



## ANALYSIS OF INNOVATIVE TWO-SPAN SUSPENSION BRIDGES

Algirdas Juozapaitis<sup>1</sup>✉, Tomas Merkevičius<sup>2</sup>, Alfonsas Daniūnas<sup>3</sup>, Romas Kliukas<sup>4</sup>,  
Giedrė Sandovič<sup>5</sup>, Ona Lukoševičienė<sup>6</sup>

<sup>1, 2, 5</sup>Dept of Bridges and Special Structures, Vilnius Gediminas Technical University,  
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

<sup>3</sup>Dept of Steel and Timber Structures, Vilnius Gediminas Technical University,  
Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

<sup>4, 6</sup>Dept of Strength of Materials, Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania

<sup>5</sup>Dept of Roads, Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania

E-mails: <sup>1</sup>algirdas.juozapaitis@st.vgtu.lt; <sup>2</sup>tomas.merkevicius@vgtu.lt; <sup>3</sup>alfonsas.daniunas@vgtu.lt;

<sup>4</sup>romualdas.kliukas@vgtu.lt; <sup>5</sup>giedre.sandovic@vgtu.lt; <sup>6</sup>ona.lukoseviciene@vgtu.lt

**Abstract.** Recently, two-span, or the so-called single pylon suspension bridges, due to their constructing structure, have been widely applied. A reduction in deformation seems to be the main problem of the behaviour and design of such bridges. The deformation of suspension bridges is mainly determined by cable kinematic displacements caused by temporary loadings rather than by elastic deformations. Not all known methods for the stabilization of the initial form of suspension bridges are suitable for single pylon bridges. The employment of the so-called rigid cables that increase the general stiffness of the suspension bridge appears to be one of the innovative methods for stabilizing the initial form of single pylon suspension bridges. Rigid cables are designed from standard steel profiles and, compared to the common ones made of spiral and parallel wires, are more resistant to corrosion. Moreover, the construction joints, in terms of fabrication and installation, have a simpler form. However, calculation methods for such single pylon suspension bridges with rigid cables are not sufficiently prepared. Only single publications on the analysis of the behaviour of one or three-span suspension bridges with rigid cables have been available so far. The paper presents analytical expressions to calculate the displacements and internal forces of suspension bridges with rigid cables thus assessing the sequence of cable installation. Also, the paper describes the sequence of iterative calculation.

**Keywords:** suspension bridge, single pylon bridge, steel bridge, rigid cable, symmetric loadings, non-linear analysis, internal forces and displacements.

### 1. Introduction

For a long time, due to their effective behaviour and excellent architectural appearance, suspension bridges have been employed for carrying out long and average-sized spans (Ryall *et al.* 2000; Troyano 2003). By reason of dominating tension stresses, suspension bridges assure covering the longest spans in the world (Gimsing, Georgakis 2012; Strasky 2005). Recently, two-span, or the so-called single pylon suspension bridges have been introduced. The cables of these bridges are anchored either in the foundation or a stiffening girder. The latter ones are also called self-anchored bridges (Kim *et al.* 2002; Zhang *et al.* 2013). The cables of suspension bridges are manufactured from high strength steel spiral or parallel wires the flexural stiffness of which is equal to zero (Gimsing, Georgakis 2012; Kulbach 2007). It should be stressed that

such cables are subject to specific anti-corrosion protection, and their constructions joints, from the point of view of the structure, are sufficiently complex (Betti *et al.* 2005; Bloomstine, Sorensen 2006; Nakamura, Suzumura 2009; Xu, Chen 2013; Yanaka, Kitagawa 2002), which in turn, increases bridge construction and exploitation costs.

High deformability is one of the most serious disadvantage of suspension bridges (Gimsing, Georgakis 2012; Jennings 1987; Katchurin *et al.* 1971) and is mainly determined by the kinematic displacements of the suspension cable caused by asymmetric and local traffic loadings rather than by the elastic deformations of the cable (Juozapaitis, Norkus 2004; Kulbach 2007). It should be noted that the kinematic displacements of the asymmetrically loaded cable directly depend on its initial sag and do not rest on the length of its span (Jennings 1987; Juozapaitis,

Norkus 2004). The stiffening girder is the main structural element that allows ensuring the required stability of the initial form of suspension bridges (Gimsing, Georgakis 2012; Ryall *et al.* 2000). This classical structural element of stabilization is not accepted as highly effective, because, under relatively high asymmetric loadings, the mass of the girder of high rigidity is required (Grigorjeva *et al.* 2010; Katchurin *et al.* 1971; Lewis 2012; Wollman 2001). Also, some other structural measures that allow reducing kinematic displacements are known (Gimsing, Georgakis 2012; Katchurin *et al.* 1971; Strasky 2005). However, some of those are quite complex or not effective enough from a technical-economic point of view (Jennings 1987). Moreover, not all available stabilization methods for the initial form of suspension bridges can be proper to single pylon bridges.

To reduce the kinematic displacements of suspension bridges and to stabilize their initial form, a structured decision has been suggested, following which, the so-called rigid cables instead of the common ones are applied (Grigorjeva *et al.* 2010, 2015; Juozapaitis *et al.* 2006, 2010). The cables having a similar structure are designed using hot rolled or welded steel profiles. The forms of their cross-section may differ from I-beams to rectangular or round tubes and chutes (Grigorjeva *et al.* 2010; Juozapaitis *et al.* 2010, 2013). Both the constructions joints of such rigid cables are simple and reliable. Contrary to the cables made of high strength wires, dangerous local strains are not produced (Fürst *et al.* 2001; Gimsing, Georgakis 2012; Prato, Ceballos 2003; Strasky 2005). The cross-sections of rigid cables are remarkably resistant to the impact of corrosion. To achieve higher technical efficiency, high strength steel is recommended for producing rigid cables.

A number of works focus on analysing the behaviour of traditional suspension bridges, i.e. the bridges with flexible cables (Clemente *et al.* 2000; Cobo del Arco, Aparicio 2001; Gimsing, Georgakis 2012; Katchurin *et al.* 1971; Kim, Thai 2010; Wollman 2001; Wyatt 2004). Thus, it should be emphasized that for calculating internal forces and displacements of suspension bridges, numerical methods are widely applied (Nevaril, Kytir 2001; Wang *et al.* 2002). However, applying them not always assists in adequately evaluating the sequence of the suspension bridge and installation.

The number of works on the analysis of two-span (single pylon) suspension bridges is not that high. The methods for performing calculations on such innovative suspension bridges with rigid cables have not been put into practice yet. A few publications on the analysis of the behaviour of one or three-span suspension bridges have been prepared (Grigorjeva, Kamaitis 2015; Juozapaitis *et al.* 2010, 2013). Thus, an important point is the analysis of the behaviour of the innovative two-span suspension bridge with the rigid cable and the preparation of its calculation method.

The paper examines calculations on the innovative single pylon suspension bridge with the rigid cable under symmetric loadings. The proposed methodology evaluates the sequence of installing the rigid cable. Analytical expressions estimating the internal forces and displacements of such a bridge are presented thus discussing the sequence of iterative calculation.

## 2. Analysis of the two-span suspension bridge with the rigid cable

### 2.1. Specificities of calculating and designing the innovative suspension bridge

A constructional scheme of the innovative two-span suspension (single pylon) bridge is typical of the structure of a classical bridge with a flexible cable. The bridge, instead of the flexible cable, uses the rigid one, i.e. a cable having flexural stiffness  $E_c J_c \neq 0$ . The other structural parts of the suspension bridge remain the same and include stiffening girders, pylons and hangers (Fig. 1).

The main problem of suspension bridges, including the single pylon ones, is relatively high deformability under asymmetric or local loadings. The displacements of such bridges effectively reduce applying innovative decisions one of which, as mentioned above, is the employment of the so-called rigid cables instead of conventional flexible ones. Rigid cables are known as having axial  $E_c A_c \neq 0$  and flexural stiffness  $E_c J_c \neq 0$ . The value of flexural stiffness  $E_c J_c$  is selected according to eligibility for limit state conditions.

The introduced innovative cables use hot-rolled or welded steel profiles having the I-beam and box cross-section (2010; Grigorjeva *et al.* 2010). It should be noted for producing such rigid cables, high strength steel is highly recommended.

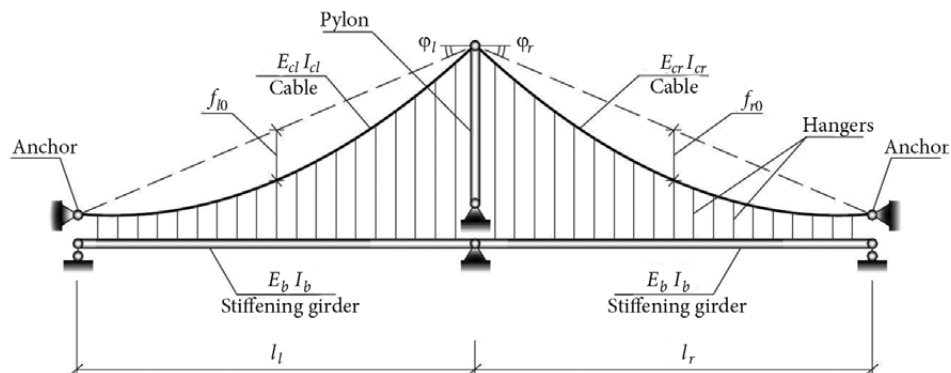


Fig. 1. The structure of suspension bridge

Obviously, the cross-sectional area of rigid cables, compared to the conventional spiral or parallel wire cable, may be slightly higher, but the cost will be significantly lower. On the other hand, compared to the suspension bridge with flexible cables, the total mass of new bridge supporting structures (cable and stiffening girder) will be lower, because the rigid cable, similarly to the stiffness girder, takes over asymmetric and concentrated loadings. Moreover, taking into account the operating costs for the anti-corrosive protection and maintenance of flexible cables made of parallel wires, the efficiency of applying rigid cables increases.

Emphasis should be placed on two main options of forming (installing) the rigid cable (Juozapaitis *et al.* 2010). In the first case, the cable acquires flexural stiffness ( $EJ_c \neq 0$ ) from the very beginning of forming the bridge and takes over both permanent  $g$  and contemporary  $v$  loadings through the processes of tension and flexure. In the latter case, the cable gains flexural stiffness after installing the bridge rather than at the very beginning. In this case, the cable takes over permanent loadings through tension only while the temporary ones as tension and as a flexural element. In the latter case, the general loadings of the rigid cable are significantly reduced.

The behaviour of such suspension bridges is qualitatively similar to that of the bridges with conventional flexible cables, and for calculating them; the same well known assumptions are applied (Gimsing, Georgakis 2012; Katchurin *et al.* 1971; Wollman 2001).

### 2.2. Calculating the two-span suspension bridge under permanent loading

A calculation scheme for the innovative two-span (single pylon) suspension bridge is presented in Fig. 2.

For making calculations on the bridge, the second more rational case of forming a rigid cable is examined, i.e. the bearing cable, as an absolutely flexible one, takes

over the whole permanent loading  $g$  and, as a rigid cable ( $EJ_c \neq 0$ ), temporary loading  $v_c$ . It should be emphasized that, in this case, the stiffening girder takes only a part of temporary loading ( $v_b$ ).

The scheme for the loaded single pylon suspension bridge shows that two main structural elements (cable and stiffening girder) are, for the sake of clarity, relatively fragmented (Fig. 2). The general case, when the lengths between spans are not equal ( $l_l \neq l_r$ ), has been investigated. The endings of both stiffening girders and bearing cables of such a bridge are pinned supported.

#### 2.2.1. The right span of the bridge

The spans of the suspension cable of the single pylon bridge are installed at different levels, i.e. the cables of both spans are inclined. It should be emphasized that the calculation of the inclined specified cables applying local coordinates is much more complex (Juozapaitis, Daniūnas 2005). The angle of the tilt of the cables is slight enough, and therefore, for the sake of simplicity, the inclined cables in global ordinates will be calculated. Next, the right and left parts of the bridge will be examined. At the first stage of formation, the rigid cable is affected by permanent loading  $g$  making an impact on the bridge.

The equilibrium condition for the right side (marked with symbol  $r$ ) cable is defined by Eq (1):

$$H_{r0} \cdot z_{r0}(x_r) - M_{rg}(x_r) = 0, \tag{1}$$

$$M_{rg}(x_r) = \frac{gl_m^2}{8} \left( \frac{4x_m}{l_m} + \frac{4x_m^2}{l_m^2} \right), \tag{2}$$

where  $H_{r0}$  – a horizontal component of cable tension under loading  $g$ ;  $z_{r0}(x_r)$  – the initial curve of the cable (quadratic parabola is accepted), calculated from the line connecting upper and lower spans (Fig. 2);  $M_{rg}(x_r)$  – the

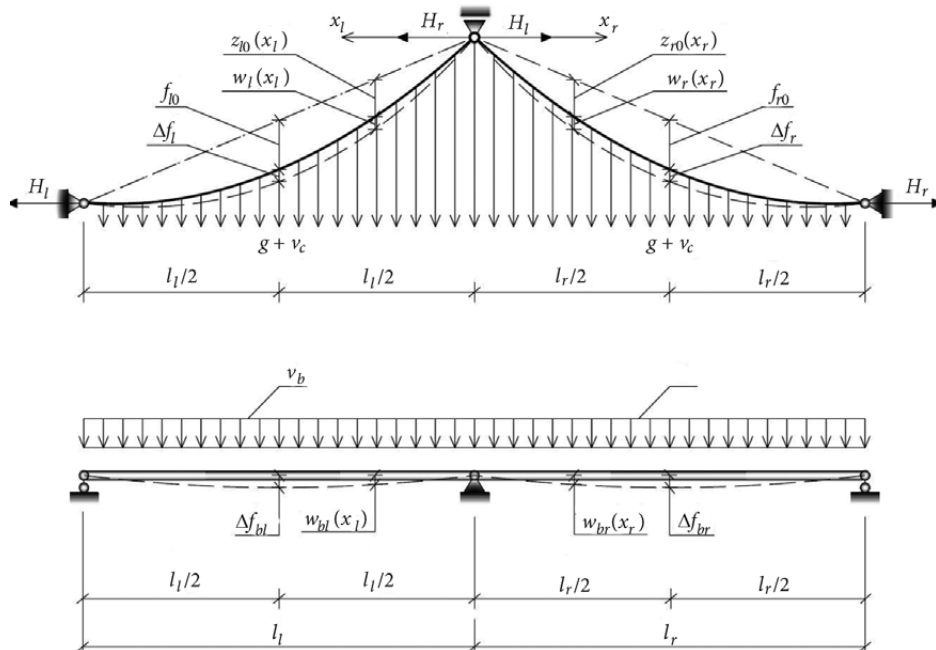


Fig. 2. Schemes for the calculations on the suspension bridge

moment caused by permanent loading  $g$  in the girder of the analogous span. The horizontal component of the tension force of this flexible cable is calculated as follows:

$$H_{m0} = \frac{gl_r^2}{8f_{r0}}, \quad (3)$$

where  $f_{r0}$  – the initial sag of the right flexible inclined cable in the middle of the span ( $x_r = 0.5l_r$ ), calculated from the line connecting upper and lower supports. It should be emphasized that the value of the tensile strength of the inclined flexible cable acting along the line will be higher and equal to  $N_r = \frac{H_{r0}}{\cos\varphi_r}$ .

### 2.2.2. The left span of the bridge

Under symmetric loadings, the calculation of the left side of the bridge (marked with symbol  $l$ ) mainly does not differ from that of the right side. The initial state of this cable, under permanent loading  $g$  is defined using an analogical equation:

$$H_{l0} \cdot z_{l0}(x_l) - M_{lg}(x_l) = 0, \quad (4)$$

$$M_{lg}(x_l) = 0.125gl_l^2 \left( \frac{4x_l}{l_l} + \frac{4x_l^2}{l_l^2} \right), \quad (5)$$

where  $H_{l0}$  – a horizontal component of cable tension force under load  $g$ ;  $z_{l0}(x_l)$  – the initial curve of the cable calculated from the line connecting the upper and lower supports of the cable (Fig. 2).  $M_{lg}(x_l)$  – the moment caused by permanent loading  $g$  in the girder of an analogous span ( $l_l$ ).

The horizontal component of the tension force of the flexible left cable is calculated as follows:

$$H_{l0} = \frac{gl_l^2}{8f_{l0}}, \quad (6)$$

where  $f_{l0}$  – the initial sag of the right flexible inclined cable in the middle of the span ( $x_l = 0.5l_l$ ), calculated from the line connecting the upper and lower supports of the cable. In parallel with the right cable, the value of the tensile strength of the inclined left cable acting along the line will be equal to  $N_l = \frac{H_{l0}}{\cos\varphi_l}$ .

### 2.2.3. Selecting composed parameters for the two-span suspension bridge

Striving for the equilibrium of the initial state of the two-span suspension bridge, the parameters of the structural elements of separate spans must be accurately selected, because, under permanent loading, the following condition must be additionally satisfied:

$$H_{r0} = \frac{gl_r^2}{8f_{r0}} = H_{l0} = \frac{gl_l^2}{8f_{l0}}. \quad (7)$$

Consequently, dependence between the values of the sags of the right and left flexible cable is obtained:

$$f_{r0} = f_{l0} \frac{l_r^2}{l_l^2}. \quad (8)$$

Eq (8) shows that a greater difference between the lengths of bridge spans results in a large difference in the values of the initial sags of the right and left cable. In the case condition (Eq (8)) is not satisfied, under loading  $g = const.$ , the top of the pylon should experience horizontal displacements, which, in turn, would cause adverse displacements of kinematic origin.

It must be emphasized that received condition (Eq (8)) is for the case when pylons are pinned supported to the foundation.

### 2.3. Calculation on the two-span suspension bridge under permanent and temporary loadings

The chapter examines the symmetric loading of the bridge when the temporary loading of the same intensity acts on both spans, i.e. when  $v = v_r = v_l$ . As mentioned above, for examining the behaviour of this bridge, the second version of forming the cable is accepted. Under permanent loading  $g$  only, the initial equilibrium state of both flexible cables of the spans is defined by Eqs (1)–(5). According to the second case of installation, uniform flexural stiffness  $E_{cr}J_{cr} = E_{cl}J_{cl} = E_cJ_c \neq 0$  is provided for flexible cables applying certain structural measures. Thus, under the action of variable loading, cables are rigid. A deformation scheme for the single pylon bridge affected by permanent  $g$  and variable  $v$  loadings is presented in Fig. 2. It should be emphasized that temporary symmetric loading  $v$  acting over the entire length of the bridge will distribute between rigid cables and the stiffening girder proportionally to their flexural rigidity. Moreover, variable loading  $v_c$  on rigid cables will spread axial forces  $H_r$  and  $H_l$  as well as bending moments  $m_{cr}(x_r)$  and  $m_{cl}(x_l)$  inside them. It should be noted that, in the meantime, stiffening girders will take the remaining part of this variable loading  $v_b$  that will cause only bending moments  $m_{br}(x_r)$  and  $m_{bl}(x_l)$  inside them.

#### 2.3.1. Calculation of the right span of the single pylon suspension bridge

As mentioned above, when the entire bridge is under symmetric temporary loading  $v$ , the rigid cables of both spans and stiffening girders will take vertical displacements  $w_{cr}(x_r)$ ;  $w_{cl}(x_l)$ ;  $w_{br}(x_r)$ ;  $w_{bl}(x_l)$  (Fig. 2). It should be remembered that rigid cables will take all permanent and a part of temporary loadings  $g + v_c$ , and stiffening girders – only a part of variable loading  $v_b$ . As for the right span, a condition for displacement equality of the stiffness girder and the rigid cable is written as:

$$w_{br}(x_r) = w_{cr}(x_r) = w_r(x_r). \quad (9)$$

The condition is correct if is accepted that suspension deformations have no influence on internal forces and displacements (Juozapaitis et al. 2010). Next, the behaviour of the stiffening girder and the rigid cable, under temporary loading, will be separately examined.

Considering Eq (9), the condition of the moment equilibrium of the stiffening girder of the right span will be obtained:

$$-E_b J_b \cdot w_r''(x_r) + M_{b,r}(x_r) = 0, \quad (10)$$

$$M_{b,r}(x_r) = 0.125 v_b l_r^2 \left( \frac{4x_r}{l_r} + \frac{4x_r^2}{l_r^2} \right), \quad (11)$$

where  $E_b J_b$  – the flexural stiffness of the stiffening girder;  $M_{b,r}(x_r)$  – the moment caused by variable loading ( $v_b$ ).

The equilibrium for the right rigid cable is as follows:

$$H_r [z_{r0}(x_r) + w_r(x_r)] + m_{cr}(x_r) + M_{c,r}(x_r) = 0, \quad (12)$$

where  $m_{cr}(x_r) = -E_c J_c \cdot w_r''(x_r)$  – the bending moment in the rigid cable;  $M_{c,r}(x_r)$  – the moment passed from permanent  $g$  and variable  $v_c$  loadings in the girder of an analogous span.

Eq (12) shows that both tensile strength and the bending moment will appear in the rigid cable.

A combination of the stiffening girder and the rigid cable for the purpose of common behaviour results in the following for equilibrium:

$$(E_b J_b + E_c J_c) w_r''(x_r) - H_r [z_{r0}(x_r) + w_r(x_r)] + M_r(x_r) = 0. \quad (13)$$

Eq (13) defines a general case of calculating the single pylon suspension bridge. In case the cable was flexible, i.e.  $E_c J_c = 0$ , the equilibrium condition of a standard single span suspension bridge from condition Eq (13) would be obtained (Juozapaitis *et al.* 2010; Wollman 2001). Eq (13) shows that the displacements of the innovative suspension bridge with the rigid cable will be smaller, because the general flexural rigidity of the bridge increases  $E_r J_r = E_b J_b + E_c J_c$ .

The application of the concept of the fictitious displacement of the rigid cable (Moskalev, Popova 2003) points to the following solution:

$$w_r(x_r) = \Delta f_{fic,r} \left[ \frac{4x_r}{l_r} - \frac{4x_r^2}{l_r^2} + \frac{8}{k_r^2 l_r^2} \lambda \right], \quad (14)$$

$$k_r^2 l_r^2 = \frac{H_r l_r^2}{(E_b J_b + E_c J_c)}, \quad (15)$$

$$\lambda = ch k_r x_r - \frac{1 - ch k_r l_r}{sh k_r l_r} sh k_r x_r - 1, \quad (16)$$

where  $\Delta f_{fic,r} = z_{fic,r}(x_r^*) - z_{0r}(x_r^*)$  – a fictitious displacement of the right rigid cable in the middle of the span ( $x_r^* = 0.5l_r$ );  $k_r^2 l_r^2$  – the general parameter of the slenderness of the right side of the bridge;  $\lambda$  – function of the slenderness parameters  $k_r l_r$  and  $k_r x_r$ .

The analysis of Eq (14) shows that the solution is analogous to the already known formula calculating

single rigid suspension elements (cables). Thus, the behaviour of the innovative suspension bridge is adequate for the behaviour of single rigid suspension elements. The flexural stiffness of the suspension bridge is made of stiffening girders and a sum of the flexural stiffness of the rigid cable. Thus, another valid conclusion can be drawn. Changes in the values of the flexural stiffness of the stiffening girder and rigid cable, i.e. their ratio, under the constant general stiffness of the bridge, may result in the adjustment of tension in the above mentioned structural elements.

For example, if variable loading on the right rigid cable  $v_c$  is known, its horizontal tension force is calculated as:

$$H_r = \frac{(g + v_c) l_r^2}{8(f_{r0} + \Delta f_{fic,r})}. \quad (17)$$

The above equation clearly indicates that the fictitious displacement may assist in establishing the horizontal tension force of the rigid cable analogically to the flexible one, what allows decreasing the volume of iterative calculations.

Eq (17) displays that the horizontal tension force of right rigid cable  $H_r$  depends on fictitious displacement  $\Delta f_{fic,r}$  while the latter, in turn, depends on horizontal tension force (Eq (14)). Therefore, an additional equation linking these two values is required:

$$s_r = s_{0r} + \Delta s_{el,r}, \quad (18)$$

where  $s_r$  – the length of the right with additional loading;  $s_{0r}$  – the initial length of the right cable;  $\Delta s_{el,r}$  – the elongation of the elastic right cable.

The expression of the deformation of the inclined rigid cable will be written down following the assessment of a possible horizontal displacement of pylon surface  $\Delta l_r$ :

$$s_r = \frac{l_r + \Delta l_r}{\cos \varphi_r} + \frac{8(f_{r0} + \Delta f_{fic,r})^2}{3(l_r + \Delta l_r)} \psi(k_r l_r \alpha_r) \cos^3 \varphi_r. \quad (19)$$

Member  $\psi(k_r l_r \alpha_r)$  in Eq (19) evaluates the impact of flexural stiffness on the deformation of the inclined cable. It should be noticed that, in this expression, member  $\alpha_r = \cos^2 \varphi_r$  specifies the parameter of the slenderness of the rigid cable moving from local to global coordinates.

The calculation of the rigid cable is performed with the help of gradual approximation. The values of the main unknown  $\Delta f_{fic,r}$  of the first step of iterative calculation is accepted as those of an absolutely flexible cable.

$$\Delta f_{fic,r} \approx \frac{3}{128} \frac{(g + v_c) l_r^4}{E_c A_c f_{0r}^2 \cos^3 \varphi_r}. \quad (20)$$

Next, according to Eq (17), the values of spreading force as well as that of slenderness parameter  $k_r^2 l_r^2$  are established. Subsequently, calculation is made using Eqs (18) and (19). The condition of iterative calculation is expressed as follows:

$$\Delta s_{el,r} - \Delta s_{gr} \leq \varepsilon, \tag{21}$$

where  $\Delta s_{gr} = s_r - s_{0r}$ .

Upon finishing calculations with the required accuracy  $\varepsilon$ , the values of the fictitious displacements of cable  $\Delta f_{fic,r}$  and its horizontal tension force  $H_r$  will be known. Then, the real vertical displacement of rigid cable  $w_r(x_r)$  and its bending moment  $m_{cr}(x_r)$  will be calculated. The bending moment of the stiffening girder in any cross-section can be established accordingly to the known expression  $m_{br}(x_r) = -E_b J_b \cdot w''_{br}(x_r)$  while its displacement, considering the accepted assumption, will be equal to the displacement of the rigid cable. The moment of the stiffening girder acting in the middle of the span, when its displacement  $\Delta f_{br}$  is known, can be estimated according to

the following simple expression  $m_{br}(x_r) = \frac{48 \cdot E_b J_b \cdot \Delta f_{br}}{5l_r^2}$ .

It should be noted that the application of the concept of the fictitious displacement of the rigid cable, compared to other calculation methods for suspension bridges allows significantly reducing the intensity of iterative calculation. On the other hand, this concept admits simple transformation to performing calculations on the standard suspension bridge with the flexible cable thus taking into account that  $E_c J_c = 0$ . The process of iterative calculation remains the same.

### 2.3.2. Calculations on the left span of the single pylon suspension bridge

The calculation of the structures of the left span of the suspension bridge is analogous to that of the structures of the right span. Only the geometrical parameters of the stiffening girder and rigid cable change. Thus, the initial equilibrium conditions of Eqs (12), (13), and (17) and their solutions Eq (14) will be analogical. Thus, the equations for calculating the horizontal tension force of the left rigid cable Eq (22) and its displacements Eq (23) is written as follows:

$$H_l = \frac{(g + v_c)l_l^2}{8 \cdot (f_{l0} + \Delta f_{fic,l})}, \tag{22}$$

$$w_l(x_l) = \Delta f_{fic,l} \left[ \frac{4x_l}{l_l} - \frac{4x_l^2}{l_l^2} + \frac{8}{k_l^2 l_l^2} \lambda \right], \tag{23}$$

$$k_l^2 l^2 = \frac{H_l l_l^2}{(E_b J_b + E_c J_c)}, \tag{24}$$

$$\lambda = ch k_l x_l - \frac{1 - ch k_l l_l}{sh k_l l_l} \cdot sh k_l x_l - 1, \tag{25}$$

where  $\Delta f_{fic,l} = z_{fic,l}(x_l^*) - z_{0l}(x_l^*)$  – a fictitious displacement of the left rigid cable in the middle of the span ( $x_l^* = 0, 5l_l$ );  $k_l^2 l_r^2$  – the general parameter of the slenderness of the left of the bridge;  $\lambda$  – function of the slenderness parameters  $k_l l_l$  and  $k_l x_l$ .

The equation for the coherence of the deformations of the left rigid cable is as follows:

$$s_l = s_{0l} + \Delta s_{el,l}, \tag{26}$$

$$s_{0l} = \frac{l_l}{\cos \varphi_l} + \frac{8 f_{l0}^2}{3 l_l} \cos^3 \varphi_l, \tag{27}$$

$$\Delta s_{el,l} = \frac{(H_l - H_{0l})}{E_c A_c \cdot \cos^2 \varphi_l}, \tag{28}$$

where  $s_l$  – the length of the left cable with additional loading;  $s_{0l}$  – the initial length of the left cable;  $\Delta s_{el,l}$  – the elastic elongation of the left cable.

The length of the left cable with added variable loading  $v_c$  is equal to:

$$s_l = \frac{l_l + \Delta l_l}{\cos \varphi_l} + \frac{8 (f_{l0} + \Delta f_{fic,l})^2}{3 (l_l + \Delta l_l)} \psi(k_l l_l \alpha_l) \cos^3 \varphi_l. \tag{29}$$

The calculation of the structures of the left span of the bridge is performed similarly to that of the right side. The path of gradual approximation is analogous.

### 3. Concluding remarks

The paper discusses the behaviour of the innovative two-span (single pylon) suspension bridge and presents calculations on symmetric loadings. The paper considers a general case when the spans of the bridge are of a different length. The paper analyses the sequence of construction phase such a bridge when the rigid cable is formed adding permanent loading. This allows reducing the initial strains on the rigid cable. The equations for calculating vertical displacements and internal forces of the rigid cable and the stiffening girder of the right and left spans of the innovative suspension bridge are presented. The concept of the fictitious displacement of the rigid cable may assist in significantly reducing the extent of the iterative calculation of the single pylon suspension bridge. The paper deals with a possibility of adjusting the internal forces and displacements of the rigid cable and stiffening girder in such a suspension bridge. The values of the ratio of the flexural rigidity of these structural elements have the highest impact. It should be emphasized that the obtained expressions of calculations define the general calculation case of the symmetrically loaded two-span (single pylon) suspension bridge. These expressions can be easily adapted to making calculations on common single pylon bridges with the flexible cable considering that the flexural stiffness of the rigid cable is equal to  $E_c J_c = 0$ . It should be also underlined that rigid cables allow reducing the general displacements of the suspension bridge.

### References

- Betti, R.; West, A. C.; Vermaas, G.; Cao, Y. 2005. Corrosion and Embrittlement in High-Strength Wires of Suspension Bridge Cables, *Journal in Bridge Engineering* 10(2): 151–162. [http://dx.doi.org/10.1061/\(ASCE\)1084-0702\(2005\)10:2\(151\)](http://dx.doi.org/10.1061/(ASCE)1084-0702(2005)10:2(151))
- Bloomstine, B.; Sorensen, O. 2006. Prevention of Main Cable Corrosion by Dehumidification, in *Advances in Cable-Sup-*

- ported Bridges. Ed. by Mahmoud, K. M. London: Taylor and Francis Group. 215–228. ISBN 10 0-415-41982-4.
- Cobo del Arco, D.; Aparicio, A. C. 2001. Preliminary Static Analysis of Suspension Bridges, *Engineering Structures* 23(9): 1096–1103. [http://dx.doi.org/10.1016/S0141-0296\(01\)00009-8](http://dx.doi.org/10.1016/S0141-0296(01)00009-8)
- Clemente, P.; Nicolosi, G.; Raitchel, A. 2000. Preliminary Design of Very Long-Span Suspension Bridges, *Engineering Structures* 22(12): 1699–1706. [http://dx.doi.org/10.1016/S0141-0296\(99\)00112-1](http://dx.doi.org/10.1016/S0141-0296(99)00112-1)
- Fürst, A.; Marti, P.; Ganz, H. 2001. Bending of Stay Cables, *Structural Engineering International* 11(1): 42–46. <http://dx.doi.org/10.2749/101686601780324313>
- Gimsing, N. J.; Georgakis, Ch. T. 2012. *Cable Supported Bridges: Concept and Design*. 3<sup>rd</sup> ed. John Wiley & Sons. 599 p. ISBN 9780470666289.
- Grigorjeva, T.; Kamaitis, Z. 2015. Numerical Analysis of the Effects of the Bending Stiffness of the Cable and the Mass of Structural Members on Free Vibrations of Suspension Bridges, *Journal of Civil Engineering and Management* 21(7): 948–957. <http://dx.doi.org/10.3846/13923730.2015.1055787>
- Grigorjeva, T.; Juozapaitis, A.; Kamaitis, Z. 2010. Static Analysis and Simplified Design of Suspension Bridges Having Various Rigidity of Cables, *Journal of Civil Engineering and Management* 16(3): 363–371. <http://dx.doi.org/10.3846/jcem.2010.41>
- Jennings, A. 1987. Deflection Theory Analysis of Different Cable Profiles for Suspension Bridges, *Engineering Structures* 9(2): 84–94. [http://dx.doi.org/10.1016/0141-0296\(87\)90002-2](http://dx.doi.org/10.1016/0141-0296(87)90002-2)
- Juozapaitis, A.; Kliukas, R.; Merkevičius, T.; Lukoševičienė, O.: 2013. Analysis of Modern Suspension Multi-Spans Bridges with Stiff in Bending Cables, *The Baltic Journal of Road and Bridge Engineering* 8(3): 205–211. <http://dx.doi.org/10.3846/bjrbe.2013.26>
- Juozapaitis, A.; Idnurm, S.; Kaklauskas, G.; Idnurm, J.; Gribniak, V. 2010. Non-Linear Analysis of Suspension Bridges with Flexible and Rigid Cables, *Journal of Civil Engineering and Management* 16(1): 149–154. <http://dx.doi.org/10.3846/jcem.2010.14>
- Juozapaitis, A.; Vainiūnas, P.; Kaklauskas, G. 2006. A New Steel Structural System of a Suspension Pedestrian Bridge, *Journal of Constructional Steel Research* 62(12): 1257–1263. <http://dx.doi.org/10.1016/j.jcsr.2006.04.023>
- Juozapaitis, A.; Daniūnas A. 2005. Analiz povedeniya i raschet gibkoy visyachey naklonnoy niti. *Theoretical Foundations of Civil Engineering: Polish-Ukrainian Transactions*. T. 13. Warsaw–Dnepropetrovsk: Wydawnictwo Politechniki Warszawskiej, 299–306 (in Russian).
- Juozapaitis, A.; Norkus, A. 2004. Displacement Analysis of Asymmetrically Loaded Cable, *Journal of Civil Engineering and Management* 10(4): 277–284. <http://dx.doi.org/10.1080/13923730.2004.9636320>
- Katchurin, V.; Bragin, A.; Erunov, B. 1971. *Proektirovanie visyachikh i vantovykh mostov*. Moskva: Transport. 280 s. (in Russian).
- Kim, H. K.; Lee, M. J.; Chang, S. P. 2002. Non-linear Shape-Finding Analysis of a Self-Anchored Suspension Bridge, *Engineering Structures* 24(12): 1547–1559. [http://dx.doi.org/10.1016/S0141-0296\(02\)00097-4](http://dx.doi.org/10.1016/S0141-0296(02)00097-4)
- Kim, S. E.; Thai, H. T. 2010. Nonlinear Inelastic Dynamic Analysis of Suspension Bridges, *Engineering Structures* 32(12): 3845–3856. <http://dx.doi.org/10.1016/j.engstruct.2010.08.027>
- Kulbach, V. 2007. *Cable Structures. Design and Analysis*. Tallin: Estonian Academy Publisher. 224 p.
- Lewis, W. J. 2012. A Mathematical Model for Assessment of Material Requirements for Cable Supported Bridges: Implications for Conceptual Design, *Engineering Structures* 42: 266–277. <http://dx.doi.org/10.1016/j.engstruct.2012.04.018>
- Moskalev, N. S., Popova, R. A. 2003. *Stal'nye konstruktsii legkikh zdaniy*. Moskva: ACB. 216 p. (in Russian). ISBN 5930932026.
- Nakamura, Sh.; Suzumura, K. 2009. Hydrogen Embrittlement and Corrosion Fatigue of Corroded Bridge Wires, *Journal of Constructional Steel Research* 65(2): 269–277. <http://dx.doi.org/10.1016/j.jcsr.2008.03.022>
- Nevaril, A.; Kyttyr, J. 2001. FEM Analysis of Bridge-Type Cable System, in *Proc. of IABSE Conference Cable Supported Bridges – Challenging Technical Limits*, 12–14 June 2001, Seoul, Korea. IABSE Reports 84: 154–155. <http://dx.doi.org/10.2749/222137801796349989>
- Prato, C. A.; Ceballos, M. A. 2003. Dynamic Bending Stresses Near the Ends of Parallel Bundle Stay Cables, *Structural Engineering International* 13(1): 42–46. <http://dx.doi.org/10.2749/101686603777965008>
- Ryall, M. J.; Parke, G. A. R.; Harding, J. E. 2000. *Manual of Bridges Engineering*. London: Tomas Telford Ltd. 1012 p. ISBN 0727727745.
- Strasky, J. 2005. *Stress-Ribbon and Supported Cable Pedestrian Bridges*. London: Thomas Telford Ltd. 240 p. ISBN 072773282X.
- Troyano, L. F. 2003. *Bridge Engineering. A Global Perspective*. London: Tomas Telford Ltd. 775 p. ISBN 0727732153.
- Wang, H.; Li, A.; Li, J. 2002. Progressive Finite Element Model Calibration of a Long-Span Suspension Bridge Based on Ambient Vibration and Static Measurements, *Engineering Structures* 32(9): 2546–2556. <http://dx.doi.org/10.1016/j.engstruct.2010.04.028>
- Wollman, G. P. 2001. Preliminary Analysis of Suspension Bridges, *Journal of Bridge Engineering* 6(4): 227–233. [http://dx.doi.org/10.1061/\(ASCE\)1084-0702\(2001\)6:4\(227\)](http://dx.doi.org/10.1061/(ASCE)1084-0702(2001)6:4(227))
- Wyatt, T. A. 2004. Effect of Localised Loading on Suspension Bridges, in *Proc. of the ICE – Bridge Engineering* 157(2): 55–63. <http://dx.doi.org/10.1680/bren.2004.157.2.55>
- Xu, J.; Chen, W. 2013. Behavior of Wires in Parallel Wire Stayed Cable under General Corrosion Effects, *Journal of Constructional Steel Research* 85: 40–47. <http://dx.doi.org/10.1016/j.jcsr.2013.02.010>
- Yanaka, Y.; Kitagawa, M. 2002. Maintenance of Steel Bridges on Honshu-Shikoku Crossing, *Journal of Constructional Steel Research* 58(1): 131–150. [http://dx.doi.org/10.1016/S0143-974X\(01\)00031-1](http://dx.doi.org/10.1016/S0143-974X(01)00031-1)
- Zhang, J.; Liu, A.; Ma, Z. J. 2013. Behavior of Self-Anchored Suspension Bridges in the Structural System Transformation, *Journal of Bridge Engineering* 18(8): 712–721. [http://dx.doi.org/10.1061/\(ASCE\)BE.1943-5592.0000422](http://dx.doi.org/10.1061/(ASCE)BE.1943-5592.0000422)