and after construction phase and get rid of the dependence on the construction process simulation when calculating the problem by FEM.

4. Engineering application

4.1. FEM pre-processing for cable structure

Some structural analysis software packages, such as SAP2000, Midas/civil supply catenary element which is used to model cable structure. The key and difficult issue adopting FEM to analyze cable structure is the initial cable-shape, unstrained cable lengths and equilibrium forces of each element. Some software specifically developed a form-finding tool, such as Midas/civil supplies suspension bridge analysis tool based on the node update method. Taking the single cable shown in Fig. 5, for example, adopting FEM to analyze node 2 displacements under concentrated P = 35.586 kN, the FEM pre-processing for cable structure uses the form-finding tool and the present work. The initial form used in Midas/civil form-finding tool is the broken-line shown in Fig. 5. Three FEM models were established and there were 30 elements and 31 nodes. The models described are as follows:

Midas: acquiring the nodes and internal forces information using Midas/civil form-finding tool, and Midas/civil was used to analyze the single cable adopting catenary element.

ANSYS: acquiring the nodes and internal forces information using the present work, ANSYS was used to analyze the single cable adopting Link10 element.

Present work: acquiring the nodes and internal forces information using the present work, Midas was used to analyze the single cable adopting catenary element.

Table 3. Comparison of displacements of isolated cable under concentrated load

Different method	Element type	Displacements, m	
		vertical	horizontal
Midas	Elastic catenary	–1.069	–6.106
ANSYS	Link10	–0.867	–5.715
Present work	Elastic catenary	–0.859	–5.627

According to Example 2, the exact vertical and horizontal displacements of the issue are –0.859 m, –5.626 m. As shown in Table 3, the present work model acquired a high accuracy results. Adopting straight link (Link10) element to model the cable also get a good agreement with the exact results. Using the form-finding tool supplied by Midas, in despite of the cetanary element was adopted, the results are far from the exact solution.

4.2. Cable system analysis considering the sliding effect

The precise main cable and hangers unstrained lengths, the cable curve under cable completion state are the key issue during the construction control. Currently, the back-running analysis method is adopted for the construction stage analysis using FEM. However, the professional bridge analysis FEM software, such as Midas, don't supply saddle element, so it is difficult to calculate the sliding effect between main cable and saddle and the excursion of the saddle accurately. The selection of saddle's parameters and arrangement of the saddles are depended on the designers' experience, the sliding effect always was neglected and the proper parameters were adopted according to the main cable's angle. By this way, calculation of the main cable's cutting length will inevitably lead to errors and the excursion of the saddle during the construction stage cannot be precisely calculated. The Yingwuzhou Yangtze River Bridge is taken as an example to illustrate the advantages of the proposed method.

Fig. 6. Side view of the Yingwuzhou Yangtze River Bridge

Yingwuzhou Yangtze River Bridge shown in Fig. 6 is the 8th bridge across the Yangtze River in Wuhan, with a span of (225 + 850 + 850 + 255) m and rise-span ratio 1/8. The stiffness girder adopted steel-concrete composite girder, the equivalent moment of transverse and vertical inertia, the equivalent cross sectional area are 207.598 m^4, 2.091 m^4 and 1.563 m^2. The spacing between the intermediate hangers is 15 m, the spacing between the side hangers and tower is 20 m, the saddle's radius is 6.3 m, the splay's radius is 8 m. The coordinates of the fixed points between main cable and saddles are (0; 0; 161.878), (±850; 0; 143.783), (±1075; 0; 39.0). The fixed points on splay saddles are the same as the design points, theoretical intersections of the main cable in finished bridge state are (0; 0; 162.5), (± 850; 0; 144.5). Dead load is 342.82 kN/m. For the sake of convenience, some parameters of the bridge were simplified.

The main cable-shape under bridge completion stage and cable finished state considering the sliding effect (present work) and not considering the sliding effect (traditional method) are shown Fig. 7. Some key parameters under two stages are listed in Table 4.

As shown in Fig. 7 and Table 4, there are some differences between the present work and the traditional method. Compared with the traditional work, the main cable's unstained length decreases about 1.2 m. Under the bridge finished stage, the differences of the tangential angles saddle is smaller acquired by the two methods, compared with the cable finished stage. Ignoring the towers' longitudinal displacement under eccentric load, the excursion of the side tower saddle is 0.865 m in the side span direction

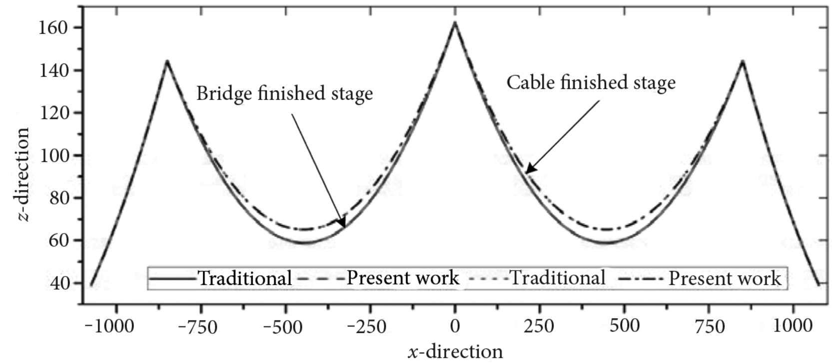

Fig. 7. Comparison of the calculated cable-shape under two different construction stages

Table 4. Comparison of key parameters between finished bridge state and cable finished stage

Load case		Finished bridge state		Cable finished stage	
Item		Traditional method	Present work	Traditional method	Present work
Total unstressed cable length, m		2245.165	2243.937	2245.165	2243.937
The tangential angle between cable and saddle, °	vice cable saddle	20.674	20.667	19.461	19.517
	side tower (left)	30.017	29.987	30.612	30.464
	side tower (right)	22.504	22.482	21.700	21.543
	main tower (left)	24.639	24.622	23.873	23.732
	main tower (right)	24.639	24.622	23.873	23.732

and the main cable's horizontal force is 24158.19 kN calculated by the traditional method, which are 0.858 m and 24174.97 kN calculated by the present work. The difference of the excursion of the saddle between the two methods is 0.7 cm. If the error is taken into account in considering sliding effect model, 189.4 kN unbalanced horizontal force in the main span direction will be produced. According to the initial design parameters, to the 132 m height side-tower, 2 cm longitudinal displacement will be produced at the top of the side tower. The side cable horizontal force is very sensitive to the excursion of the saddle, the error produced without considering the sliding effect cannot be ignored.

In general, the differences of the internal forces and geometric parameters between the two methods are small under bridge finished stage and cable finished stage. Without considering the sliding effect, it will produce larger error of the saddles' excursion leading to the tower in unfavourable force state during the construction stage. It illustrated that the traditional method is able to be used to analyze the main cable's internal force, but the sliding effect should be considered during calculation of the excursion of the saddles. According to the proposed method, the arrangement and parameters of the saddles can be determined before the form-finding analysis while accurately modelling the sliding effect, which greatly increases the designers' initiative.

5. Conclusions

1. According to the arrangement of the hangers and the suspension bridge's geometric parameters, the main cable's unstrained length, geometric configuration, equilibrium forces and hangers' unstrained lengths under bridge finished state are obtained. The analytical results compared with typical numerical examples illustrated that the proposed method is accurate and effective for the shape-finding analysis of suspension bridges.

2. Setting the results acquired from the proposed form-finding analysis method as the construction analysis initial values, the main cable's internal forces and displacements during the stiffness girder erection process can be calculated using the load increment method. This method is independent of the before and after construction phase, and gets rid of the dependence on the construction process when solving the problem by FEM.

3. Adopting the proposed method as a FEM pre-processing tool for cable structure acquired a more accuracy analysis result, compared with the shape-finding tool of commercial software Midas. Even though ordinary truss element is adopted to model the cable structure, the obtained result is also sufficiently accurate for engineering application.

4. The traditional method can be used to analyze the main cable's internal force, but the sliding effect cannot be calculated considering the excursion of the saddles. But according to the proposed method, the sliding effect is considered accurately, thus, the arrangement and parameters of the saddles will be determined before the form-finding analysis, which greatly increases the designers' initiative.

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